IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received January 27, 2020, accepted February 19, 2020, date of publication March 2, 2020, date of current version March 17, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2977833

Efficient Control Charts for Monitoring Process CV Using Auxiliary Information

SADDAM AKBAR ABBASI
Department of Mathematics, Statistics and Physics, Qatar University, Doha, Qatar
e-mail: sabbasi@qu.edu.qa

This work was supported by the Qatar National Library, Qatar.

ABSTRACT Coefficient of variation (CV) control charts are a suitable choice for the monitoring of variation in cases when the process standard deviation is proportional to the process mean. This study is aimed at enhancing the detection ability of usual CV chart by incorporating the use of auxiliary information. In recent years, researchers investigated the use of auxiliary information for improving the sensitivity of location and dispersion charts. However, no study has investigated the CV control charts in presence of auxiliary information. In this study, I propose and investigate a set of auxiliary information based charts for efficient monitoring of process CV by considering a variety of CV estimators. The auxiliary information is used in terms of regression, ratio and hybrid forms. A real life example concerning the monitoring of air quality is also presented to illustrate the application of proposed charts.

INDEX TERMS Auxiliary information, coefficient of variation, control chart, power, air quality.

I. INTRODUCTION

Control charts act as the most important tool in the statistical process control (SPC) tool-kit. The main purpose of their implementation is the timely detection of assignable causes that can affect the quality of a product or state of a process. The quick detection of these assignable causes can greatly improve the quality standards of a product/process (cf. [1]). Walter A. Shewhart did the pioneer work by proposing the control charts for the monitoring of manufacturing processes but soon their use is extended to the monitoring of other processes such as in nuclear engineering, health care, analytical laboratories, environmental monitoring etc (cf. [2]).

Control charts are mostly used for the monitoring of process location and dispersion. When the process mean levels are constant and the process standard deviation is independent of mean, the variability of the process is usually monitored using range (R) or standard deviation (S) charts. For cases, when the process location is not stable, and process standard deviation is proportional to the mean, the process CV is mostly constant and hence CV control charts are a preferred choice, for the the monitoring of process variability (cf. Kang *et al.* [3]). For many real life processes, we can observe this phenomenon. For example, Castagliola *et al.* [4] showed that the standard deviation of the pressure test drop time in a sintering process (that manufactures mechanical parts) is proportional to its mean. Abbasi and Adegoke [5] indicates a proportional relationship (in a multivariate setup) between the covariance matrix and the mean vector for the monitoring of inner diameter and average length of carbon fiber tubes, considering the pultrusion process. Moreover, Nguyen *et al.* [6] observed that in sanitary sector, the standard deviation of the weight of scrap zinc alloy material is proportional to its mean.

Kang *et al.* [3] did the initial work for the monitoring of process CV by presenting a Shewhart type CV chart. They provided control limits using Monte Carlo simulations as well as the quantile points from the non-central t distribution. A number of studies then enhanced the performance of the CV chart proposed by Kang *et al.* [3], by considering a variety of design structures. Castagliola *et al.* [7] and Yeong *et al.* [8] proposed the adaptive Shewhart CV and EWMA CV control charts, respectively, based on variable sampling interval and compared its performance with the SH-CV, synthetic CV (Syn-CV) and VSI-CV charts. Calzada and Scariano [9] proposed a synthetic control chart for the monitoring of process CV. Amdouni *et al.* [10] proposed the use of a variable sample size (VSS) to monitor CV in short production runs. Menzefricke [11] proposed a CV control chart for log normally distributed quality characteristic, considering Bayesian framework. Zhang *et al.* [12] proposed a modified EWMA chart to further enhance

The associate editor coordinating the review of this manuscript and approving it for publication was Jenny Mahoney.

TABLE 1. d_2 **values for the different CV estimators considering varying levels of** n **and** ρ.

ρ	n	U	Reg_1	Reg_2	Reg_3	R	H_1	H_2	H_3
0.3	5	0.942	1.012	0.995	1.055	0.944	1.022	1.002	1.012
	7	0.960	0.989	0.981	0.998	0.962	1.016	1.002	0.989
	10	0.973	0.986	0.981	0.988	0.975	1.011	1.002	0.986
	12	0.978	0.986	0.983	0.988	0.979	1.008	1.001	0.986
	15	0.982	0.987	0.985	0.988	0.983	1.006	1.001	0.987
0.5	5	0.941	1.002	0.984	1.042	0.944	1.022	1.002	1.002
	7	0.960	0.984	0.975	0.996	0.962	1.016	1.002	0.984
	10	0.973	0.984	0.978	0.988	0.975	1.011	1.002	0.984
	12	0.978	0.985	0.981	0.988	0.979	1.008	1.001	0.985
	15	0.982	0.987	0.984	0.989	0.983	1.006	1.001	0.987
0.7	5	0.941	0.989	0.973	1.033	0.944	1.022	1.002	0.989
	7	0.960	0.981	0.972	0.998	0.962	1.016	1.002	0.981
	10	0.973	0.984	0.978	0.992	0.975	1.011	1.001	0.984
	12	0.978	0.985	0.981	0.992	0.979	1.008	1.001	0.985
	15	0.982	0.988	0.984	0.993	0.983	1.006	1.001	0.988
0.9	5	0.941	0.986	0.975	1.042	0.944	1.022	1.002	0.986
	7	0.959	0.988	0.981	1.018	0.962	1.016	1.002	0.988
	10	0.973	0.991	0.987	1.009	0.974	1.011	1.001	0.991
	12	0.978	0.992	0.989	1.006	0.979	1.008	1.001	0.992
	15	0.982	0.994	0.991	1.005	0.984	1.007	1.001	0.994
0.95	5	0.941	0.990	0.982	1.055	0.944	1.022	1.002	0.990
	7	0.959	0.993	0.988	1.030	0.962	1.015	1.002	0.993
	10	0.972	0.995	0.992	1.017	0.974	1.011	1.001	0.995
	12	0.978	0.996	0.993	1.013	0.979	1.008	1.001	0.996
	15	0.982	0.997	0.995	1.010	0.984	1.007	1.001	0.997

the sensitivity of EWMA originally proposed by [7]. Hong *et al.* [13] proposed the generally weighted moving average (CV-GWMA) chart and showed a better performance of CV-GWMA over CV-EWMA and CV-DEWMA (double exponentially weighted moving average) charts (cf. [14]), particularly for the detection of small shifts. Recently, a lot of researchers proposed new structures for efficient monitoring of process CV. Dawod *et al.* [15] and Abbasi and Adegoke [5] investigated univariate and multivariate CV control charts, respectively, for Phase I of SPC. They considered both diffuse symmetric and localized CV disturbance scenarios. Abbasi *et al.* [16] proposed the design of CV control chart using the progressive mean technique. Chen *et al.* [17] developed a generally weighted moving average control chart for the monitoring of process CV. Chew *et al.* [18] proposed a variable parameter control chart for monitoring the multivariate coefficient of variation. Nguyen *et al.* [6] investigated the performance of Shewhart type VSI control chart for CV monitoring in presence of measurement error. Haq and Khoo [19] proposed new adaptive EWMA control charts for monitoring both univariate and multivariate CV. Moreover, Abbasi *et al.* [20] enhanced the performance of CV control chart by incorporating ranked set sampling schemes in the design structures.

To enhance the detection ability of the charts, researchers made use of different methods in recent past. One such method is to use the auxiliary information that can help in better estimation of process parameters and in return enhancing the detection ability of control charts. Riaz [21] proposed V_r chart for efficient monitoring of process variability by using the regression type auxiliary estimator of population variance. Riaz *et al.* [22] investigated the estimation effects of auxiliary information based location charts considering normal and non-normal processes. Ahmad *et al.* [23] investigated a variety of auxiliary information based variability charts. Abbas *et al.* [24] proposed an efficient EWMA location chart by making use of auxiliary information. Recently, Sanusi *et al.* [25] and Adegoke *et al.* [26] enhanced the performance of location EWMA and HWMA charts by making use of auxiliary information based ratio and regression estimators, respectively. Adegoke *et al.* [27] enhanced the EWMA location chart by using the auxiliary information with different ranked set sampling schemes. Further studies on the use of auxiliary information based

TABLE 2. d_3 values for the different CV estimators considering varying levels of n and ρ.

ρ	n	U	Reg_1	Reg_2	Reg_3	R	H_1	H_2	H_3
0.3	5	0.344	0.504	0.436	1.445	0.350	0.544	0.464	0.504
	7	0.285	0.320	0.309	0.347	0.290	0.450	0.384	0.320
	10	0.234	0.246	0.242	0.250	0.238	0.371	0.316	0.246
	12	0.213	0.219	0.217	0.221	0.216	0.336	0.287	0.219
	15	0.188	0.192	0.191	0.193	0.191	0.298	0.254	0.192
0.5	5	0.344	0.442	0.410	0.758	0.351	0.499	0.424	0.442
	7	0.285	0.311	0.301	0.342	0.291	0.412	0.350	0.311
	10	0.234	0.241	0.238	0.248	0.239	0.339	0.288	0.241
	12	0.213	0.216	0.214	0.220	0.217	0.307	0.261	0.216
	15	0.189	0.189	0.188	0.192	0.192	0.272	0.231	0.189
0.7	5	0.344	0.400	0.369	0.586	0.352	0.419	0.353	0.400
	7	0.285	0.286	0.279	0.325	0.292	0.346	0.291	0.286
	10	0.234	0.223	0.220	0.238	0.239	0.284	0.239	0.223
	12	0.213	0.199	0.197	0.211	0.218	0.257	0.217	0.199
	15	0.189	0.174	0.173	0.183	0.193	0.228	0.192	0.174
0.9	5	0.343	0.274	0.267	0.448	0.353	0.275	0.221	0.274
	7	0.284	0.203	0.201	0.272	0.292	0.227	0.182	0.203
	10	0.234	0.156	0.156	0.196	0.240	0.186	0.149	0.156
	12	0.213	0.138	0.138	0.172	0.219	0.168	0.135	0.138
	15	0.189	0.120	0.120	0.149	0.194	0.149	0.119	0.120
0.95	5	0.343	0.208	0.205	0.369	0.353	0.214	0.162	0.208
	7	0.284	0.151	0.151	0.236	0.293	0.177	0.133	0.151
	10	0.234	0.115	0.116	0.172	0.240	0.145	0.109	0.115
	12	0.213	0.102	0.102	0.152	0.219	0.131	0.099	0.102
	15	0.189	0.088	0.088	0.132	0.194	0.116	0.087	0.088

control charts can be seen in [28]–[30], and references therein.

All these studies are using auxiliary information for enhancing the efficiency of location or dispersion charts. No study, as of yet, enhanced the efficiency of CV charts with the use of auxiliary information. The purpose of this study is to enhance the detection ability of process CV charts by making use of auxiliary information. The auxiliary information is used in the form of ratio, regression and hybrid forms. The rest of the article is detailed as: Section II presents a set of auxiliary information based CV estimators, a general control chart structure is presented in Section III. The performance comparison of different auxiliary information based CV charts is presented in Section IV. Performance comparison of the proposed CV charts is made with some existing CV charts in Section V. Illustrative example is presented in Section VI and finally conclusions in Section VII.

II. AUXILIARY INFORMATION BASED CV ESTIMATORS

The purpose of this study is to enhance the detection ability of usual CV chart by incorporating the auxiliary information. I use a set of auxiliary information based estimators to serve this purpose.

Let Y represents the quality characteristic of interest which is correlated with an auxiliary variable X. The pairs (Y_i, X_i) are assumed to follow bivariate normal distribution with mean vector μ and variance covariance matrix Σ (i.e $(Y_i, X_i) \sim N_2(\mu, \Sigma)$), where $\mu = \begin{pmatrix} \mu_y \\ \mu_x \end{pmatrix}$ and $\Sigma = \begin{pmatrix} \sigma_y^2 & \rho\sigma_y\sigma_x \\ \rho\sigma_y\sigma_x & \sigma_x^2 \end{pmatrix}$. Here ρ represents the correlation coefficient between the study and the auxiliary variables Y and X, respectively. Let $(x_1, y_1), \ldots (x_n, y_n)$ represents a sample of size n from the bivariate normal distribution. Let $\bar{y}$ and $\bar{x}$ represent sample means, s_y^2 and s_x^2 represent sample variances, s_{xy} represent sample covariance, c_y and c_x represents sample CVs and r_{xy} represents sample coefficient of correlation. Based on these notations, a set of estimators for process CV, are defined below:

Usual CV Estimator (CV_U)

All the existing CV charts are based on the usual definition of CV, as given below:

$$CV_U = \frac{s_y}{\bar{y}} \tag{1}$$

TABLE 3. $V_{\alpha/2}$ **for the different CV estimators considering varying levels of** n **and** ρ**.**

ρ	n	U	Reg_1	Reg_2	Reg_3	R	H_1	H_2	H_3
0.3	5	0.165	0.167	0.166	0.169	0.167	0.000	0.000	0.167
	7	0.267	0.269	0.268	0.269	0.264	0.000	0.000	0.269
	10	0.374	0.379	0.378	0.380	0.375	0.000	0.054	0.379
	12	0.416	0.421	0.421	0.420	0.417	0.000	0.146	0.421
	15	0.479	0.480	0.481	0.480	0.475	0.084	0.236	0.480
0.5	5	0.167	0.176	0.175	0.175	0.168	0.000	0.000	0.176
	7	0.268	0.276	0.274	0.276	0.263	0.000	0.000	0.276
	10	0.379	0.384	0.384	0.386	0.374	0.000	0.000	0.384
	12	0.422	0.426	0.425	0.425	0.414	0.070	0.225	0.426
	15	0.479	0.487	0.485	0.483	0.476	0.169	0.308	0.487
0.7	5	0.160	0.178	0.176	0.175	0.157	0.000	0.000	0.178
	7	0.264	0.281	0.279	0.275	0.259	0.000	0.135	0.281
	10	0.375	0.394	0.390	0.389	0.370	0.146	0.294	0.394
	12	0.421	0.444	0.442	0.438	0.418	0.224	0.360	0.444
	15	0.478	0.502	0.499	0.494	0.474	0.314	0.427	0.502
0.9	5	0.162	0.209	0.201	0.208	0.160	0.181	0.353	0.209
	7	0.266	0.336	0.327	0.336	0.262	0.325	0.466	0.336
	10	0.368	0.475	0.459	0.466	0.364	0.453	0.566	0.475
	12	0.414	0.543	0.531	0.528	0.410	0.503	0.604	0.543
	15	0.475	0.608	0.597	0.588	0.471	0.556	0.649	0.608
0.95	5	0.164	0.238	0.222	0.216	0.161	0.370	0.531	0.238
	7	0.266	0.427	0.398	0.404	0.261	0.483	0.613	0.427
	10	0.367	0.589	0.568	0.541	0.366	0.576	0.684	0.589
	12	0.413	0.651	0.637	0.591	0.410	0.614	0.711	0.651
	15	0.474	0.703	0.694	0.637	0.468	0.656	0.744	0.703

where $\bar{y}$ and s_y respectively represent the mean and sample standard deviation of the study variable Y, defined as:

$$\bar{y} = \frac{\sum y_i}{n} \quad and \quad s_y^2 = \frac{\sum (y_i - \bar{y})^2}{n-1}.$$

A set of auxiliary information based estimators for process CV are proposed by [31]–[34]. Below, I am describing some of their best estimators, that are used in this study:

Regression Estimator 1 (CV_{Reg_1})

This estimator of CV is based on using the ratio of the auxiliary information based regression estimators of process variance and process mean:

$$CV_{Reg_1} = \frac{\left({s_y}^2 + b_2\left({\sigma_x}^2 - {s_x}^2\right)\right)^{\frac{1}{2}}}{\left(\bar{y} + b_1\left(\mu_x - \bar{x}\right)\right)} \tag{2}$$

Under normal distribution, the expression for b_1 and b_2 simplifies to (cf. [34]):

$$b_1 = \frac{s_{xy}}{s_x^2} \quad \text{and} \quad b_2 = \frac{\left(s_{xy}\right)^2}{\left(s_x^2\right)^2}.$$

Regression Estimator 2 (CV_{Reg_2})

CV_{Reg_2} estimator of CV is using the population CV information of the auxiliary variable (X). The estimator is defined as:

$$CV_{Reg_2} = \frac{s_y}{\bar{y}} + b_3\left(\frac{\sigma_x}{\mu_x} - \frac{s_x}{\bar{x}}\right) \tag{3}$$

Under normal distribution b_3 simplifies to (cf. [34]):

$$b_3 = \frac{\left\{\frac{\left(s_{xy}^2\right)}{2\bar{x}\bar{y}s_x s_y} + \frac{s_{xy}s_x s_y}{\bar{x}^2\bar{y}^2}\right\}}{\left\{\frac{s_x^2}{2\bar{x}^2} + \left(\frac{s_x}{\bar{x}}\right)^4\right\}}$$

Regression Estimator 3 (CV_{Reg_3})

Archaana and Rao [34] proposed a new regression estimator of CV by improving the CV estimator of [32].

$$CV_{Reg_3} = \frac{s_y}{\bar{y}} - \hat{\partial}_1(\bar{x} - \mu_x) - \hat{\partial}_2(s_{xx} - \sigma_{xx}) + b_7(\sigma_x^2 - s_x^2) \tag{4}$$

TABLE 4. $V_{(1-\alpha/2)}$ for the different CV estimators considering varying levels of n and ρ.

ρ	n	U	Reg_1	Reg_2	Reg_3	R	H_1	H_2	H_3
0.3	5	2.132	4.026	3.538	6.721	2.162	2.600	2.416	4.026
	7	1.940	2.535	2.320	3.126	1.963	2.326	2.166	2.535
	10	1.754	1.953	1.889	2.029	1.775	2.102	1.963	1.953
	12	1.672	1.768	1.735	1.795	1.693	1.992	1.863	1.768
	15	1.598	1.656	1.641	1.672	1.614	1.901	1.777	1.656
0.5	5	2.131	3.749	3.327	6.474	2.189	2.480	2.299	3.749
	7	1.936	2.480	2.286	3.019	1.978	2.224	2.064	2.480
	10	1.754	1.932	1.871	2.035	1.779	2.013	1.887	1.932
	12	1.674	1.757	1.729	1.833	1.701	1.907	1.787	1.757
	15	1.595	1.668	1.644	1.698	1.620	1.825	1.707	1.668
0.7	5	2.132	3.370	2.961	5.788	2.205	2.248	2.082	3.370
	7	1.926	2.283	2.160	2.869	1.988	2.036	1.890	2.283
	10	1.751	1.843	1.792	2.011	1.788	1.856	1.740	1.843
	12	1.673	1.701	1.670	1.803	1.702	1.774	1.658	1.701
	15	1.593	1.607	1.591	1.666	1.626	1.695	1.587	1.607
0.9	5	2.139	2.395	2.243	4.267	2.222	1.828	1.684	2.395
	7	1.918	1.849	1.797	2.564	2.006	1.690	1.564	1.849
	10	1.747	1.560	1.538	1.829	1.786	1.567	1.463	1.560
	12	1.674	1.470	1.457	1.641	1.712	1.515	1.414	1.470
	15	1.602	1.399	1.394	1.528	1.627	1.459	1.369	1.399
0.95	5	2.122	2.000	1.898	3.821	2.215	1.651	1.506	2.000
	7	1.917	1.629	1.582	2.329	1.998	1.546	1.416	1.629
	10	1.747	1.416	1.399	1.732	1.790	1.450	1.344	1.416
	12	1.671	1.348	1.337	1.558	1.712	1.402	1.307	1.348
	15	1.600	1.292	1.289	1.460	1.630	1.360	1.273	1.292

where

$$b_7 = \frac{\left(\frac{s_{xy}^2}{\bar{y}s_y} - 2\hat{\partial}_2 s_{xx}^2\right)}{2\, s_{xx}^2}$$

Under normal distribution (cf. [32]):

$$\hat{\partial}_1 = \frac{-\sigma_{xy}^2 c_y}{2\sigma_x^3\sigma_{yy}} \quad and \quad \hat{\partial}_2 = \frac{c_y\sigma_{xy}}{\sigma_x^3\mu_y}$$

Ratio estimator (CV_R). The ratio estimator of CV uses the information of the ratio of population mean of the auxiliary variable and its sample mean ([33]):

$$CV_R = \frac{s_y}{\bar{y}} \times \frac{\mu_x}{\bar{x}} \tag{5}$$

A class of hybrid estimators were proposed by Tripathi et al. [32], that are based on the mixture of ratio and regression estimators. In this study, I will also use some of these hybrid estimators:

Hybrid Estimator 1 (CV_{H_1}).

$$CV_{H_1} = \frac{s_y - \alpha_2\,(s_x - \sigma_x)}{\bar{y}} \tag{6}$$

where $\alpha_2 = 1.32105$

Hybrid Estimator 2 (CV_{H_2}).

$$CV_{H_2} = \frac{s_y - (s_x - \sigma_x)}{\bar{y}} \tag{7}$$

Hybrid Estimator 3 (CV_{H_3}).

$$CV_{H_3} = \frac{\left({s_y}^2 + b_5\left({\sigma_x}^2 - {s_x}^2\right)\right)^{\frac{1}{2}}}{(\bar{y} + b_4\,(\mu_x - \bar{x}))} \tag{8}$$

where

$$b_4 = \frac{s_{xy}}{s_{xx}} \quad \text{and} \quad b_5 = \left(\frac{s_{xy}}{s_{xx}}\right)^2.$$

Next section will describe a general control chart structure for evaluating the performance of CV charts based on these estimators.

III. CONTROL CHART STRUCTURE

To obtain the control limits of the auxiliary information based CV charts, I define a general statistic CV_A; $\forall A = U, Reg_1, Reg_2, Reg_3, R, H_1, H_2, H_3$, i.e. CV_A can be any of the CV estimators, defined in Section II. Using CV_A, a standardized CV statistic can be defined as $V_A = CV_A/\gamma$; where

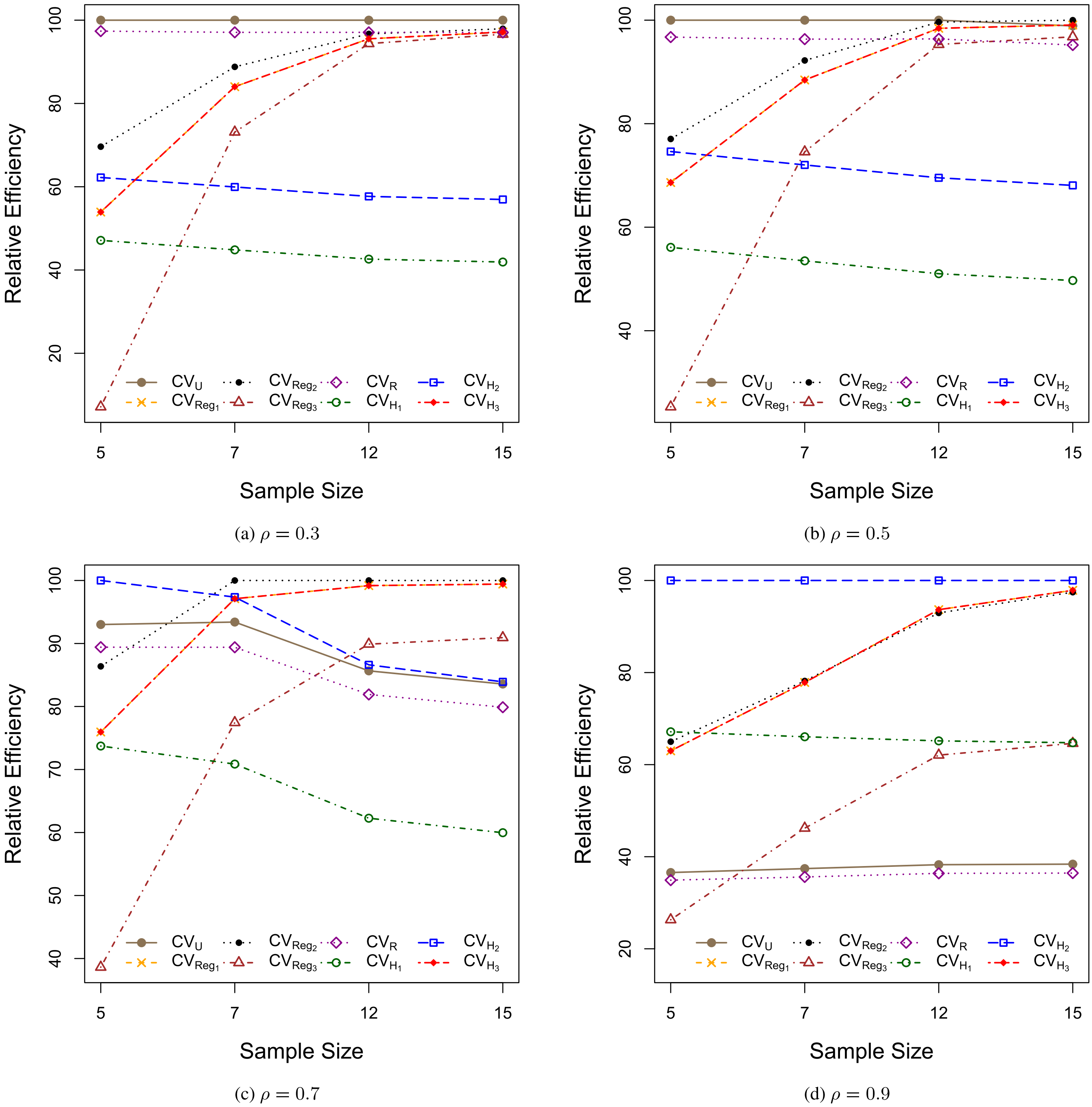


FIGURE 1. Relative efficiency for the different CV estimators at varying levels of *n* and ρ.

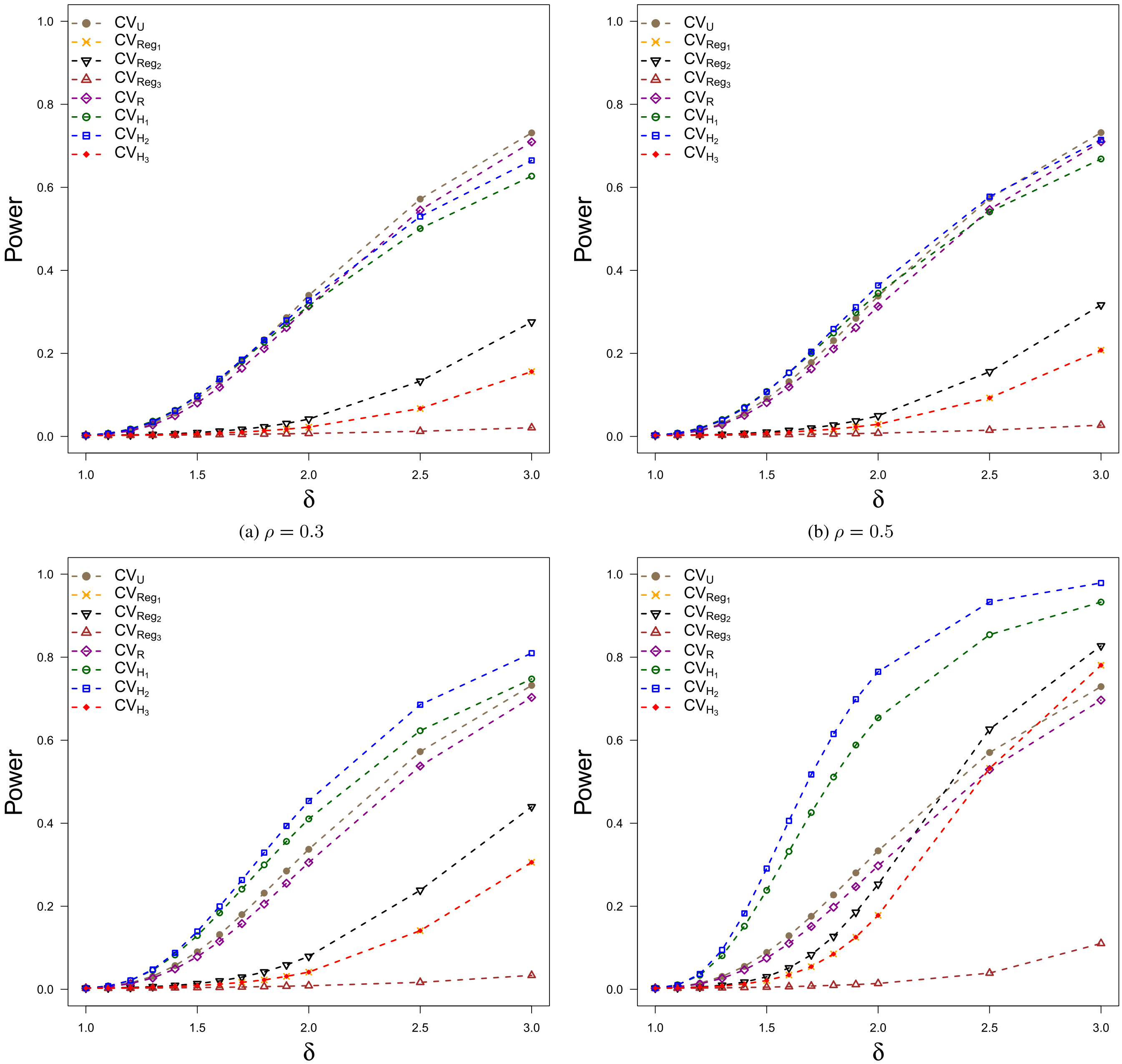
γ is the process coefficient of variation. By taking the expectation on both sides of V_A, we get $E(V_A) = E(CV_A)/\gamma = d_{2,A,n,\rho}$. Similarly, $\sigma_{V_A} = d_{3,A,n,\rho}$. For a specific CV_A estimator, d_2 and d_3 entirely depend on sample size n and correlation coefficient ρ. The $E(CV_A)$ can be replaced with the average of sample CV_A estimates, (i.e. $\overline{CV_A} = \sum_{j=1}^{m} CV_{A_j}/m$). Hence, an unbiased estimator of process coefficient of variation (γ) can be defined as $\hat{\gamma_A} = \overline{CV_A}/d_{2,A,n,\rho}$. Moreover, to maintain the false alarm rate α, the lower and upper quantile points of the distribution of V_A estimates can be defined as $V_{A,\alpha/2}$ and $V_{A,(1-\alpha/2)}$, respectively. Using these notations, the probability limits for the CV_A charts are defined as:

$$\begin{aligned} LPL_{CV_A} &= V_{A,\alpha/2}\hat{\gamma_A} \\ &= V_{A,\alpha/2}\frac{\overline{CV_A}}{d_{2,A,n,\rho}} \end{aligned} \tag{9}$$

$$\begin{aligned} UPL_{CV_A} &= V_{A,(1-\alpha/2)}\hat{\gamma_A} \\ &= V_{A,(1-\alpha/2)}\frac{\overline{CV_A}}{d_{2,A,n,\rho}} \end{aligned} \tag{10}$$

TABLE 5. Power of detection for the different CV charts using $n = 10$ at varying levels of ρ.

ρ	δ	CV_U	CV_{Reg_1}	CV_{Reg_2}	CV_{Reg_3}	CV_R	CV_{H_1}	CV_{H_2}	CV_{H_3}
0.3	1.0	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
	1.1	0.0080	0.0043	0.0050	0.0037	0.0077	0.0081	0.0081	0.0043
	1.2	0.0255	0.0106	0.0138	0.0081	0.0247	0.0230	0.0245	0.0106
	1.3	0.0610	0.0259	0.0343	0.0185	0.0582	0.0509	0.0559	0.0259
	1.4	0.1198	0.0548	0.0704	0.0398	0.1130	0.0929	0.1035	0.0548
	1.5	0.1992	0.1001	0.1260	0.0748	0.1879	0.1476	0.1673	0.1001
	1.6	0.2892	0.1624	0.1978	0.1264	0.2746	0.2097	0.2391	0.1624
	1.7	0.3847	0.2396	0.2802	0.1920	0.3681	0.2746	0.3138	0.2396
	1.8	0.4791	0.3215	0.3668	0.2680	0.4598	0.3403	0.3879	0.3215
	1.9	0.5675	0.4055	0.4536	0.3486	0.5485	0.4027	0.4577	0.4055
	2.0	0.6445	0.4893	0.5384	0.4291	0.6261	0.4600	0.5239	0.4893
	2.5	0.8792	0.7946	0.8245	0.7546	0.8690	0.6814	0.7533	0.7946
	3.0	0.9606	0.9257	0.9386	0.9070	0.9563	0.8019	0.8656	0.9257
0.5	1.0	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
	1.1	0.0078	0.0046	0.0054	0.0041	0.0078	0.0084	0.0079	0.0046
	1.2	0.0253	0.0112	0.0141	0.0084	0.0242	0.0250	0.0256	0.0112
	1.3	0.0613	0.0265	0.0347	0.0182	0.0579	0.0564	0.0607	0.0265
	1.4	0.1195	0.0567	0.0722	0.0382	0.1120	0.1043	0.1151	0.0567
	1.5	0.1974	0.1041	0.1278	0.0718	0.1845	0.1653	0.1881	0.1041
	1.6	0.2901	0.1685	0.2015	0.1213	0.2716	0.2351	0.2689	0.1685
	1.7	0.3857	0.2471	0.2865	0.1850	0.3649	0.3079	0.3526	0.2471
	1.8	0.4797	0.3322	0.3756	0.2608	0.4568	0.3788	0.4322	0.3322
	1.9	0.5672	0.4186	0.4656	0.3413	0.5427	0.4452	0.5092	0.4186
	2.0	0.6448	0.5050	0.5509	0.4241	0.6219	0.5082	0.5780	0.5050
	2.5	0.8793	0.8092	0.8357	0.7551	0.8663	0.7306	0.8038	0.8092
	3.0	0.9609	0.9327	0.9437	0.9080	0.9555	0.8436	0.9031	0.9327
0.7	1.0	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
	1.1	0.0081	0.0051	0.0055	0.0041	0.0078	0.0092	0.0091	0.0051
	1.2	0.0254	0.0134	0.0160	0.0082	0.0232	0.0299	0.0319	0.0134
	1.3	0.0620	0.0327	0.0405	0.0177	0.0558	0.0712	0.0807	0.0327
	1.4	0.1200	0.0708	0.0859	0.0378	0.1080	0.1348	0.1572	0.0708
	1.5	0.1992	0.1303	0.1560	0.0729	0.1788	0.2141	0.2538	0.1303
	1.6	0.2918	0.2130	0.2467	0.1243	0.2658	0.3013	0.3582	0.2130
	1.7	0.3887	0.3084	0.3466	0.1941	0.3582	0.3906	0.4600	0.3084
	1.8	0.4818	0.4092	0.4501	0.2752	0.4491	0.4730	0.5558	0.4092
	1.9	0.5692	0.5076	0.5479	0.3629	0.5349	0.5488	0.6383	0.5076
	2.0	0.6467	0.5983	0.6352	0.4531	0.6148	0.6159	0.7071	0.5983
	2.5	0.8803	0.8724	0.8882	0.7877	0.8622	0.8279	0.9006	0.8724
	3.0	0.9613	0.9605	0.9660	0.9266	0.9541	0.9161	0.9626	0.9605
0.9	1.0	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
	1.1	0.0083	0.0075	0.0078	0.0042	0.0077	0.0133	0.0153	0.0075
	1.2	0.0262	0.0275	0.0314	0.0103	0.0240	0.0556	0.0740	0.0275
	1.3	0.0632	0.0839	0.0960	0.0273	0.0565	0.1466	0.2033	0.0839
	1.4	0.1225	0.2001	0.2223	0.0654	0.1088	0.2796	0.3829	0.2001
	1.5	0.2025	0.3638	0.3922	0.1330	0.1818	0.4276	0.5659	0.3638
	1.6	0.2959	0.5388	0.5671	0.2275	0.2676	0.5662	0.7189	0.5388
	1.7	0.3906	0.6901	0.7124	0.3434	0.3580	0.6839	0.8269	0.6901
	1.8	0.4850	0.7996	0.8150	0.4632	0.4490	0.7744	0.8993	0.7996
	1.9	0.5727	0.8733	0.8831	0.5790	0.5368	0.8412	0.9422	0.8733
	2.0	0.6490	0.9193	0.9245	0.6789	0.6141	0.8894	0.9675	0.9193
	2.5	0.8819	0.9892	0.9890	0.9347	0.8627	0.9802	0.9980	0.9892
	3.0	0.9620	0.9975	0.9972	0.9868	0.9534	0.9958	0.9998	0.9975



(a) $\rho = 0.3$

(b) $\rho = 0.5$

(c) $\rho = 0.7$

(d) $\rho = 0.9$

FIGURE 2. Power comparison of different auxiliary information based CV Charts when $n = 5$.

After fixing the control limits for a specific CV_A estimator, the corresponding CV_A estimates are plotted to identify the state of the process. If all the CV_A statistics fall inside the probability limits, the process is declared to be in-control. If any of the plotting statistic falls outside the limits, the process is said to be in out-of-control state.

For the rest of the study, the control charts based on the different choices of A as $U, Reg_1, Reg_2, Reg_3, R, H_1, H_2$ and H_3 are named as CV_U, CV_{Reg_1}, CV_{Reg_2}, CV_{Reg_3}, CV_R, CV_{H_1}, CV_{H_2} and CV_{H_3} charts, respectively.

IV. PERFORMANCE EVALUATION AND COMPARISON

To evaluate the performance of a wide range of CV charts, as described in Section II, power of detection is used a performance measure. Power of detection is defined as the probability of detecting an out-of-control signal, when the process CV shifts from an in-control level γ_0 to a shifted level γ_1, where γ_1 is defined as $\gamma_1 = \delta\gamma_0$. The power of detection may vary with a change in design parameters such as shift δ, sample size n and correlation coefficient ρ. I noticed that changing γ_0 doesn't effect the performance of

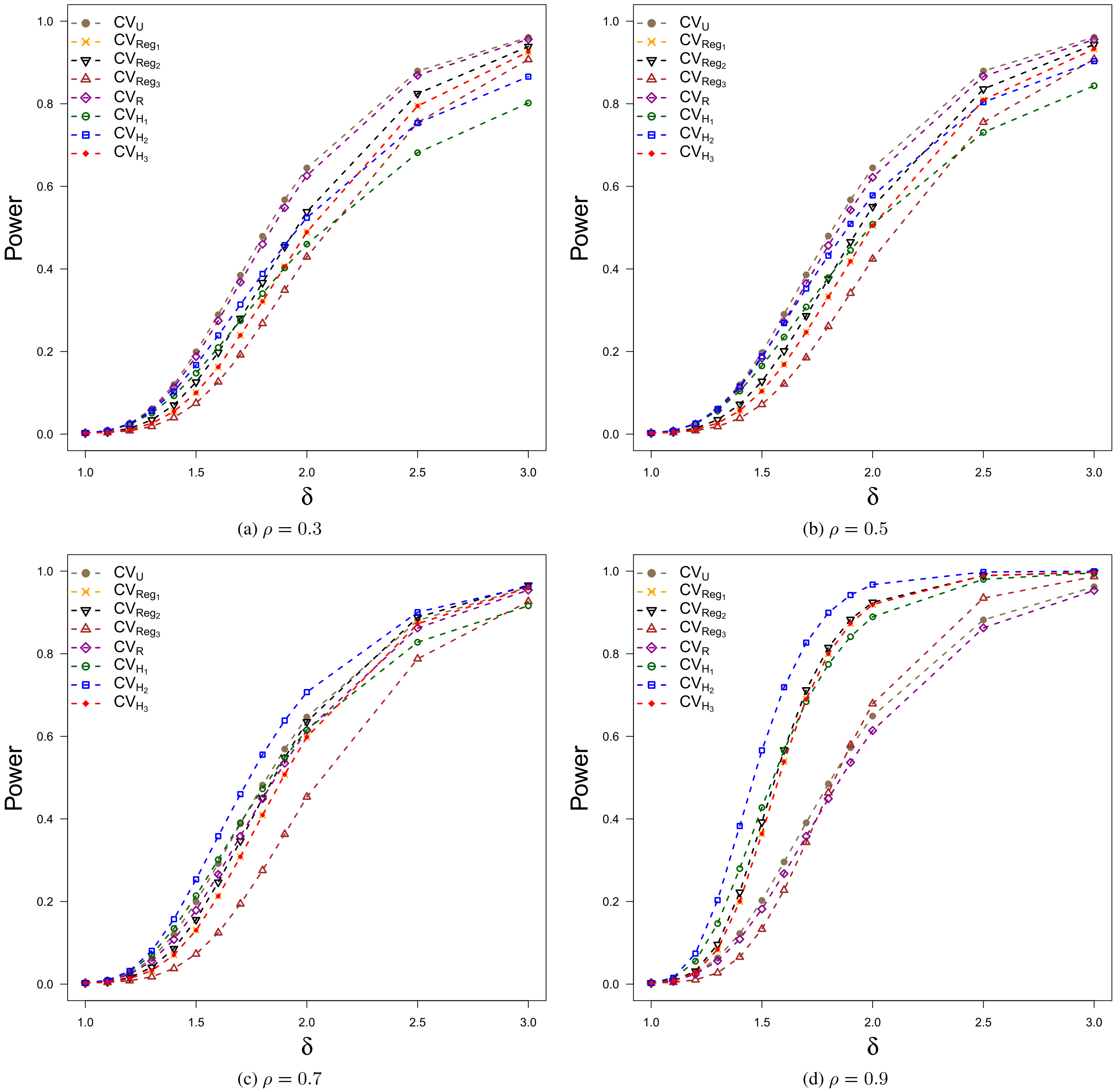


FIGURE 3. **Power comparison of different auxiliary information based CV Charts when $n = 10$.**

the CV charts as the general control chart structure is based on the standardized CV statistic. For the performance evaluation of all the CV charts, I used $\delta = 1.0, 1.1, 1.2, \ldots, 3$, $n = 5, 10, 15$, $\rho = 0.3, 0.5, 0.7, 0.9$ and $\gamma_x = \gamma_y = \gamma_0 = 0.10$. This will help us in identifying the best chart for a variety of design parameters. Note that, $\delta = 1.0$ represents that there is no shift in the process CV. A detailed Monte Carlo simulation study is conducted for performance evaluation of the different CV charts. The steps taken in simulation study are described below:

Firstly, to get the control chart constants (d_2 and d_3) and the quantile ($V_{A,\alpha/2}$ and $V_{A,(1-\alpha/2)}$), the following procedure is adopted.

- One million samples of size n are generated from an in-control bivariate normal distribution $N_2(\mu_0, \gamma_0\Sigma_0)$.
- The sample means $\bar{y}, \bar{x}$, the sample standard deviations s_y, s_x, the sample coefficient of variations c_y, c_x and the sample covariance s_{xy} are estimated from each sample.

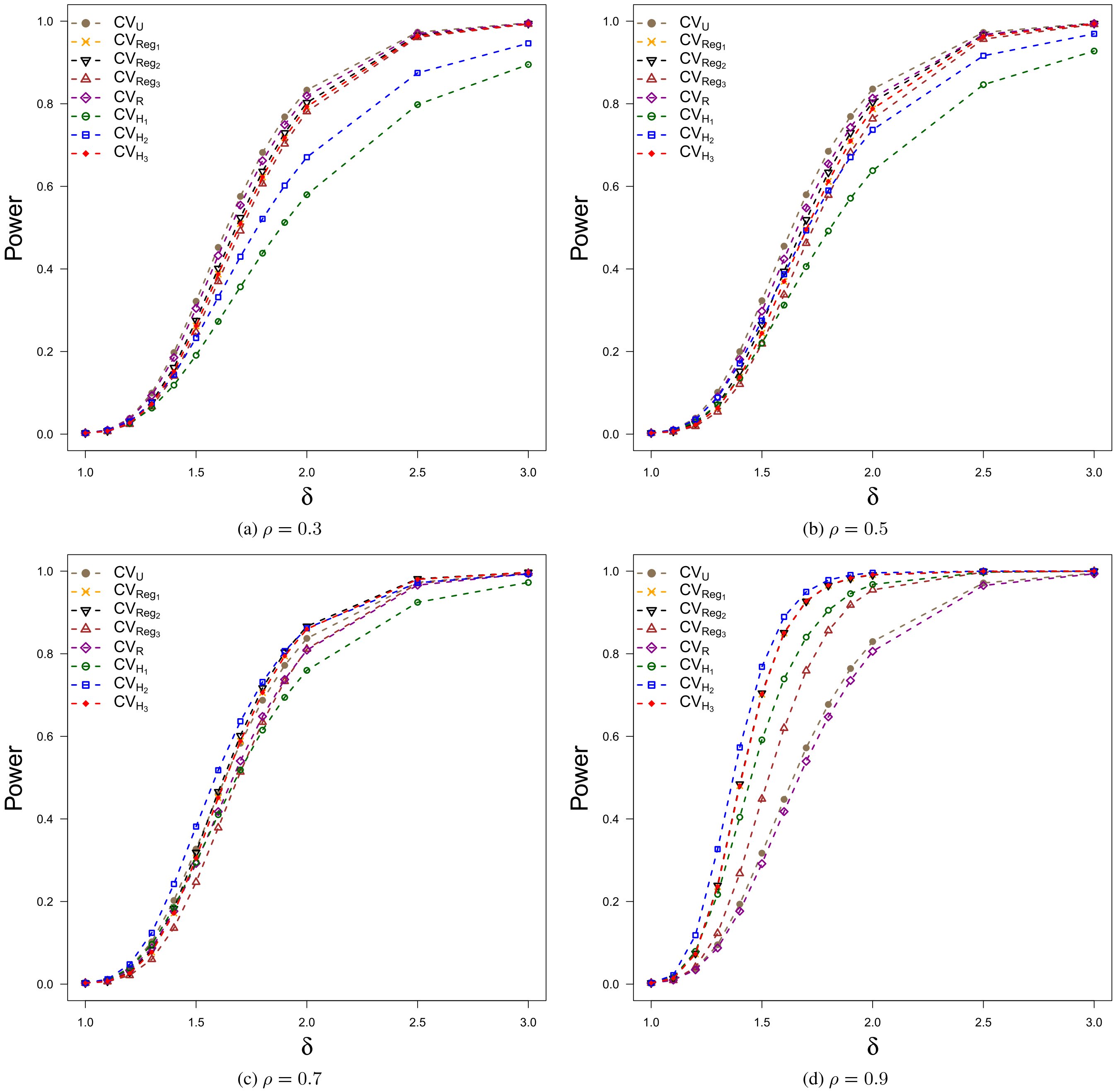


FIGURE 4. **Power comparison of different auxiliary information based CV Charts when $n = 15$.**

- Using these estimated measures, the $CV_A(\forall A = U, Reg_1, Reg_2, Reg_3, R, H_1, H_2, H_3)$ estimators are computed.
- Constants d_2 and d_3 are obtained as the mean and standard deviation of these one million CV_A estimates, and are provided in Tables 1-2, respectively, for varying levels of n and ρ.
- Moreover, $V_{A,\alpha/2}$ and $V_{A,(1-\alpha/2)}$ are obtained as the $(\alpha/2)$ and $(1 - \alpha/2)$ quantile points of the distributions of corresponding CV_A estimates, and are provided in Tables 3-4, respectively, for varying levels of n and ρ.

After finding the control chart constants and the probability points for the CV_A charts, I firstly compared the efficiency of the different CV estimators, as described below:

A. EFFICIENCY COMPARISON

The relative efficiency (RE) of the different CV estimators is computed for evaluating their precision following Rousseeuw

TABLE 6. Power of detection for the different CV charts, considering both positive and negative shifts, when $n = 10$ and $\rho = 0.90$.

δ	CV_U	CV_{Reg_1}	CV_{Reg_2}	CV_{Reg_3}	CV_R	CV_{H_1}	CV_{H_2}	CV_{H_3}
0.1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.2	0.9996	1.0000	1.0000	0.9998	0.9990	1.0000	1.0000	1.0000
0.3	0.8595	0.9989	0.9986	0.9925	0.8387	0.9963	1.0000	0.9989
0.4	0.4275	0.9027	0.8598	0.8027	0.4117	0.7432	0.9970	0.9027
0.5	0.1558	0.3904	0.3189	0.3572	0.1508	0.2859	0.8098	0.3904
0.6	0.0531	0.0914	0.0719	0.1060	0.0523	0.0837	0.3512	0.0914
0.7	0.0186	0.0223	0.0185	0.0291	0.0183	0.0252	0.0958	0.0223
0.8	0.0074	0.0072	0.0063	0.0087	0.0073	0.0085	0.0226	0.0072
0.9	0.0032	0.0030	0.0030	0.0036	0.0033	0.0032	0.0052	0.0030
1.0	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
1.1	0.0083	0.0075	0.0078	0.0042	0.0077	0.0133	0.0153	0.0075
1.2	0.0262	0.0275	0.0314	0.0103	0.0240	0.0556	0.0740	0.0275
1.3	0.0632	0.0839	0.0960	0.0273	0.0565	0.1466	0.2033	0.0839
1.4	0.1225	0.2001	0.2223	0.0654	0.1088	0.2796	0.3829	0.2001
1.5	0.2025	0.3638	0.3922	0.1330	0.1818	0.4276	0.5659	0.3638
1.6	0.2959	0.5388	0.5671	0.2275	0.2676	0.5662	0.7189	0.5388
1.7	0.3906	0.6901	0.7124	0.3434	0.3580	0.6839	0.8269	0.6901
1.8	0.4850	0.7996	0.8150	0.4632	0.4490	0.7744	0.8993	0.7996
1.9	0.5727	0.8733	0.8831	0.5790	0.5368	0.8412	0.9422	0.8733
2.0	0.6490	0.9193	0.9245	0.6789	0.6141	0.8894	0.9675	0.9193

TABLE 7. Comparison of proposed auxiliary information based CV charts with CV_U, CV_{EWMA} and ICV_{EWMA} charts, using $n = 5, 10, 15$ and $\gamma = 0.1$.

n	δ	CV_U	CV_{EWMA}	ICV_{EWMA}	CV_{Reg_2}	CV_R	CV_{H_1}	CV_{H_2}
5	1.0	370.41	370.14	370.38	370.37	370.37	370.37	370.37
	1.1	163.05	57.25	65.10	242.13	175.44	78.31	73.48
	1.2	66.28	19.00	20.36	128.04	75.36	20.24	16.13
	1.3	32.43	9.86	10.37	61.50	36.28	8.08	6.00
	1.4	17.82	6.37	6.58	29.73	20.89	4.29	3.11
	1.5	10.95	4.64	4.73	14.84	12.97	2.80	2.04
	2.0	3.33	2.00	2.02	1.72	3.33	1.22	1.06
	3.0	1.37	1.22	1.23	1.04	1.43	1.01	1.00
10	1.0	370.65	370.66	370.48	370.37	370.37	370.37	370.37
	1.1	122.91	31.40	32.71	87.11	132.63	58.14	42.43
	1.2	38.63	9.93	10.05	16.16	42.25	11.56	7.19
	1.3	16.05	5.24	5.35	4.56	18.02	4.25	2.64
	1.4	8.27	3.47	3.49	2.12	9.32	2.31	1.55
	1.5	4.87	2.57	2.58	1.40	5.57	1.61	1.20
	2.0	1.62	1.29	1.29	1.01	1.64	1.03	1.00
	3.0	1.04	1.02	1.02	1.00	1.05	1.00	1.00
15	1.0	371.19	370.41	370.29	370.37	370.37	370.37	370.37
	1.1	98.96	21.86	22.30	40.93	103.20	41.63	26.44
	1.2	26.18	6.95	7.01	5.76	29.28	7.33	4.13
	1.3	9.96	3.73	3.76	1.93	11.60	2.74	1.69
	1.4	5.01	2.52	3.02	1.23	5.74	1.62	1.17
	1.5	3.12	1.91	2.52	1.06	3.47	1.24	1.04
	2.0	1.20	1.11	2.19	1.00	1.24	1.00	1.00
	3.0	1.01	1.00	1.92	1.00	1.01	1.00	1.00

and Croux [35] and Abbasi [36]. RE is defined as the ratio of minimum standardized variance (SV_{min}) to the standardized variance (SV) of a specific estimator. Mathematically, we can define RE as:

$$RE_{CV_A} = \frac{min(SV_{CV_A})}{SV_{CV_A}} \times 100 \tag{11}$$

TABLE 8. Comparison of proposed auxilairy information based CV charts with CV_{sync} chart, using $n = 5, 10, 15$ and $\gamma = 0.1$.

n	δ	CV_{sync}	CV_{Reg_2}	CV_R	CV_{H_1}	CV_{H_2}
5	1.0	370.40	370.37	370.37	370.37	370.37
	1.5	*5.85*	14.84	12.97	2.80	2.04
	2.0	2.11	1.72	3.33	1.22	1.06
	2.5	1.46	1.12	1.87	1.05	1.01
	3.0	1.24	1.04	1.43	1.01	1.00
10	1.0	370.40	370.37	370.37	370.37	370.37
	1.5	*2.76*	1.40	5.57	1.61	1.20
	2.0	1.27	1.01	1.64	1.03	1.00
	2.5	1.07	1.00	1.16	1.00	1.00
	3.0	1.02	1.00	1.05	1.00	1.00
15	1.0	370.40	370.37	370.37	370.37	370.37
	1.5	*1.89*	1.06	3.47	1.24	1.04
	2.0	1.08	1.00	1.24	1.00	1.00
	2.5	1.01	1.00	1.04	1.00	1.00
	3.0	1.00	1.00	1.01	1.00	1.00

where SV_{CV_A} is defined as:

$$SV_{CV_A} = \frac{n\, var(CV_A)}{[ave(CV_A)]^2} \quad (12)$$

Relative efficiency of the different estimators is graphically compared in Figure 1, using one million samples of size $n = 5, 7, 12, 15$ considering $\rho = 0.3, 0.5, 0.7$ and 0.9. In each plot, relative efficiency is plotted on y-axis and the sample size on x-axis. The plots indicate the following: For low correlation levels (i.e. $\rho = 0.3$ and 0.5):

- The best best way of estimating CV is by using the usual estimator (i.e. CV_U).
- CV_R estimator is also maintaining high relative efficiency.
- CV_{H_1} estimator is the least efficient estimator.

As the correlation between study and auxiliary variables increase, the auxiliary information based CV estimators are becoming more efficient as compared to the usual CV estimator.

At moderate to high correlation levels:

- The usual CV estimator quickly looses its efficiency, compared to the auxiliary information based CV estimators.
- The most efficient way of estimating CV is by using the CV_{H_2} estimator.
- At large sample sizes, the CV_{H_3} and CV_{Reg_3} estimators are also performing relatively well.

After the comparison of the relative efficiency, the performance of the charts is evaluated as the power of detecting shifts in process CV. The power of the charts is computed using the following procedure:

- One million samples of size n are generated from $N_2(\mu_0, \delta\gamma_0\Sigma_0)$, where δ represent the shift in the process CV level.
- The CV_A estimates are computed for each sample and are plotted against the respective probability limits.
- The detection power of the charts is computed as the proportion of sample CVs plotted outside the probability limits.

The chart with the higher power of detection will be considered better than the competing charts. The power of detection is computed for all the CV_A charts considering varying levels of design parameters n, δ and ρ. To save space, the power results using $n = 10$ at varying levels of ρ are provided in Table 5. For other combinations of n and ρ, the results are presented graphically in Figures 2-4. The comparison indicates that:

- The power of detection increases with an increase in the level of δ for all the charts.
- The detection ability of the auxiliary information based charts increases with an increase in the level of ρ.
- At low correlation levels (i.e. $\rho = 0.3$), the usual CV chart based on CV_U is the best performing chart.
- As ρ increases, the performance of the CV charts based on auxiliary information gets better and better, as compared to the CV_U chart.
- At moderate to high correlation levels, the CV_{H_2} chart is the best performing chart, considering the different sample sizes.
- The CV_{H_1} and CV_{Reg_2} are the second best choices for small and large sample sizes, respectively, at high correlation levels.
- The comparative performance of the CV_{H_1} chart deteriorates significantly for large sample sizes, particularly at low correlation levels.

Detection of positive shifts in process CV are usually of more interest, as it indicates a reduction in quality of process/products. On the other hand, detection of negative shifts in process CV are also important as it reflects improvement in process or quality of products. Table 6 reports the power of detection for all the charts considering positive and negative shifts when $\rho = 0.90$ and $n = 10$. I can observe from Table 6 that:

- The detection power of almost all the proposed charts is greater for the detection of negative shifts, compared to the detection of positive shifts of same magnitude.
- The CV_{H2} is the best performing chart for the detection of both negative and positive shifts process CV.
- For the detection of negative shifts, the CV_{H_3} and CV_{Reg_1} charts show better detection ability, compared to the CV_{H_1}, CV_{Reg_2} and CV_{Reg_3} charts.
- The CV_R and CV_U charts are the worst performing charts for the detection of negative shifts.

The power comparisons in Figures 2 - 4 and Table 6 advocates that the CV_{H_2} chart is the best performing chart at moderate to high correlation between the study and the auxiliary variable.

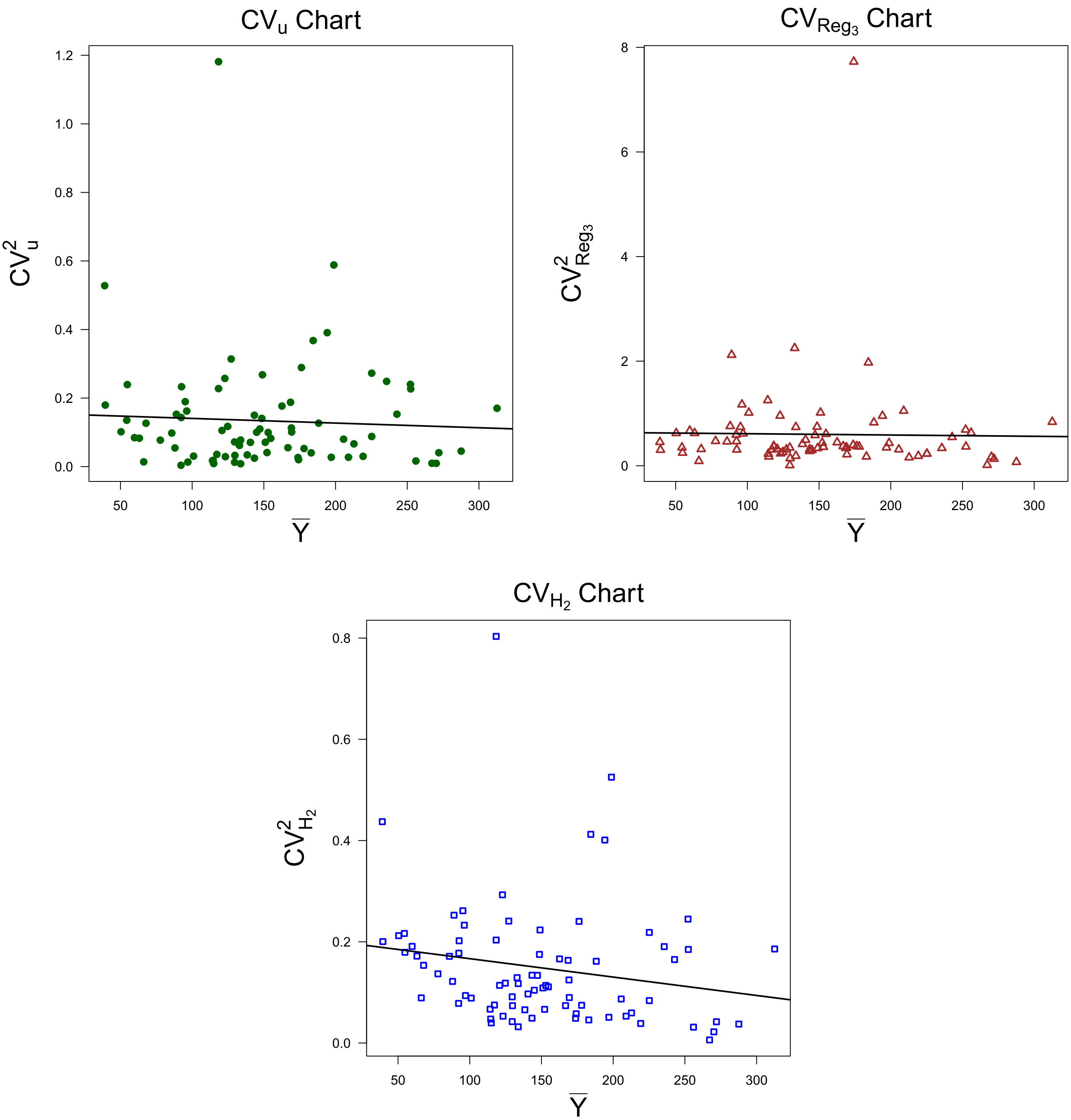


FIGURE 5. $\hat{\gamma}_A{}^2$ versus $\bar{Y}$ for the CV_U, CV_{Reg_3} and CV_{H_2} charts.

V. COMPARISON WITH EXISTING CHARTS

In this section the performance of the auxiliary information based CV charts is compared with some existing CV charts, proposed in SPC literature. This will highlight the benefit of using auxiliary information for efficient monitoring of process CV. Specifically, the performance of proposed charts is compared with the CV_U chart proposed by Kang *et al.* [3], the CV_{EWMA} chart proposed by Hong *et al.* [37], the synthetic CV (CV_{sync}) control chart proposed by Calzada and Scariano [9], and the improved CV-EWMA (ICV_{EWMA}) chart proposed by Park *et al.* [38]. Average run length is used as a performance measure for the comparison of these charts. For a fair comparison, in-control ARL (ARL_0) of all the charts are kept at a fixed level of 370 using $\gamma = 0.1$ and $n = 5, 10, 15$. At a fix ARL_0, the chart with the smallest out-of-control ARL (ARL_1) will be considered better than

TABLE 9. Regression test results for the Air Quality data.

	Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F-value	p-value
CV_U	Model	1	0.0057	0.0057	0.2087	0.6490
	Error	78	2.1446	0.0275		
	Total	79	2.1503			
CV_{Reg_3}	Model	1	0.0189	0.0189	0.0227	0.8806
	Error	78	65.0310	0.8337		
	Total	79	65.0499			
CV_{H_2}	Model	1	0.0410	0.0410	2.7669	0.1002
	Error	78	1.1556	0.0148		
	Total	79	1.1966			

other charts. Table 7 reports the average run length comparison of the proposed CV charts with CV_U, CV_{EWMA} and ICV_{EWMA} charts whereas Table 8 provides comparison with CV_{sync} chart. The comparison reveals the following:

- The proposed CV_{H_1} and CV_{H_2} are performing better than the existing CV_U, CV_{EWMA} and ICV_{EWMA} for all choice of n with $\delta \geq 1.2$.
- The proposed CV_{H_1} and CV_{H_2} charts are performing better than the CV_{sync} chart at all levels of n and δ.
- The proposed CV_{Reg_2} chart is better than the CV_{sync} chart for $\delta \geq 2$.

From the comparisons, I revealed that the proposed CV_{H_1} and CV_{H_2} charts are performing better than all the other competing charts. Moreover, it is to be noted that I am using Shewhart structure for my proposed charts, in this study. EWMA or CUSUM versions of these charts will be even more efficient, for the detection of shifts in process CV.

VI. ILLUSTRATIVE EXAMPLE

In this section, I will present an illustrative example using a real data set on air quality to show the application of auxiliary information based CV charts. Atmosphere is a thick layer of air that exists around the globe. The air pollution directly and indirectly effects the environment through different sources, such as: i) combustion of fossil fuels, like coal and oil for electricity and road transport that produce nitrogen and sulfur dioxide, ii) the release of the heavy amount of carbon monoxide, hydrocarbon, chemicals and organic compounds into the atmosphere by industries and factories iii) the emission of excessive use of pesticides, insecticides, and fertilizers at the agriculture lands etc.

In result, the polluted air seriously affects the human health for example, Carbon Monoxide (CO) reduces the amount of hemoglobin in blood; Nitrogen Oxide effects the respiratory system; Sulfur Dioxide SO2 creates smog and cause of global warming; Lead (Pb) may cause neurological damages including nervous system failures, cardiovascular dysfunctions and skin problems. In such situations, it is necessary to monitor and control the pollution levels in air, to improve its quality. For the illustration of this study, I consider a real-life Air Quality data that contains average responses recorded on hourly basis from an array of 5 metal oxide chemical sensors embedded in an air quality chemical multi-sensor device. Originally, this data set contains 9358 observations but after discarding missing values (that are tagged with the value −200), I considered 827 valid observations. The data set consists of 13 process variables, out of which I considered the true hourly averaged tungsten oxide (NO_x) concentration (in ppb) as the study variable (Y) and the true hourly averaged tungsten oxide (NO_2) concentration (in $\mu g/m^3$) as the auxiliary variable (X). Further details about other variables of the data set may be seen in De *et al.* [39] or on the website: (cf.*https* : //*archive.ics.uci.edu*/*ml*/*datasets*/*Air* + *quality*).

The overall estimates for this data are considered as the process parameters. I found out that $\mu_y = 143.502$; $\mu_x = 100.26$; $\gamma_x = 0.21788$; $\gamma_y = 0.37817$; and $\rho = 0.8574$. Moreover, the 827 observations are distributed in to 165 samples of size 5. Based on these process parameters, the control limits for all the CV_A charts are computed to fix the false alarm rate $\alpha = 0.0027$. From the set of these 165 samples, 80 samples are selected at random to represent the in-control process. As recommended by [3] and [5], the first step should be to check the constancy of CV and suitability of the proposed charts. For the illustrative example, I will use the three CV charts, namely, CV_U, CV_{Reg_3} and CV_{H_2} charts. I plotted $\hat{\gamma_A}^2$ against the sample means using all the three CV estimators and the plots are provided in Figure 5. All the plots in Figure 5 show that the CV is constant against the means, considering the three estimators (CV_U, CV_{Reg_3} and CV_{H_2}). Moreover, to test it formally, the regression results are provided in Table 9. The reported p-values are all greater than 0.05, indicating the independence of CV and mean levels.

For fixing the false alarm rate $\alpha = 0.0027$ for the three charts, the selected control limits are (0.161, 2.673) for the CV_U chart, (0.202, 4.742) for the CV_{Reg_3} chart and (0.295, 2.429) for the CV_{H_2} chart. Further, to check the detection ability of the three charts, shift of magnitude $\delta = 2.5$ is introduced in the next 20 random selected samples. The plotting statistics for the three charts are reported in Table 10. These statistics are plotted against their respective control limits in Figure 6. The plots show that the CV_U detects 5 (at sample points 85-87, 91, 92 and 98), the CV_{Reg_3} detects 2 (at sample points 81 and 97) whereas the CV_{H_2} chart detects 10 (at sample points 83, 85-88, 91, 92, 94, 97 and 98) out-of-control

TABLE 10. Plotting Statistics of the CV_U, CV_{Reg_3} and CV_{H_2} charts for the Air Quality data.

Sample #	CV_U	CV_{Reg_3}	CV_{H_2}	Sample #	CV_U	CV_{Reg_3}	CV_{H_2}
1	2.0281	1.7427	1.9164	51	0.8588	1.4965	0.8926
2	0.3388	2.0989	0.4686	52	0.4157	1.4849	0.5859
3	0.3054	2.0733	0.8097	53	0.6215	1.6167	0.7189
4	0.7098	1.5545	0.7987	54	0.6575	3.9665	0.9502
5	0.7351	1.8166	0.9777	55	0.7840	1.2719	0.7658
6	1.2936	1.3178	1.1196	56	0.5357	1.7513	0.6823
7	0.6817	1.0548	0.6450	57	2.8739	1.6003	2.3701
8	0.2990	0.2490	0.5440	58	1.3806	1.2663	1.2359
9	1.2593	1.5984	1.1368	59	0.8884	1.5611	0.9336
10	1.4217	1.6140	1.2962	60	0.2591	1.0831	0.3936
11	0.7614	2.0898	1.0957	61	0.7687	2.1622	1.1553
12	1.0909	2.4204	1.1398	62	1.0240	1.3993	0.9679
13	0.5328	0.9591	0.5423	63	0.9415	2.4052	1.0627
14	0.8402	1.2319	0.7926	64	0.8432	2.0878	1.2179
15	1.1510	2.2767	1.3518	65	0.9407	1.4919	1.0361
16	0.6077	1.5980	0.7220	66	0.5301	1.1050	0.5650
17	0.8765	2.0213	0.9668	67	0.2627	0.3244	0.2071
18	1.1458	1.5398	1.0688	68	1.3187	1.5372	1.1540
19	1.3413	2.5858	1.4308	69	0.7034	1.8619	0.8233
20	0.7362	2.2750	0.9061	70	0.3112	0.7918	0.7898
21	0.4625	2.6627	0.7878	71	0.3506	2.9593	0.6842
22	0.8274	1.7989	1.0944	72	1.0647	2.8611	1.2759
23	0.6177	2.3005	0.9226	73	0.4584	1.1440	0.5196
24	0.2422	1.1454	0.4736	74	0.4998	1.4697	0.7245
25	0.3562	1.2634	0.5755	75	0.5622	0.7039	0.5115
26	0.9058	1.3510	0.9095	76	1.1206	1.4565	1.1838
27	1.2617	1.6270	1.1926	77	1.2953	2.1961	1.3085
28	1.6036	3.7145	1.6979	78	1.0344	1.9569	1.0738
29	0.1653	2.0373	0.7397	79	0.4311	1.6659	0.5852
30	1.4816	1.4735	1.2978	80	0.2503	1.1050	0.5276
31	0.9725	1.5631	1.2302	81	1.2007	6.8996	1.9899
32	1.9213	1.7805	1.7486	82	2.0284	2.1746	1.7467
33	1.3688	1.5343	1.2500	83	2.4944	2.8693	2.6224
34	0.4367	1.5600	0.5965	84	2.0105	3.1528	2.2479
35	0.4889	1.7022	0.6776	85	3.7994	3.7022	3.9647
36	0.7048	2.6647	0.8724	86	3.4645	2.9104	3.3320
37	0.8343	1.5856	0.8919	87	4.4499	2.8465	4.1119
38	1.6531	2.5829	1.6745	88	2.1997	2.4766	2.4698
39	1.2767	1.4664	1.1885	89	1.8315	2.2790	1.8227
40	0.7481	1.4764	0.7801	90	1.6778	1.9435	1.6502
41	0.9917	2.2824	1.1064	91	4.7958	3.0929	4.0691
42	1.1113	1.7688	1.0780	92	4.0631	4.1199	3.9571
43	0.8375	1.4451	0.8543	93	2.3232	2.2941	2.2088
44	0.3764	7.3496	0.6356	94	2.2982	3.0689	2.5396
45	0.4785	0.9991	0.7183	95	1.5760	2.4593	2.0161
46	0.4380	2.7091	0.6091	96	1.3373	2.1829	1.4843
47	0.7596	2.0604	0.8811	97	1.9429	7.1076	2.7799
48	1.0007	1.8089	1.1138	98	4.2805	3.0217	3.8725
49	0.4503	1.2823	0.6088	99	2.1717	1.9270	1.9653
50	1.0335	3.8503	1.3289	100	1.9543	2.7494	2.2697

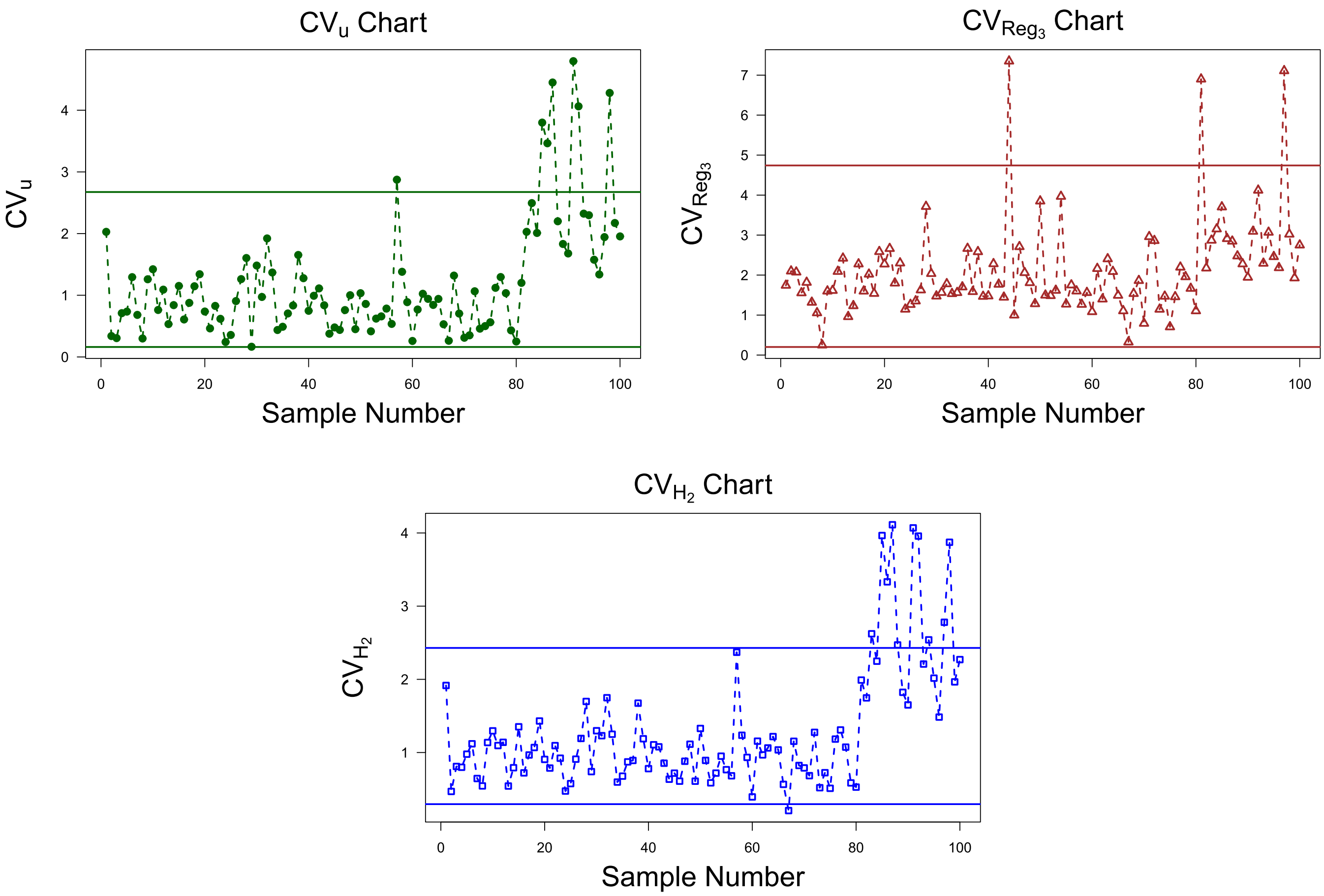


FIGURE 6. Control chart plots of the CV_U, CV_{Reg_3} and CV_{H_2} charts for the air quality data.

signals, after the occurrence of shift. This clearly shows the superiority of the CV_{H_2} chart over the other competing charts. This is also in accordance with the findings of Section IV.

VII. CONCLUSION

In this study, a set of auxiliary information based control charts are proposed for efficient monitoring of process CV. The auxiliary information is used in regression, ratio and hybrid forms. The performance of these charts is evaluated using power of detection as the performance measure. Further, the performance of auxiliary information based charts is compared with the CV_U, CV_{EWMA}, ICV_{EWMA} and CV_{sync} charts. It has been observed that the performance of a wide range of the auxiliary information based CV charts improve with an increase in the correlation between auxiliary and study variable. The newly proposed charts, based on auxiliary information, are significantly efficient than the competing charts, particularly for moderate to high correlation levels. The best performing chart is best on the H_2 auxiliary estimator (i.e. the CV_{H_2} chart). This study will help quality practitioners to choose an efficient control chart for the monitoring of process CV.

CV_{EWMA} and CV_{CUSUM} charts based on auxiliary information can be investigated as future research directions.

REFERENCES

[1] D. C. Montgomery, *Introduction to Statistical Quality Control*, 6th ed. New York, NY, USA: Wiley, 2009.
[2] S. A. Abbasi and A. Miller, "On proper choice of variability control chart for normal and non-normal processes," *Qual. Rel. Eng. Int.*, vol. 28, no. 3, pp. 279–296, Apr. 2012.
[3] C. W. Kang, M. S. Lee, Y. J. Seong, and D. M. Hawkins, "A control chart for the coefficient of variation," *J. Qual. Technol.*, vol. 39, no. 2, pp. 151–158, Apr. 2007.
[4] P. Castagliola, G. Celano, and S. Psarakis, "Monitoring the coefficient of variation using EWMA charts," *J. Qual. Technol.*, vol. 43, no. 3, pp. 249–265, Jul. 2011.
[5] S. A. Abbasi and N. A. Adegoke, "Multivariate coefficient of variation control charts in phase I of SPC," *Int. J. Adv. Manuf. Technol.*, vol. 99, nos. 5–8, pp. 1903–1916, Nov. 2018.
[6] H. D. Nguyen, Q. T. Nguyen, K. P. Tran, and D. P. Ho, "On the performance of VSI shewhart control chart for monitoring the coefficient of variation in the presence of measurement errors," *Int. J. Adv. Manuf. Technol.*, vol. 104, nos. 1–4, pp. 211–243, Sep. 2019.
[7] P. Castagliola, A. Achouri, H. Taleb, G. Celano, and S. Psarakis, "Monitoring the coefficient of variation using a variable sampling interval control chart," *Qual. Rel. Eng. Int.*, vol. 29, no. 8, pp. 1135–1149, 2012.
[8] W. C. Yeong, M. B. C. Khoo, L. K. Tham, W. L. Teoh, and M. A. Rahim, "Monitoring the coefficient of variation using a variable sampling interval EWMA chart," *J. Qual. Technol.*, vol. 49, no. 4, pp. 380–401, Oct. 2017.
[9] M. E. Calzada and S. M. Scariano, "A synthetic control chart for the coefficient of variation," *J. Stat. Comput. Simul.*, vol. 83, no. 5, pp. 853–867, May 2013.

[10] A. Amdouni, P. Castagliola, H. Taleb, and G. Celano, "Monitoring the coefficient of variation using a variable sample size control chart in short production runs," *Int. J. Adv. Manuf. Technol.*, vol. 81, nos. 1–4, pp. 1–14, Oct. 2015.

[11] U. Menzefricke, "Control charts for the variance and coefficient of variation based on their predictive distribution," *Commun. Statist.-Theory Methods*, vol. 39, no. 16, pp. 2930–2941, Aug. 2010.

[12] J. Zhang, Z. Li, B. Chen, and Z. Wang, "A new exponentially weighted moving average control chart for monitoring the coefficient of variation," *Comput. Ind. Eng.*, vol. 78, pp. 205–212, Dec. 2014.

[13] E. P. Hong, H. W. Kang, C. W. Kang, and J. W. Baik, "CV control chart using GWMA technique," *Adv. Mater. Res.*, vol. 337, pp. 247–254, Sep. 2011.

[14] L. Zhang and G. Chen, "An extended EWMA mean chart," *Qual. Technol. Quant. Manage.*, vol. 2, no. 1, pp. 39–52, Jan. 2005.

[15] A. B. A. Dawod, S. A. Abbasi, and M. Al-Momani, "On the performance of coefficient of variation control charts in phase I," *Qual. Rel. Eng. Int.*, vol. 34, no. 6, pp. 1029–1040, Oct. 2018.

[16] S. A. Abbasi, M. A. Mohamed, M. A. Ahmed, R. J. Lajara, and H. F. Hadi, "Monitoring coefficient of variation using progressive mean technique," in *Proc. 8th Int. Conf. Ind. Technol. Manage. (ICITM)*, 2019.

[17] R. Chen, Z. Li, and J. Zhang, "A generally weighted moving average control chart for monitoring the coefficient of variation," *Appl. Math. Model.*, vol. 70, pp. 190–205, Jun. 2019.

[18] X. Chew, M. B. C. Khoo, K. W. Khaw, W. C. Yeong, and Z. L. Chong, "A proposed variable parameter control chart for monitoring the multivariate coefficient of variation," *Qual. Rel. Eng. Int.*, vol. 35, no. 7, pp. 2442–2461, 2019.

[19] A. Haq and M. B. C. Khoo, "New adaptive EWMA control charts for monitoring univariate and multivariate coefficient of variation," *Comput. Ind. Eng.*, vol. 131, pp. 28–40, May 2019.

[20] S. A. Abbasi, T. Abbas, and N. A. Adegoke, "Efficient CV control charts based on ranked set sampling," *IEEE Access*, vol. 7, pp. 78050–78062, 2019.

[21] M. Riaz, "Monitoring process variability using auxiliary information," *Comput. Statist.*, vol. 23, no. 2, pp. 253–276, Apr. 2008.

[22] M. Riaz, R. Mehmood, S. Ahmad, and S. A. Abbasi, "On the performance of auxiliary-based control charting under normality and nonnormality with estimation effects," *Qual. Rel. Eng. Int.*, vol. 29, no. 8, pp. 1165–1179, Dec. 2013.

[23] S. Ahmad, S. A. Abbasi, M. Riaz, and N. Abbas, "On efficient use of auxiliary information for control charting in SPC," *Comput. Ind. Eng.*, vol. 67, pp. 173–184, Jan. 2014.

[24] N. Abbas, M. Riaz, and R. J. M. M. Does, "An EWMA-type control chart for monitoring the process mean using auxiliary information," *Commun. Statist.-Theory Methods*, vol. 43, no. 16, pp. 3485–3498, Aug. 2014.

[25] R. A. Sanusi, M. Riaz, N. A. Adegoke, and M. Xie, "An EWMA monitoring scheme with a single auxiliary variable for industrial processes," *Comput. Ind. Eng.*, vol. 114, pp. 1–10, Dec. 2017.

[26] N. A. Adegoke, A. N. H. Smith, M. J. Anderson, R. A. Sanusi, and M. D. M. Pawley, "Efficient homogeneously weighted moving average chart for monitoring process mean using an auxiliary variable," *IEEE Access*, vol. 7, pp. 94021–94032, 2019.

[27] N. A. Adegoke, A. N. H. Smith, M. J. Anderson, S. A. Abbasi, and M. D. M. Pawley, "Shrinkage estimates of covariance matrices to improve the performance of multivariate cumulative sum control charts," *Comput. Ind. Eng.*, vol. 117, pp. 207–216, Mar. 2018.

[28] S. A. Abbasi and M. Riaz, "On dual use of auxiliary information for efficient monitoring," *Qual. Rel. Eng. Int.*, vol. 32, no. 2, pp. 705–714, Mar. 2016.

[29] S. Hussain, L.-X. Song, S. Ahmad, and M. Riaz, "A new auxiliary information based cumulative sum median control chart for location monitoring," *Frontiers Inf. Technol. Electron. Eng.*, vol. 20, no. 4, pp. 554–570, Apr. 2019.

[30] M. Riaz, R. Mehmood, N. Abbas, and S. A. Abbasi, "On effective dual use of auxiliary information in variability control charts," *Qual. Rel. Eng. Int.*, vol. 32, no. 4, pp. 1417–1443, Jun. 2016.

[31] A. Das and T. Tripathi, "A class of estimators for coefficient of variation using knowledge on coefficient of variation of an auxiliary character," presented at the Annu. Conf. Ind. Soc. Agricult. Statist., New Delhi, India, 1981.

[32] T. P. Tripathi, H. P. Singh, and L. N. Upadhyaya, "A general method of estimation and its application to the estimation of co-efficient of variation," *Statist. Transition*, vol. 5, no. 6, pp. 887–909, 2002.

[33] V. Archana and K. A. Rao, "Ratio estimators for the co-efficient of variation in a finite population," *Pakistan J. Statist. Oper. Res.*, vol. 7, no. 2, pp. 265–281, 2011.

[34] V. Archaana and K. A. Rao, "Some improved estimators of co-efficient of variation from bi-variate normal distribution: A Monte Carlo comparison," *Pakistan J. Statist. Oper. Res.*, vol. 10, no. 1, pp. 87–105, 2014.

[35] P. J. Rousseeuw and C. Croux, "Alternatives to the median absolute deviation," *J. Amer. Stat. Assoc.*, vol. 88, no. 424, pp. 1273–1283, Dec. 1993.

[36] S. A. Abbasi, "Enhancements to control charts for monitoring process dispersion and location," Ph.D. dissertation, Univ. Auckland, Auckland, New Zealand, 2014.

[37] E. Hong, C. Kang, J. Baek, and H. Kang, "Development of CV control chart using EWMA technique," *J. Soc. Korea Ind. Syst. Eng.*, vol. 31, no. 4, pp. 114–120, 2008.

[38] Y. S. Park, E. P. Hong, K. Y. Park, and W. H. Shon, "Performance improvement study of CV-EWMA control chart to detect small shifts of CV," *Adv. Mater. Res.*, vol. 1051, pp. 1016–1022, Oct. 2014.

[39] S. De Vito, E. Massera, M. Piga, L. Martinotto, and G. Di Francia, "On field calibration of an electronic nose for benzene estimation in an urban pollution monitoring scenario," *Sens. Actuators B, Chem.*, vol. 129, no. 2, pp. 750–757, Feb. 2008.

SADDAM AKBAR ABBASI received the Ph.D. degree in statistics from The University of Auckland, New Zealand, in 2013. He served as an Assistant Professor at the King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia, for three years. He is currently an Associate Professor with the Department of Mathematics, Statistics and Physics, Qatar University, Doha, Qatar. His research interests include SPC, time series analysis, profile monitoring, and non-parametric statistics.