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Multivariate Mixed EWMA-CUSUM Control Chart for Monitoring the Process Variance-Covariance Matrix

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ABSTRACT The dispersion control charts monitor the variability of a process that may increase or decrease. An increase in dispersion parameter implies deterioration in the process for an assignable cause, while a decrease in dispersion indicates an improvement in the process. Multivariate variability control charts are used to monitor the shifts in the process variance-covariance matrix. Although multivariate EWMA and CUSUM dispersion control charts are designed to detect the small amount of change in the covariance matrix but to gain more efficiency, we have developed a Mixed Multivariate EWMA-CUSUM (MMECD) chart. The proposed MMECD chart is compared with its existing counterparts by using some important performance run length-based properties such as ARL, SDRL, EQL, SEQL, and different quantile of run length distribution. A real application related to carbon fiber tubing process is presented for practical considerations.

INDEX TERMS Control charts, dispersion parameter, mixed EWMA-CUSUM, memory type, multivariate normality.

I. INTRODUCTION

Control charts are widely used to detect changes in a process location and/or dispersion parameter. These charts are categorized as memory and memoryless charts. Shewhart [1] initiated the idea of a control chart named by Shewhart chart, which is a memoryless control chart; it identifies large shifts in a process and uses only the current information. Memory type control charts are efficient in identifying small changes in the process parameter(s). The most common examples include Cumulative sum (CUSUM) control chart proposed by Page [2] and Exponentially Weighted Moving Average (EWMA) control chart by Roberts [3]. The afore-mentioned charts are univariate charts that monitor a single quality characteristic of interest.

Sometimes, we are interested in the monitoring of more than one correlated quality characteristics like the hardness and tensile strength of steel; thus multivariate control charts are employed. Hotelling [4] introduced a chart that monitors two or more correlated quality characteristics and named it as Chi-squared control chart. Shewhart control chart (location) in the univariate set-up is an analog of Chi-squared control chart (mean vector). Pignatiello Jr and Runger [5] and Crosier [6] proposed memory type multivariate control charts. They offered Multivariate CUSUM (MCUSUM) control charts that monitor the mean vector. Lowry *et al.* [7] developed a Multivariate EWMA (MEWMA) control chart; this chart follows a direct analog of univariate EWMA. Multivariate memory-type control charts are efficient to identify small changes in the process mean vector.

Alt [8] proposed a multivariate control chart that monitored the variance-covariance matrix and named it as generalized variance chart. This chart is not effective to detect small shifts in the process variance-covariance matrix. Djauhari *et al.* [9] introduced vector variance control chart, which can be employed when the variance-covariance matrix is singular. This chart monitors both rational subgroups and individual observations. It was also combined with the generalized variance chart to produce an effective detecting ability of the variance-covariance matrix chart. Memar and Niaki [10] proposed multivariate charts used to monitor the variance-covariance matrix with individual observations. Healy [11] developed two charts that monitor process mean vector and process variance-covariance

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matrix by using MCUSUM statistics. Also, Chen *et al.* [12] developed a MEWMA (Max-MEWMA) chart that monitors shift in both process parameters such as location and dispersion simultaneously. Recently, Adegoke *et al.* [13] proposed a multivariate version of Homogeneously Exponentially Weighted Moving Average (HEWMA) control chart for the monitoring of process mean vector.

Abbas *et al.* [14], [15] combined the structure of EWMA and CUSUM charts to gain sensitive scheme for the monitoring of process parameter(s). Ajadi *et al.* [16] extended this idea by raising the sensitivity of mixed EWMA-CUSUM (MEC) chart in the univariate set-up. Later, Ajadi and Riaz [17] introduced a multivariate MEC chart for the monitoring the process mean vector. Following the same inspiration, we intend to design, in this article, a multivariate MEC control chart for the monitoring of process variance-covariance matrix. The study proposal will serve the purpose for different kind of processes such as carbon fiber tubing process, material flow controlling process and bayer process.

The rest of this study is organized as: Section II presents the information of the existing multivariate control charts for monitoring the process variance-covariance matrix, along with the newly proposed control chart. Section III offers the performance evaluations and comparison of the proposed chart and its counterparts. Section IV provides a real application to validate the superiority of the proposed scheme to its counterparts. Finally, Section V gives the summary, conclusions and recommendation of this study.

II. CONTROL CHARTS FOR THE PROCESS VARIANCE-COVARIANCE MATRIX

This section discusses some useful control charts used to monitor process variance-covariance matrix, such as generalized variance chart, multivariate EWMA and CUSUM control charts for monitoring the process variance-covariance matrix. The design structures of these charts will be given, and it will be discussed how the process is declared in-control (IC) or out-of-control (OOC).

A. PRELIMINARIES

Let X be a p dimensional vector $(X_{p\times 1})$ following a multivariate normal distribution with mean vector μ and variance-covariance matrix Σ. Symbolically, we may write it as: $X \sim N_p(\mu, \Sigma)$, where μ is p dimensional mean vector $\left(\mu_{p\times 1}\right)$ and Σ is p dimensional variance-covariance matrix $(\Sigma_{p\times p})$. The mean and variance-covariance matrix are defined as follows:

$$\mu_i = \begin{bmatrix} \mu_1 & \mu_2 & \cdots & \mu_p \end{bmatrix}'$$

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & & \rho\sigma_1\sigma_{p-1} & \rho\sigma_1\sigma_p \\ \rho\sigma_2\sigma_1 & \sigma_2^2 & \cdots & \rho\sigma_2\sigma_{p-1} & \rho\sigma_2\sigma_p \\ \vdots & & \ddots & \vdots & \\ \rho\sigma_p\sigma_1 & \rho\sigma_p\sigma_2 & \cdots & \rho\sigma_p\sigma_{p-1} & \sigma_p^2 \end{pmatrix}_{p\times p}$$

For our study purposes, we will use μ_0 and Σ_0 as the known mean vector and variance-covariance matrix, respectively. Let X_i be the i^{th} sample matrix consisting of the x_{ijk} as the i^{th} $(i = 1, 2, \ldots, n)$ observation of the j^{th} $(j = 1, 2, \cdots, p)$ quality characteristic on the k^{th} $(k = 1, 2, \ldots, m)$ sample. Let $\bar{X}_i$ and S_i are p dimensional i^{th} sample mean vector and sample variance-covariance matrix ($\bar{X}_{p\times 1}$ and $S_{p\times p}$) respectively, defined as:

$$\overline{X}_i = \begin{bmatrix} \overline{X}_1 & \overline{X}_2 & \cdots & \overline{X}_p \end{bmatrix}',$$

and

$$S_i = \begin{bmatrix} S_1^2 & S_{12} & \cdots & S_{1p} \\ S_{21} & S_2^2 & \cdots & S_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ S_{p1} & S_{p2} & \cdots & S_p^2 \end{bmatrix}$$

Based on these terminologies, we outline brief details of some commonly used multivariate control charts for dispersion and propose a new control chart in the following subsections.

B. GENERALIZED VARIANCE CONTROL CHART

Generalized variance (GenVar) chart, proposed by Alt [8], was developed for monitoring the determinant of the sample variance-covariance matrix $|S|$. The decision limits including upper control limit (UCL), center line (CL) and lower control limit (LCL) for this chart are given as

$$UCL = |\Sigma_0|\left(b_1 + L_1\sqrt{b_2}\right), \quad (1)$$

$$CL = |\Sigma_0|\, b_1, \quad (2)$$

$$LCL = max\left\{\begin{matrix} |\Sigma_0|\left(b_1 - L_1\sqrt{b_2}\right) \\ 0 \end{matrix}\right\}. \quad (3)$$

where

$$b_1 = \frac{1}{(n-1)^p}\prod_{i=1}^{p}(n-i),$$

$$b_2 = \frac{1}{(n-1)^{2p}}\prod_{i=1}^{p}(n-i)\left[\prod_{i=1}^{p}(n-i+2) - \prod_{i=1}^{p}(\text{n-i})\right],$$

and L_1 is the width of the control limit. In most of the time when the actual value of Σ_0 is unknown then, it is estimated by $|\Sigma_0| = |\hat{\Sigma}|\big/ b_1$ where $\hat{\Sigma}$ is the Phase I estimate for the variance-covariance matrix. The plotting statistic is taken as $|S_i|$ which is compared against the above-mentioned control limits. If $|S_i|$ falls outside UCL or LCL, then the process is declared as OOC, otherwise IC.

C. MULTIVARIATE EWMA CONTROL CHART

Chen *et al.* [12] proposed a multivariate chart based on EWMA statistic (MEWMAD) for the simultaneous monitoring of process mean vector and variance-covariance matrix. In this study, we are only interested in the process dispersion, and the variability statistic of the MEWMAD control chart is given as:

$$W_i = \sum\nolimits_{j=1}^{n}\left(X_{ij} - \bar{X}_i\right)'\Sigma_0^{-1}\left(X_{ij} - \bar{X}_i\right), \quad (4)$$

$$Y_i = (1-\lambda) Y_{i-1} + \lambda \Phi^{-1} \left[H(W_i); p(n-1)\right], \quad (5)$$

where H(.;$p(n-1)$) represents the Chi-squared distribution with $p(n-1)$ degrees of freedom, λ is the smoothing constant which always lies between zero and one, and $\Phi^{-1}(.)$ is the normal inverse cumulative distribution function. The other notations are defined in Section II(A).

Based on Y_i, we may define a new statistics V_i as (that will be used as plotting statistic):

$$V_i = \left(\sqrt{\frac{2-\lambda}{\lambda\left[1-(1-\lambda)^{2i}\right]}}\right) Y_i, \quad (6)$$

The plotting statistic $|V_i|$ is compared against the control limit (h_1). If $|V_i|$ exceeds h_1, the process is declared OOC, otherwise IC.

D. MULTIVARIATE CUSUM CONTROL CHART

The MCUSUM control chart for monitoring the process variability is named by MCUSUMD control chart, proposed by Healy [11] and defined as:

$$S_i = max\left(0, \sum_{j=1}^{n} \left(X_{ij}-\bar{X}_i\right)' \Sigma_0^{-1} \left(X_{ij}-\bar{X}_i\right) - k_1 + S_{i-1}\right),$$

where $k_1 = pn\left(\delta/\delta - 1\right)\log\delta$ and δ refers to the amount of shift (see section III(A)).

According to Cheng and Thaga [18], the statistic

$$\sum_{j=1}^{n} \left(X_{ij}-\bar{X}_i\right)' \Sigma_0^{-1} \left(X_{ij}-\bar{X}_i\right),$$

was standardized by using the following expression

$$N_i = \Phi^{-1}\left(H\left[\sum_{j=1}^{n}\left(X_{ij}-\bar{X}_i\right)' \Sigma_0^{-1}\left(X_{ij}-\bar{X}_i\right); p(n-1)\right]\right).$$

Therefore, the plotting statistic is defined as

$$S_i = max\left(0, N_i - k_1 + S_{i-1}\right).$$

The process is stated as the IC state as long as the S_i is below the control limit h_2, otherwise, it is considered as OOC. It is to be mentioned that for our study purposes, we have fixed the value of $k_1 = 0.5$, in order to make the chart more sensitive for the smaller shift.

E. THE PROPOSED MULTIVARIATE MIXED EWMA-CUSUM CONTROL CHART

In this section, we propose a new multivariate dispersion chart by integrating the effects of MEWMAD and MCUSUMD control charts into a single structure. This idea was initially developed in the univariate setup by Abbas *et al.* [14],[15]. Later, Ajadi *et al.* [16] and Ajadi and Riaz [17] made further developments on it. This study follows their inspirations and develops a new multivariate dispersion chart, namely Multivariate Mixed EWMA-CUSUM (MMECD) control chart. The methodological details of the proposed MMECD chart are as follows:

Firstly, we compute the W_i statistic given in (4) and convert it into Chi-squared value with $p(n-1)$ degrees of freedom. After this, we applied normal inverse cumulative distribution function to obtained the standardized statistic such as:

$$M_i = \Phi^{-1}\left[H(W_i); p(n-1)\right]. \quad (7)$$

Next, M_i is transformed into the MEWMA statistic as given below:

$$Z_i = (1-\lambda) Z_{i-1} + \lambda M_i, \quad (8)$$

$$U_i = \left(\sqrt{\frac{2-\lambda}{\lambda\left[1-(1-\lambda)^{2i}\right]}}\right) Z_i. \quad (9)$$

We can integrate U_i into MCUSUM dispersion statistics as:

$$MMECD_i = max\left(0, U_i - k_2 + MMECD_{i-1}\right),$$
$$k_2 = k_2^* \sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}, \quad (10)$$

where k_2^* is chosen equal to half of the shift in terms of standard deviation. The statistic $MMECD_i$ is compared with the control limit (h_3) and the process is declared OOC when MMECD_i is greater than h_3.

III. PERFORMANCE EVALUATION AND COMPARISONS

This section will serve the following purposes: discuss the performance measures used to evaluate the performance of the charts under investigation; describe the construction of the control limits of various charts of this study; outline the algorithm of run length, and design of control charting constants; provide a detail comparison between the proposed multivariate variance-covariance matrix chart (MMECD) and its various counterparts.

A. PERFORMANCE MEASURES

In this study, we use various run length (RL) properties to assess the performance of the control charts under discussion, by considering different amounts of shifts (δ) in a process. Following Chen *et al.* [12], the shift in the variance-covariance matrix is defined as follows:

$$\Sigma_1 = \delta \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \cdots & \rho\sigma_1\sigma_{p-1} & \rho\sigma_1\sigma_p \\ \rho\sigma_2\sigma_1 & \sigma_2^2 & \cdots & \rho\sigma_2\sigma_{p-1} & \rho\sigma_2\sigma_p \\ \vdots & & \ddots & \vdots & \\ \rho\sigma_p\sigma_1 & \rho\sigma_p\sigma_2 & \cdots & \rho\sigma_p\sigma_{p-1} & \sigma_p^2 \end{pmatrix}_{p\times p}$$

where $\delta = 1$ refers to an IC state, otherwise OOC. For the sake of simplicity and a fair comparison with existing charts, we have used $\rho = 0.2$ and the case of equal variances. However, one may expect similar findings for the other choices of ρ and variances. For OOC, we have considered the case of an increase in variability (i.e. $\delta > 1$).

The measures covered in this study include average run length (ARL), standard deviation run length (SDRL), median run length (MDRL), extra quadratic loss (EQL), and sequential extra quadratic loss (SEQL), and some useful percentiles/quantiles (Q_is) of the run length distribution. These measures are briefly described as:

- A series of points in an IC state until an OOC signal is received referred to a run. The number of points in a run is termed as run length.
- ARL represents the average number of sample points awaited until the first OOC signal is received. It is classified into two types, ARL_0 (i.e. IC state) and ARL_1(i.e. OOC state) [19].
- SDRL is another useful measure used to assess the spread of the run length distribution.
- MDRL refers to the midpoint of run length distribution (i.e. the point that covers 50% of the area).
- EQL is defined as the weighted ARL with respect to the range of shift ($\delta_{\min}$ to $\delta_{\max}$) by considering the square of shift (δ^2) as weight. Mathematically, it is defined as:

$$EQL = \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \delta^2 ARL(\delta) d\delta$$

A discrete form of the *EQL* measure may be defined as:

$$EQL \cong \frac{1}{q} \sum\nolimits_{j=1}^{q} \delta_j^2 ARL\left(\delta_j\right),$$

where q refers to the number of shifts covered in the performance evaluation.

- SEQL is the cumulative measure that refers to the *EQL* up to a certain shift (say δ_i), mathematically defined as:

$$SEQL_i = \frac{1}{\delta_i - \delta_{min}} \int_{\delta_{min}}^{\delta_i} \delta^2 ARL(\delta) d\delta, \ \forall i = 2, 3, \ldots\ldots, \delta_{max}.$$

A discrete form of the *SEQL* measure may also be defined as:

$$SEQL_i \cong \frac{1}{q_i} \sum\nolimits_{j=1}^{q_i} \delta_j^2 ARL\left(\delta_j\right)$$

For more details on these performance measures, one may be seen in [20]–[24] and the references therein.

B. ALGORITHM FOR CHOOSING THE CONTROL LIMITS OF MMECD CHART

Step 1 Algorithm for Run Length:

(i) Generate a sample from the multivariate normal distribution and calculate the sample statistic (W_i) and its inverse normal using (4) and (7) respectively.

(ii) Calculate Z_i and substitute its value in U_i using (8) and (9) respectively; then substitute U_i in (10).

(iii) Evaluate statistic $MMECD_i$ as given in (10) and plot it against the control limit h_3. If $MMECD_i$ is plotted beyond the control limit, then the process is declared OOC and the corresponding sample number (which is one in this case) is the run length. On the other hand, we proceed to (iv) if $MMECD_i$ is plotted inside the control limit h_3.

(iv) We generate another sample from the multivariate normal distribution. Compute the plotting statistic and compare it with the control limit, as we did in (ii) and (iii) above. If the process is declared OOC, then stop at

this stage and report 2 as run length, otherwise continue this method for several iterations.

Step 2 Iterative Procedure:

Repeat step 1 iteratively to get a large number of RL values (say 10,000 run lengths), and calculate the average of these RL values, producing ARL. If the process in the IC state, then the resulting ARL will be ARL_0 and for OOC state the resulting ARL will be ARL_1.

C. DESIGN STRUCTURE OF CHARTING CONSTANT AND LIMITS

The design structures of the proposed chart and its counterparts depend on the sample size (n) and the number of correlated quality characteristics to be monitored simultaneously (p). We have evaluated the performance of the charts as a function of n and p. For our study purposes, we have evaluated the results for $n = 5$ and $p = 2, 3, 4$ for the proposed MMECD control chart. For the comparison purpose with the existing counterparts, we have covered the case of $p = 2$. Some selective results for control limits are given below for different charts of this study.

- In generalized variance (GenVar) control chart, the width of the control limits depends on charting constant L_1, which is $L_1 = 5.394$ and $L_1 = 6.23$ when $p = 2$ and $p = 3$, respectively for the prefixed $ARL_0 = 250$.
- The UCL (h_1) of multivariate EWMA dispersion (MEWMAD) chart depends on the smoothing constant λ. For the fixed $ARL_0 = 250$, $h_1 = 2.57, 2.73, 2.794$ and 2.856 with respect to $\lambda = 0.10, 0.20, 0.30$ and 0.50 when $p = 2$.
- For the MCUSUMD chart, the control limit ($h_2 = 3.725$) with prefixed $ARL_0 = 250$ is used when the reference parameter, k_1, is 0.5 and $S_0 = 0$.
- The control limit (h_3) of the proposed MMECD chart relies on four designing parameters n,p, λ and k_2^*. First, we fixed n = 5, p = 2,$k_2^* = 0.5$ and $ARL_0 = 250$, and the resulting values of control limit (h_3) are $34.7, 24.2, 18$ and 10.75 with respect to $\lambda = 0.10, 0.20, 0.30$ and 0.50. Similarly, for p = 3, the values of control limit (h_3) are 34.9 and 18 with respect to $\lambda = 0.10$ and 0.30, while for p = 4, the values of control limit (h_3) are 34.9 and 18 with respect to $\lambda = 0.10$ and 0.30.

D. COMPARATIVE ANALYSIS

The run length profile (i.e., ARL, SDRL, and percentiles values) with respect to various shifts in the process variance-covariance matrix ($1 \leq \delta \leq 2$) for all charts under considerations are provided in Tables 1-5. The results of the generalized variance control chart are reported in Table 1. The run length profile of the MEWMAD control chart with respect to different choices of smoothing parameters ($\lambda = 0.10, 0.20, 0.30$ and 0.50) is given in Table 2. The findings of MCUSUMD control chart are provided in Table 3. The run length profile of the proposed MMECD control chart with respect to different choices of λ and quality characteristics ($p = 2$,3 and 4) is given in Tables 4 and 5.

TABLE 1. Run length profile of the generalized variance chart at fixed $p = 2$, when $ARL_0 = 250$.

δ	ARL	SDRL	$Q_{0.05}$	$Q_{0.25}$	$Q_{0.50}$=MDRL	$Q_{0.75}$	$Q_{0.95}$
1.00	249.4119	248.6994	13	71	173	347	743
1.05	172.1502	171.1894	9	50	120	238	511
1.10	124.6748	123.5462	7	36	87	172	371
1.15	92.47498	91.98941	5	27	64	128	276
1.20	70.28158	70.14763	4	21	49	97	211
1.30	43.61294	43.74393	3	13	30	60	130
1.40	29.53602	28.93799	2	9	21	41	87
1.50	21.00708	20.4521	2	7	15	29	62
1.60	15.7665	15.25515	1	5	11	22	46
1.70	12.4051	11.90472	1	4	9	17	36
1.80	9.94478	9.47369	1	3	7	14	29
1.90	8.16916	7.651528	1	3	6	11	24
2.00	6.91642	6.39336	1	2	5	9	20

The findings of the GenVar control chart revealed that 20% increase in the dispersion parameter might cause 71.46% decrease in the ARL_1(cf. Table 1). The run length profile of MEWMAD control chart with respect to different choices of smoothing parameters ($\lambda = 0.10, 0.20, 0.30$ and 0.50) depicted that 86.77% decrease reported in the ARL_1 of MEWMAD chart due to 20% shift in the dispersion parameter (*i.e.* $\delta = 1.2$)(cf. Table 2). The findings of the MCUSUMD control chart exhibited that there is 86.56% decrease in the ARL_1 with the 20% increase of dispersion parameter (*i.e.*, $\delta = 1.2$)(cf. Table 2). The run length profile of the MMECD control chart with respect to different choices of λ and quality characteristics ($p = 2$,3 and 4) is revealed that 88.34% decrease is reported in ARL_1 of MMECD chart due to 20% shift in the dispersion parameter.

In addition, the run length curves, along with the box plots, are presented in Figure 1. When the process is in a stable state or IC state (*i.e.* $\delta = 1$), all charts have similar performance. For a shifted environment in dispersion parameter (e.g. $\delta = 1.5$), then the proposed MMECD chart outperforms the other competitive charts under study, as may be seen in Figure 1.

Further, other performance measures, such as EQL and SEQL are reported in Table 6. The SEQL is employed to check the performance of a chart over a specific range of shifts in the dispersion parameter. It is to be mentioned that EQL is based on all the shifts in the process dispersion (that is the last column of Table 6). The findings depict that MMECD chart having smoothing parameter ($\lambda = 0.5$) offers the minimum SEQL and EQL as compared to all the competitive charts under consideration.

Moreover, the prime findings of this study are summarized as follows:

- The proposed MMECD scheme is better than the GenVar chart for small and moderate shifts in the process dispersion for all values of λ.
- The proposed MMECD chart outperforms MCUSUMD chart for the monitoring of process variance-covariance matrix when the shifts in the process are very small. It holds true for all values of λ.
- The proposed structure is better than the MEWMAD chart at small shifts when $\lambda < 0.5$. When $\lambda \geq 0.5$ (for the sake of brevity, we did not include results in Tables), the proposed MMECD is better than MEWMAD chart for small and moderate shifts of the process dispersion.
- The sensitivity of the proposed chart to detect small and moderate shifts in the variance-covariance matrix increases as p increases.

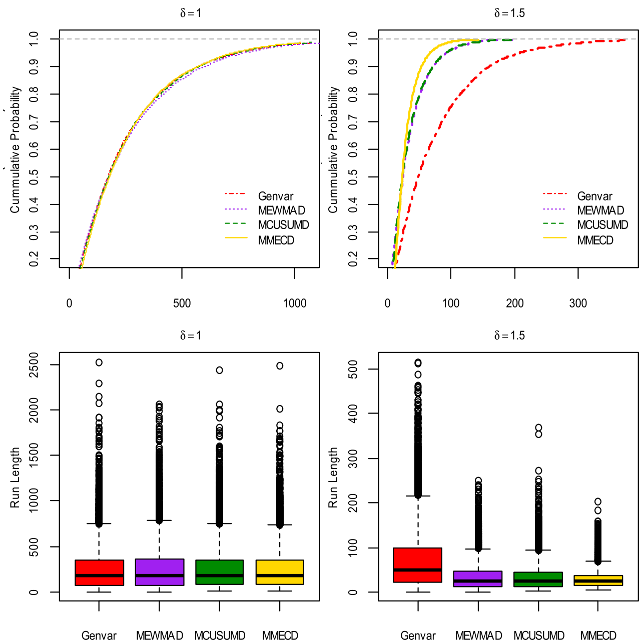

FIGURE 1. RL curves and box plots of the proposed MMECD control chart and its counterparts.

IV. AN APPLICATION

In this section, we provide an application of the proposed chart for the manufacturing process. We outline brief details

TABLE 2. Run length profile of the MEWMAD control chart for the monitoring of dispersion at fixed $p = 2$ and $ARL_0 = 250$.

	δ	ARL	SDRL	$Q_{0.05}$	$Q_{0.25}$	$Q_{0.50}$ =MDRL	$Q_{0.75}$	$Q_{0.95}$
$\lambda = 0.10$ $h_1 = 2.57$	1.00	254.7117	258.7215	9	69	174	358	780
	1.05	167.3683	169.2136	7	46	116	235	505
	1.10	90.8772	88.21587	5	28	65	125	265
	1.15	53.0809	50.24277	4	18	38	73	153
	1.20	33.9435	30.44225	3	12	25	47	94
	1.30	18.63015	15.69073	2	7	15	25	49
	1.40	12.0963	9.569828	2	5	10	16	31
	1.50	8.7257	6.742917	1	4	7	12	22
	1.60	6.815	5.051351	1	3	6	9	17
	1.70	5.4745	4.006489	1	2	5	7	13
	1.80	4.6575	3.330338	1	2	4	6	11
	1.90	4.0377	2.840578	1	2	3	5	10
	2.00	3.57485	2.445158	1	2	3	5	8
$\lambda = 0.20$ $h_1 = 2.73$	1.00	251.9869	257.2832	12	72	173	345	772
	1.05	181.9365	180.6236	9	52	126	252	544
	1.10	108.4391	107.2587	7	32	74	151	323
	1.15	66.3498	63.94976	5	21	47	91	192
	1.20	43.1293	40.58517	3	14	31	60	123
	1.30	22.0334	19.53681	2	8	16	30	61
	1.40	13.94225	11.55171	2	6	11	19	37
	1.50	9.82905	7.770473	1	4	8	13	25
	1.60	7.52515	5.718584	1	3	6	10	19
	1.70	6.04195	4.401211	1	3	5	8	15
	1.80	5.0788	3.586953	1	2	4	7	12
	1.90	4.35645	3.026574	1	2	4	6	10
	2.00	3.8308	2.611669	1	2	3	5	9
$\lambda = 0.30$ $h_1 = 2.794$	1.00	247.6967	250.54	13	71	170	341	749
	1.05	187.387	186.7722	10	55	131	260	554
	1.10	120.1797	120.1936	7	35	83	166	357
	1.15	76.37805	74.66793	5	23	54	106	224
	1.20	50.03515	47.91077	4	16	35	69	145
	1.30	25.92315	23.89356	3	9	19	35	73
	1.40	16.01985	14.03289	2	6	12	22	44
	1.50	10.83165	8.965357	2	5	8	15	29
	1.60	8.17405	6.483738	1	4	6	11	21
	1.70	6.49415	4.966367	1	3	5	9	16
	1.80	5.42985	3.973911	1	3	4	7	13
	1.90	4.59945	3.324961	1	2	4	6	11
	2.00	4.06865	2.835196	1	2	3	5	9
$\lambda = 0.50$ $h_1 = 2.856$	1.00	252.242	248.8667	14	73	176	348	756
	1.05	197.9535	197.3766	10	57	137	276	598
	1.10	138.7828	139.5844	8	40	96	192	415
	1.15	94.06115	92.60537	6	28	66	130	278
	1.20	64.48505	62.97933	5	19	45	89	190
	1.30	34.0636	32.75663	3	11	24	47	99
	1.40	20.76115	19.35889	2	7	15	28	60
	1.50	13.8152	12.48033	2	5	10	19	39
	1.60	9.95335	8.833826	1	4	7	13	27
	1.70	7.8038	6.634622	1	3	6	10	21
	1.80	6.2156	5.123693	1	3	5	8	16
	1.90	5.2223	4.061503	1	2	4	7	13
	2.00	4.4822	3.381872	1	2	4	6	11

TABLE 3. Run length profile for the MCUSUMD control chart at fixed $p = 2$, $ARL_0 = 250$ and $h_1 = 3.725$.

δ	ARL	SDRL	$Q_{0.05}$	$Q_{0.25}$	$Q_{0.50}$ =MDRL	$Q_{0.75}$	$Q_{0.95}$
1	251.7577	246.6743	17	75	176	347	757
1.05	131.1152	127.0055	11	41	92	179	388
1.1	76.42915	72.27781	8	25	54	105	220
1.15	48.31865	44.17811	6	17	35	66	138
1.2	33.79105	29.66911	5	13	25	45	94
1.3	19.22465	15.51715	4	8	15	25	50
1.4	12.85185	9.424412	3	6	10	17	31
1.5	9.56815	6.45521	3	5	8	12	22
1.6	7.7126	4.867729	3	4	6	10	17
1.7	6.4304	3.832831	2	4	5	8	14
1.8	5.4766	3.042304	2	3	5	7	11
1.9	4.86345	2.582099	2	3	4	6	10
2	4.41905	2.258274	2	3	4	5	9

of the process related to carbon fiber tubes, followed by the implementation of the proposed and some existing charts.

A. CARBON FIBER TUBING PROCESS

The carbon fibers making is a partly chemical and partly mechanical process. Carbon fibers are long and thin strands of material (about 0.005-0.010 mm in diameter), composed mostly of carbon atoms. The raw material used to make carbon fiber is called the precursor. Several thousand carbon fibers are twisted together to form a yarn, which may be used by itself or woven into a fabric. The yarn or fabric is combined with epoxy and wound or molded into shape to form various composite materials. Carbon fiber-reinforced composite materials are used to make aircraft and spacecraft parts, racing car bodies, golf club shafts, bicycle frames, fishing rods, automobile springs, sailboat masts, and many other components where lightweight and high strength are needed. The latest development in carbon fiber technology is tiny carbon tubes called nanotubes. These hollow tubes, some as small as 0.001 mm in diameter, have unique mechanical and electrical properties that may be useful in making new high-strength fibers, submicroscopic test tubes, or possibly new semiconductor materials for integrated circuits. (http://www.madehow.com/Volume-4/Carbon-Fiber.html ixzz5oE8ZmqNw).

An image of a carbon fiber tube from a tubing process is shown in Figure 2, where three characteristics (inner diameter, thickness and length of the carbon fiber tubes) is labelled. (https://rcmarketpuss-rcmarket.netdna-ssl.com/images/D/white%20carbon%20fiber%20tube-01.jpg, with some modifications for our study purpose).

B. A QUALITY CONTROL APPLICATION

In this Section, we apply a dataset related to the industrial manufacturing of a carbon fiber tubing process. In an application, it was observed that the quality of carbon fiber tubes might depend on three variables, including inner diameter,

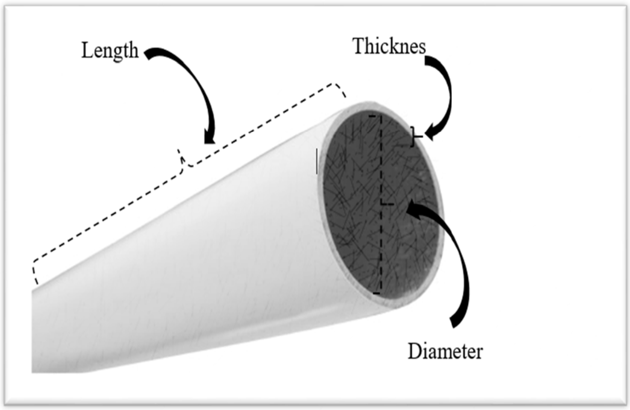

FIGURE 2. An image of a carbon fiber tube from a tubing process.

thickness and length [25], as shown in Figure 2. The dataset is categorized as carbon1 and carbon2; where carbon1 and carbon2 datasets contain 30 and 25 multivariate samples, respectively, of the three correlated quality characteristics, each of size 8. This data can be obtained by installing the MSQC package in R program. The implementation procedure used for this dataset is outlined below.

First, we used carbon1 to get the Phase I estimates for the required parameters and employed carbon2 for Phase II monitoring of the multivariate variability of the process. We noticed that all the data points are in-control in Phase II for each chart. We have considered the GenVar, MEWMAD, MCUSMD, MMECD control charts. Thus, we combined both carbon1 and carbon2 datasets, computed the sample variance-covariance matrix of the combined dataset as provided in (11) to obtain a new Phase I estimate:

$$S \times 100 = \begin{bmatrix} 0.24 & 0.35 & 0.67 \\ 0.35 & 1.44 & 1.15 \\ 0.67 & 1.15 & 6.48 \end{bmatrix}. \quad (11)$$

TABLE 4. Run length profile of the MMECD control chart for $p = 2$ at fixed $ARL_0 = 250$.

	δ	ARL	SDRL	$Q_{0.05}$	$Q_{0.25}$	$Q_{0.50}$=MDRL	$Q_{0.75}$	$Q_{0.95}$
$\lambda = 0.10$, $h_3 = 34.7$	1	252.2496	223.9771	38	94	184	338	701
	1.05	106.0152	79.31398	27	50	83	138	264
	1.1	62.57016	39.43867	22	35	52	79	140
	1.15	44.35546	23.14256	19	28	39	55	90
	1.2	34.73182	15.80737	17	23	31	42	65
	1.3	25.053	9.154358	14	19	23	30	42
	1.4	20.3011	6.333514	12	16	19	24	32
	1.5	17.37136	4.866765	11	14	17	20	27
	1.6	15.36956	3.914625	10	13	15	17	23
	1.7	13.9508	3.310232	10	12	13	16	20
	1.8	12.86418	2.871087	9	11	12	14	18
	1.9	11.96104	2.548013	8	10	12	13	17
	2	11.2401	2.325399	8	10	11	13	15
$\lambda = 0.20$, $h_3 = 24.2$	1	252.6598	228.5118	34	91	183	341	708
	1.05	106.6179	84.18543	24	47	82	140	276
	1.1	62.0194	42.02007	19	33	51	79	145
	1.15	42.76746	24.92882	16	25	36	54	92
	1.2	32.5648	16.80904	14	21	29	40	66
	1.3	22.76062	9.612103	11	16	21	27	41
	1.4	17.79754	6.445642	10	13	17	21	30
	1.5	15.01306	4.862781	9	12	14	18	24
	1.6	13.06718	3.846105	8	10	12	15	20
	1.7	11.75198	3.194137	8	9	11	13	18
	1.8	10.73168	2.753012	7	9	10	12	16
	1.9	9.94944	2.427262	7	8	10	11	14
	2	9.28576	2.17753	6	8	9	10	13
$\lambda = 0.30$, $h_3 = 18$	1	257.2159	236.0945	32	89	185	349	730
	1.05	109.6768	91.65577	21	46	82	145	290
	1.1	61.64732	44.14792	16	30	49	80	149
	1.15	41.86888	26.45432	14	23	35	53	94
	1.2	31.2763	17.75527	12	19	27	39	66
	1.3	21.15212	9.957405	10	14	19	26	40
	1.4	16.26344	6.633307	8	12	15	19	29
	1.5	13.4623	4.89305	7	10	13	16	23
	1.6	11.63008	3.8852	7	9	11	14	19
	1.7	10.33628	3.20577	6	8	10	12	16
	1.8	9.37562	2.716652	6	7	9	11	14
	1.9	8.64086	2.388328	6	7	8	10	13
	2	8.0484	2.140866	5	7	8	9	12
$\lambda = 0.50$, $h_3 = 10.75$	1	250.9925	236.9971	25	82	178	344	726
	1.05	111.0526	97.84321	17	42	81	149	307
	1.1	62.29336	50.27892	13	27	47	82	162
	1.15	40.66604	29.69112	10	20	32	53	99
	1.2	29.70482	19.8487	9	16	24	38	69
	1.3	19.00814	10.72226	7	11	16	24	40
	1.4	14.1173	6.913151	6	9	13	17	27
	1.5	11.37454	5.046988	5	8	10	14	21
	1.6	9.69288	3.996573	5	7	9	12	17
	1.7	8.45538	3.206252	5	6	8	10	15
	1.8	7.55676	2.713228	4	6	7	9	13
	1.9	6.89878	2.362551	4	5	6	8	11
	2	6.38738	2.077374	4	5	6	7	10

TABLE 5. Run length profile of the MMECD control chart for p =3 and 4 at fixed $ARL_0 = 250$.

		δ	ARL	SDRL	$Q_{0.05}$	$Q_{0.25}$	$Q_{0.50}$=MDRL	$Q_{0.75}$	$Q_{0.95}$
p=3	$\lambda = 0.10$, $h_3 = 34.9$	1.0	258.84	229.56	39	95	187	352	719.5
		1.1	52.319	29.938	20	31	44	65	112
		1.2	28.881	11.52	15	21	26	34	51
		1.3	21.287	6.7637	13	16	20	25	34
		1.4	17.274	4.641	11	14	17	20	26
		1.5	14.842	3.5418	10	12	14	17	21
		1.6	13.171	2.9081	9	11	13	15	19
		1.7	12.015	2.4872	9	10	12	13	17
		1.8	11.045	2.1418	8	10	11	12	15
		1.9	10.302	1.9465	8	9	10	11	14
		2.0	9.6805	1.7278	7	8	9	11	13
	$\lambda = 0.30$, $h_3 = 18$	1.0	258.2	238.39	32	89	186	352	736.5
		1.1	50.538	34.635	15	27	41	64	118
		1.2	25.093	12.778	11	16	22	31	50
		1.3	17.207	7.1151	9	12	16	21	31
		1.4	13.314	4.7154	7	10	12	16	22
		1.5	11.086	3.4667	7	9	10	13	18
		1.6	9.629	2.767	6	8	9	11	15
		1.7	8.6506	2.3174	6	7	8	10	13
		1.8	7.8479	1.9631	5	6	8	9	11
		1.9	7.2626	1.7587	5	6	7	8	10.5
		2.0	6.7712	1.537	5	6	7	8	10
p=4	$\lambda = 0.10$, $h_3 = 34.9$	1.0	258.19	227.48	38	96	188	352	716.5
		1.1	45.874	24.194	19	29	40	57	93
		1.2	25.483	9.1289	14	19	24	30	43
		1.3	18.903	5.3428	12	15	18	22	29
		1.4	15.396	3.7136	10	13	15	17	22
		1.5	13.283	2.8542	9	11	13	15	19
		1.6	11.83	2.3411	9	10	12	13	16
		1.7	10.729	1.9537	8	9	10	12	14
		1.8	9.924	1.7546	7	9	10	11	13
		1.9	9.2834	1.5701	7	8	9	10	12
		2.0	8.7079	1.3947	7	8	9	10	11
	$\lambda = 0.30$, $h_3 = 18$	1.0	256	235.42	30	89	185	343	727
		1.1	43.288	27.129	14	24	37	55	96
		1.2	21.472	9.759	10	14	19	26	40
		1.3	14.789	5.4768	8	11	14	17	25
		1.4	11.522	3.6013	7	9	11	13	18
		1.5	9.7011	2.7481	6	8	9	11	15
		1.6	8.4693	2.1833	6	7	8	10	12
		1.7	7.6179	1.8085	5	6	7	9	11
		1.8	6.933	1.5326	5	6	7	8	10
		1.9	6.4386	1.3826	5	5	6	7	9
		2.0	6.0388	1.2425	4	5	6	7	8

TABLE 6. SEQL of the proposed MMECD control chart and its Counterparts for $p = 2$ at fixed $ARL_0 = 250$.

Chart		δ								
		1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
GenVar		126.8534	107.6626	93.88305	83.56643	75.61256	69.371	64.36502	60.22711	56.73811
MCUSUMD		69.99166	55.18605	46.36523	40.61775	36.62666	33.70897	31.50449	29.79776	28.44862
MEWMAD	λ=0.10	78.48672	59.15192	48.48826	41.74614	37.0965	33.68628	31.08324	29.04122	27.40488
	λ=0.20	98.22072	74.11974	60.30016	51.43109	45.31824	40.85527	37.45581	34.7911	32.6564
	λ=0.30	109.0312	83.71507	68.28204	58.15395	51.102	45.92242	41.96061	38.84324	36.33373
	λ=0.50	130.562	103.2156	85.22776	72.86712	63.99549	57.34354	52.1991	48.11814	44.81122
MMECD	λ=0.10	62.89061	54.51338	50.03424	47.38963	45.77483	44.8049	44.27255	44.05596	44.07559
	λ=0.20	61.08067	51.85419	46.78744	43.68772	41.69428	40.39187	39.55471	39.04984	38.79308
	λ=0.30	59.50639	49.7766	44.39395	41.04799	38.83954	37.3411	36.31642	35.6293	35.19516
	λ=0.50	58.571	47.76603	41.7325	37.95414	35.40543	33.60878	32.31412	31.37454	30.69677

Based on this estimate, we generated 50 tri-variate samples (referring to the three correlated quality characteristics), each of size 8. The first 20 samples are kept in-control, while we shifted the sample variance-covariance matrix of the last 30 samples by rescaling by 1.2. The resulting Phase II dataset is presented in Table 7.

For this dataset, we constructed all the control charts under investigation in this study. For all the charts, control limits coefficients are set such that $ARL_0 = 250$. The graphical displays of the proposed chart and its counterparts are shown in Figures 3-6. The result of our findings is outlined as follows:

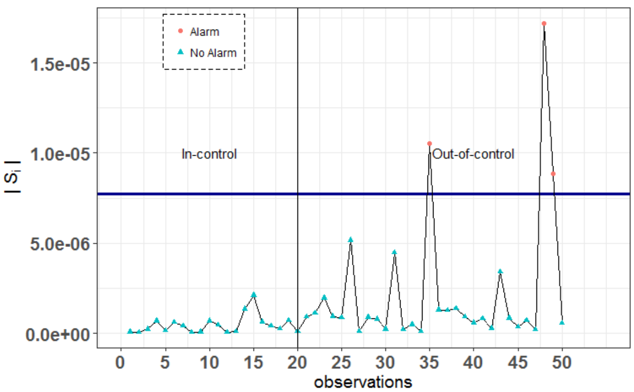

FIGURE 3. The GenVar control chart when $L_1 = 5.394$.

- The GenVar chart detected 3 out-of-control signals, at sample numbers 35, 48 and 49 (cf. Figure 3).
- The MEWMAD chart for the process dispersion alarmed 4 out-of-control signals, at sample numbers 26,48, 49 and 50 (cf. Figure 4).
- The MCUSUMD chart captured 15 out-of-control points, at sample numbers 31 and 37-50 (cf. Figure 5).
- The proposed MMECD chart detected 27 out-of-control signals at sample numbers 23-50 (cf. Figure 6).

From the analysis above, it is evident that the proposed chart is very effective in detecting small shifts (such as $\delta = 1.2$) in process variance-covariance matrix.

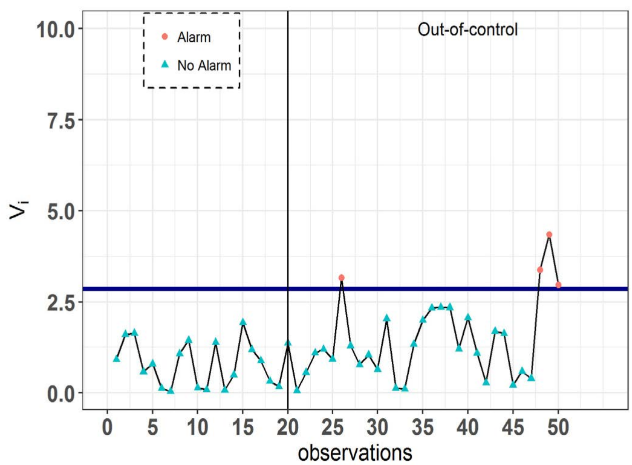

FIGURE 4. The MEWMAD chart when $h_1 = 2.86$ and $\lambda = 0.5$.

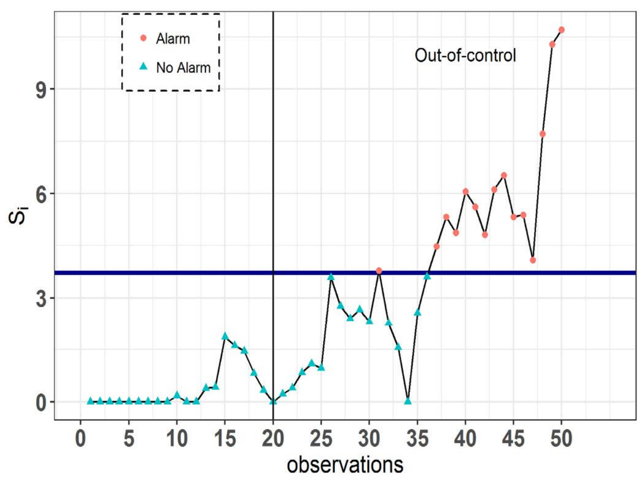

FIGURE 5. The MCUSUMD chart when $h_2 = 3.725$ and $k_1 = 0.5$.

The potential reasons for such OOC might include a special cause(s) in one of the important steps in the carbon fiber tubing process such as spinning, stabilizing, carbonizing etc. and these un-natural variations may be caused by the variables

TABLE 7. Specific carbon fiber tubing phase II dataset.

	SUBGROUP											
	1			2			3			4		
sample	inner	thickness	length	inner	thickness	length	inner	thickness	length	inner	thickness	length
1	1.06	1.16	50.02	1.04	1.04	50.14	0.90	0.75	49.72	0.95	1.00	49.89
2	0.95	0.94	50.16	1.10	1.24	50.49	0.96	0.92	49.94	1.06	1.16	50.25
3	0.97	1.01	50.09	0.95	0.86	49.96	1.05	0.93	50.23	1.01	1.10	49.85
4	0.97	0.90	49.62	1.06	1.25	50.40	1.04	1.16	50.13	1.02	1.07	50.25
5	1.03	1.14	50.08	0.96	0.77	50.03	0.95	0.83	49.77	1.00	1.03	49.72
6	0.97	0.95	49.73	1.10	1.17	50.53	0.97	1.03	50.25	1.12	1.13	49.98
7	1.12	1.28	50.51	1.00	1.05	49.47	1.06	1.13	50.34	1.01	1.09	50.35
8	1.04	0.99	50.44	1.04	1.05	50.14	0.93	0.96	49.55	0.95	1.08	49.84
9	1.01	1.04	49.81	0.98	1.00	49.69	1.01	1.10	49.95	1.04	1.21	50.19
10	0.99	1.06	49.62	0.96	1.03	50.12	1.04	1.10	49.86	1.03	1.18	50.38
11	1.04	1.13	50.70	0.96	1.12	50.32	0.98	0.98	49.77	1.00	0.94	50.38
12	0.95	1.01	50.07	0.98	0.93	50.04	0.99	0.98	49.98	0.98	1.09	50.14
13	0.95	0.87	49.80	1.02	1.00	50.25	0.97	1.02	49.73	0.97	0.93	49.80
14	0.90	0.79	49.89	0.94	0.85	49.93	1.07	1.08	50.38	0.99	1.16	50.18
15	1.01	1.04	49.94	1.05	1.11	49.72	1.02	1.24	49.98	1.01	0.95	50.55
16	0.91	0.87	49.75	0.95	1.01	49.89	0.98	1.07	49.70	0.99	0.96	50.12
17	1.05	1.20	50.09	0.99	1.10	50.17	0.93	1.02	49.69	0.96	0.89	50.06
18	0.97	0.99	50.13	0.96	1.09	50.13	0.99	1.19	49.79	0.91	0.87	49.66
19	0.98	1.15	50.04	0.99	1.22	50.05	1.02	0.99	49.76	1.02	1.09	49.91
20	1.01	1.24	50.20	1.05	1.09	50.05	1.04	1.07	50.16	1.02	0.97	49.92
21	1.06	1.28	50.44	0.98	1.08	49.92	0.91	1.00	49.37	0.97	1.17	50.26
22	1.02	0.84	49.96	1.00	0.98	49.85	1.03	1.24	49.84	0.97	1.10	49.91
23	0.94	0.86	49.98	1.03	0.88	49.96	0.95	1.06	49.77	0.86	0.82	49.30
24	0.99	1.06	49.89	0.91	0.91	49.95	1.00	1.22	49.93	1.02	1.13	50.44
25	1.01	1.01	50.13	0.99	0.96	50.01	0.89	0.77	49.95	0.99	1.10	50.17
26	0.98	1.13	50.61	0.98	1.05	49.91	0.99	1.08	49.74	0.90	0.98	49.22
27	0.92	1.00	49.83	0.96	0.97	50.02	0.93	1.01	49.75	1.03	1.07	50.24
28	0.99	1.05	49.63	0.97	0.90	50.02	1.07	0.95	50.26	1.01	0.96	49.63
29	0.99	1.19	50.27	0.96	0.92	49.77	1.02	0.90	49.85	1.01	0.93	50.42
30	0.94	1.06	50.00	1.01	1.02	50.17	0.98	0.96	49.90	1.04	1.01	50.00
31	0.96	1.04	49.32	1.02	1.08	49.99	0.99	1.39	50.14	1.00	1.08	50.17
32	1.03	1.03	50.22	1.01	1.06	50.00	0.95	0.91	49.95	1.03	1.09	49.79
33	1.01	1.17	50.18	1.03	1.10	49.88	1.00	0.95	50.19	1.07	1.34	50.43
34	0.98	0.88	49.77	1.01	0.94	49.87	1.01	0.96	50.13	0.97	1.07	49.90
35	0.95	1.05	49.83	0.93	0.93	49.92	1.02	1.00	50.36	1.07	0.80	50.21
36	1.04	1.09	50.47	1.00	1.14	49.96	0.91	0.77	49.52	1.10	1.15	50.47
37	1.02	1.06	50.08	1.01	1.10	50.22	0.99	1.06	49.78	1.03	1.34	50.24
38	1.00	1.08	50.18	1.06	1.12	50.42	0.90	0.78	49.88	1.13	1.01	50.27
39	0.99	0.88	50.06	0.96	0.89	50.12	1.04	1.02	49.69	1.08	1.23	50.43
40	1.10	1.32	50.31	1.00	1.30	49.72	1.00	0.93	50.21	1.03	0.91	50.61
41	0.98	1.06	49.69	0.94	0.81	49.79	0.94	0.95	49.97	1.01	0.94	50.39
42	0.98	0.90	50.02	1.01	1.02	49.86	1.04	0.92	50.13	0.97	1.00	49.99
43	1.00	1.09	50.09	0.90	0.96	49.36	1.08	0.94	49.71	1.07	1.31	50.08
44	0.93	1.01	50.05	0.91	0.86	49.89	1.04	1.10	50.13	1.08	1.00	50.00
45	1.01	1.19	50.38	1.06	0.99	50.35	0.99	1.05	50.29	0.95	0.91	50.15
46	0.96	1.02	49.72	1.03	1.16	50.09	0.89	0.78	49.78	1.05	1.25	49.88
47	1.03	1.10	50.01	1.02	1.13	50.33	0.87	0.89	49.82	1.02	1.08	50.11
48	1.11	1.29	50.26	1.00	1.03	50.16	0.97	1.10	49.22	1.08	1.10	50.01
49	1.04	1.16	50.08	0.95	0.85	49.72	0.98	0.94	49.64	1.12	1.31	49.95
50	1.01	1.19	49.73	1.01	0.97	50.20	0.94	0.69	49.90	0.95	0.86	50.16

TABLE 7. *(Continued.)* **Specific carbon fiber tubing phase II dataset.**

	SUBGROUP											
	5			6			7			8		
sample	inner	thickness	length	inner	thickness	length	inner	thickness	length	inner	thickness	length
1	1.08	1.09	50.21	0.97	1.00	50.06	1.01	1.06	49.95	0.97	0.92	49.87
2	1.01	1.05	49.94	0.98	0.98	49.82	1.01	1.07	50.18	1.03	1.20	50.34
3	1.00	1.04	49.80	0.95	0.93	49.78	0.93	1.02	49.94	1.06	1.15	50.16
4	0.99	1.21	49.65	0.93	0.95	49.91	1.02	0.94	50.27	0.97	0.91	49.95
5	1.00	1.04	49.75	0.93	0.95	49.87	1.04	1.14	49.85	1.03	1.16	49.90
6	1.04	1.01	50.29	1.02	0.94	50.15	1.01	1.05	49.77	0.95	0.99	49.91
7	1.06	1.27	50.34	1.03	1.05	50.04	0.96	0.92	50.10	1.03	1.13	49.97
8	0.96	1.03	49.99	0.98	1.07	50.15	0.95	0.94	49.86	1.00	1.04	50.12
9	0.99	1.00	50.09	0.94	1.10	50.32	0.96	0.99	50.04	0.98	1.12	50.22
10	1.06	0.99	50.12	1.02	1.23	50.13	0.96	1.07	49.54	1.02	1.18	50.58
11	1.05	1.14	50.02	0.99	1.11	49.88	1.04	1.02	50.16	1.00	1.06	50.27
12	0.94	0.90	49.72	0.96	1.15	49.80	0.96	0.93	49.87	1.01	1.18	50.37
13	0.98	1.04	49.87	1.09	1.41	50.05	0.94	0.83	50.00	1.00	1.24	49.68
14	0.97	0.83	49.91	0.97	0.95	49.38	1.00	0.96	50.04	0.98	1.07	50.25
15	1.05	1.12	50.31	1.05	1.14	50.18	0.87	0.82	49.48	0.85	0.79	49.71
16	1.01	1.02	49.45	0.91	0.76	49.75	0.96	1.11	50.23	1.00	1.08	50.28
17	0.86	0.87	49.27	0.95	0.95	49.75	0.94	0.97	49.76	1.03	1.33	50.13
18	1.00	1.21	50.30	0.92	0.94	50.18	1.04	1.25	49.95	0.95	0.96	49.85
19	0.89	0.86	49.42	0.97	0.87	49.84	0.94	1.12	49.67	1.01	1.15	49.66
20	0.96	0.95	50.21	1.03	1.09	50.37	1.05	1.15	50.27	0.99	1.07	50.06
21	0.90	1.08	49.56	0.93	0.99	49.80	1.00	0.98	49.76	0.99	0.92	49.99
22	0.96	0.93	50.19	0.99	1.16	50.20	1.11	1.10	50.22	1.05	1.13	50.27
23	1.04	1.12	49.45	0.95	1.02	49.65	0.95	1.03	49.78	0.99	1.17	49.96
24	1.00	1.05	49.82	0.97	1.29	49.58	0.99	0.91	49.88	0.99	0.92	49.96
25	1.08	1.15	50.22	0.97	1.13	49.78	0.91	0.90	49.46	1.05	0.98	50.12
26	1.04	1.25	50.41	1.00	1.21	49.82	0.96	0.96	50.27	0.99	0.68	49.74
27	1.01	1.01	50.02	1.07	1.22	49.77	0.97	0.90	50.19	0.95	1.00	49.92
28	1.04	1.23	50.03	1.01	0.91	50.05	1.06	1.14	50.35	0.93	0.87	49.74
29	0.97	1.08	49.86	1.01	0.88	50.41	1.01	1.15	50.00	0.93	1.00	49.69
30	1.03	1.11	49.94	0.98	0.74	50.44	1.03	1.17	50.09	1.00	1.07	50.04
31	1.12	1.33	50.59	0.97	0.94	49.85	1.02	1.19	50.01	1.14	1.41	50.14
32	1.03	1.01	49.96	1.04	1.21	50.21	1.06	1.12	50.20	1.02	1.30	49.84
33	0.94	1.00	50.20	1.00	0.99	49.87	1.01	1.06	49.96	1.10	1.18	50.29
34	0.95	1.03	49.99	1.00	1.02	50.05	1.04	1.14	50.12	1.08	1.15	50.16
35	0.89	0.82	49.67	1.10	1.35	49.96	1.02	1.23	50.66	0.97	0.97	49.82
36	0.97	0.92	49.97	0.87	0.91	49.31	1.01	0.97	49.72	1.03	1.15	50.29
37	0.91	1.14	49.71	1.05	0.97	50.37	0.97	0.79	49.91	1.09	1.23	50.18
38	0.93	0.86	49.77	1.08	1.06	49.77	0.99	1.03	50.26	1.01	0.94	50.07
39	1.00	1.03	50.12	0.95	1.09	50.04	0.97	1.03	49.80	1.08	1.15	50.46
40	1.08	1.17	50.20	0.99	0.90	50.26	1.00	0.90	50.02	1.02	1.06	49.97
41	0.96	1.03	49.54	1.02	0.91	49.78	1.00	1.02	50.12	0.88	0.79	49.67
42	0.93	0.92	49.64	0.99	1.19	49.65	0.94	0.98	50.05	0.99	1.13	49.87
43	0.95	1.04	49.69	1.03	1.04	50.34	1.00	1.16	49.75	1.05	1.03	50.41
44	1.04	1.08	50.33	0.97	0.89	50.22	0.92	1.05	49.74	1.07	1.16	50.04
45	0.96	0.97	49.74	1.00	0.97	50.02	0.99	1.11	49.80	0.99	1.07	49.93
46	0.97	0.97	50.22	1.07	1.07	50.10	1.07	1.16	50.53	1.02	1.12	49.92
47	0.96	0.99	49.93	0.96	0.79	50.07	1.03	1.14	50.13	1.00	1.06	50.03
48	0.98	0.81	49.73	1.05	0.90	49.98	0.97	0.77	50.19	0.89	1.10	50.40
49	1.10	1.29	50.59	1.03	0.98	49.68	0.95	1.19	50.16	0.98	0.80	50.42
50	1.02	0.96	50.21	1.02	1.16	50.05	1.02	1.00	49.55	0.99	0.97	50.16

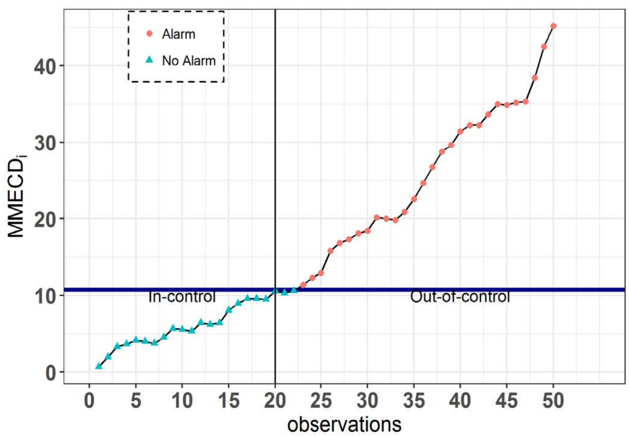

FIGURE 6. The MMECD chart when $h_3 = 10.75$, $\lambda = 0.5$ and $k_2^* = 0.5$.

that might include temperature, gas flow, and chemical composition. It is important to timely fix any issues with these stages/variables as it may lead to serious health issues such as skin allergy, lungs infections etc.

V. SUMMARY, CONCLUSION AND RECOMMENDATIONS

A new multivariate control chart, namely MMECD chart is proposed to monitor changes in the process dispersion in a multivariate setup. It is designed by mixing the features of multivariate EWMA and CUSUM charts for dispersion matrix of a process. The performance of the proposed scheme is evaluated in terms of several useful measures such as ARL, SDRL, MDRL, EQL and SEQL. The performance of the MMECD control chart is compared with the other competing charts, including GenVar control chart, MEWMAD and MCUSUMD control charts. The comparisons revealed that the proposed scheme has better performance than its counterparts for detecting the small shifts in the process dispersion in a multivariate environment. The scope of this study may be extended to investigate the Multivariate Mixed EWMA-CUSUM under contaminated normal and non-normal processes.

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