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Research Article

Improved Nonparametric Control Chart Based on Ranked Set Sampling with Application of Chemical Data Modelling

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The statistical performance of parametric control charts is questionable when the underlying process does not follow any specified probability distribution. Nonparametric control charts are the best substitute for this situation. On the other hand, the ranked set sampling technique is preferred over the simple random sampling technique because it reduces the variability of process parameters and improves the control chart's performance. This study aims to offer a nonparametric double homogeneously weighted moving average control chart under Wilcoxon signed-rank test considering the ranked set sampling technique (regarded as $\text{NPDHWMA}_{\text{RSS}}$), to further enhance the process location monitoring. The proposed chart's run-length performance is compared with competing control charts, such as DEWMA-$\overline{X}$, NPDEWMA-SR, NPRDEWMA-SR, DHWMA, and $\text{NPHWMA}_{\text{RSS}}$ control charts. The comparison revealed that the proposed $\text{NPDHWMA}_{\text{RSS}}$ control chart outperformed the other competing control charts, particularly for small to moderate shifts in process location. Finally, a real-life application is also offered for quality practitioners to show the strength of the proposed control chart.

1. Introduction

A quality assurance system offers a variety of management approaches that save time and money while producing a high-quality final product. These approaches are commonly utilized in the manufacturing process to detect irregularities and improve product quality. Statistical process control (SPC) is a key aspect in detecting abnormalities of ultimate products. SPC techniques are widely used in industrial applications, biological sciences, environmental studies, and healthcare departments to monitor the ongoing processes. The quality of the products is influenced by unnatural variation. The existence of unnatural variations causes the shift in process parameters (location and/or dispersion). Control charts are popular tools in SPC that helps in identifying the shifts in process parameters. Usually, control charts are generally classified as memoryless and memory-type charts based on their design structure. Shewhart [1] introduced the first memoryless chart known as the Shewhart chart, whereas Page [2] and Roberts [3] introduced the concept of memory charts, known as a cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) charts, respectively.

Classical parametric control charts are usually used when an ongoing process follows a predefined probability distribution. The ongoing process may not follow a specific distribution, or the distribution of the ongoing process may be in doubt. Nonparametric (NP) control charts are a reliable substitute for parametric control charts to handle such scenarios. NP control charts are convenient because their in-control (IC) run-length (RL) distribution is the same for all continuous distributions. The sign (SN) and Wilcoxon signed-rank (SR) are two well-known NP techniques practiced in statistical process monitoring (SPM) with control charts. Likewise, simple random sampling (SRS) and ranked set sampling (RSS) techniques are quite often used in SPM with both parametric and NP control charts to observe the data of the ongoing processes [4]. RSS is recommended in the SPM literature as it decreases variability and improves the efficiency of the associated control charts [5, 6]. For instance, hazardous waste sites with varying levels of contamination can be classified visually based on soil staining, whereas actual statistics of toxic chemicals and quantification of their ecological impact are prohibitively expensive.

Numerous researchers introduced a wide range of NP control charts using different NP statistics. Amin and Searcy [7] introduced an efficient NP EWMA-SR control chart to monitor the process location shift efficiently. Similarly, Bakir [8] presented NP Shewhart-SR control chart for process location monitoring. Subsequently, Yang, et al. [9] offered an EWMA-SN control chart for process location. Moreover, Graham, et al. [10] and Graham et al. [11] designed an NP EWMA-SN control chart based on a single observation and NP EWMA-SR control chart, respectively, for monitoring shifts in process location. Likewise, Lu [12] and Tsai et al. [13] presented enhanced NP EWMA control charts based on SN statistics and used RSS techniques in conjunction with NP control charts. Eventually, Chakraborty et al. [14] and Lu [15] developed a generally weighted moving average-SR (GWMA-SR) and SN statistic-based NP double GWMA control charts for process proportion, respectively, to improve the shift detection ability. Furthermore, Chakraborti and Graham [16] reviewed some latest development in both univariate and multivariate NP control charts. Also, Rasheed et al. [17] and Rasheed et al. [18] advocated RSS-based parametric and NP control charts for efficiently monitoring the process mean, respectively.

The recently introduced homogeneously weighted moving average (HWMA) control chart by Abbas [19] is efficient for monitoring process location. Adegoke et al. [20] provided an auxiliary information-based (AIB) HWMA control chart for process location more efficient than the HWMA control chart. Later, Anwar et al. [21] extended the AIB-HWMA and suggested the AIB-DHWMA control chart for improved process location monitoring. Based on a thorough literature review, it is observed that no one has developed an NP DHWMA control chart under SR along with RSS methodology to date. This is a research gap that needs to be explored. So this study proposes an NP DHWMA-SR control chart under RSS ($\text{NPDHWMA}_{\text{RSS}}$) for monitoring shifts in process location for continuous and symmetric distribution. The study's main goal is to propose a control chart for detecting small and moderate shifts in process location more quickly because small changes in process parameters can have a significant financial impact on an organization's operations. The run-length (RL) characteristics of the proposed control chart, including average run length (ARL), median run length (MDRL), and standard deviation of run-length (SDRL), are obtained using various distributions like normal, student's t, contaminated normal (CN), Laplace, and logistic distributions. The performance of proposed $\text{NPDHWMA}_{\text{RSS}}$ control chart is decided by providing a valid comparison with other existing control charts such as DEWMA-$\overline{X}$, NPDEWMA-SR, NPRDEWMA-SR, DHWMA, and $\text{NPHWMA}_{\text{RSS}}$ control charts.

The rest of the paper is organized as follows: Section 2 presents the design structure of the competing and the proposed control chart. Similarly, Section 3 describes the proposed control chart's IC and out-of-control (OOC) performance. Likewise, Section 4 contains a comparative study of the proposed control chart, whereas Section 5 provides a real-life application. Finally, concluding remarks are given in Section 6.

2. Competing and Proposed Control Charts

This section explains the design structure of the competing and the proposed control charts. These competing control charts are DEWMA-$\overline{X}$, NPDEWMA-SR, NPRDEWMA-SR, DHWMA, and $\text{NPHWMA}_{\text{RSS}}$. More detail is provided in the following subsections.

2.1. NPRDEWMA-SR Control Chart. Abbas et al. [4] presented an NP double EWMA SR under the RSS (NPRDEWMA-SR) control chart that outperforms the NPREWMA-SR control chart in terms of shift detection in process location. The plotting statistics of the NPRDEWMA-SR control chart are given as follows:

$$\left.\begin{aligned} E_{(SR_{RSS})t} &= \lambda SR_{(RSS)t} + (1-\lambda)E_{(SR_{RSS})t-1} \\ DE_{(SR_{RSS})t} &= \lambda E_{(SR_{RSS})t} + (1-\lambda)DE_{(SR_{RSS})t-1} \end{aligned}\right\}, \tag{1}$$

where $\lambda \in (0, 1]$ is a smoothing constant. The control limits of the NPRDEWMA-SR control chart can be designed as follows:

$$\left.\begin{aligned} LCL_{(\text{NPRDEWMA-SR})t} &= -L\sqrt{\left(\frac{r(r+1)(2r+1)}{6}\varpi_0^2\right)\lambda^4\frac{1+\lambda^2-\left(t^2+2t+1\right)\lambda^{2t}+\left(2t^2+2t-1\right)\lambda^{2t+2}-t^2}{\left(1-\lambda^2\right)^3}} \\ UCL_{(\text{NPRDEWMA-SR})t} &= +L\sqrt{\left(\frac{r(r+1)(2r+1)}{6}\varpi_0^2\right)\lambda^4\frac{1+\lambda^2-\left(t^2+2t+1\right)\lambda^{2t}+\left(2t^2+2t-1\right)\lambda^{2t+2}-t^2}{\left(1-\lambda^2\right)^3}} \end{aligned}\right\}. \tag{2}$$

The process goes OOC when $DE_{(SR_{RSS})t} > UCL$ (NPRDEWMA − SR)t or $DE_{(SR_{RSS})t} < LCL_{(\text{NPRDEWMA-SR})t}$; otherwise, it will remain an IC state.

2.2. DHWMA Control Chart. Abid et al. [22] developed a DHWMA control chart that detects shifts more efficiently than the HWMA control chart. The DHWMA control chart plotting statistic is given as follows:

$$DH_t = \lambda^2\overline{X}_t + \left(1-\lambda^2\right)\overline{\overline{X}}_{t-1}, \tag{3}$$

where $\overline{X}_t$ and $\overline{\overline{X}}_{t-1}$ are the mean of t^{th} and mean of the previous $t-1$ samples, respectively. The control limits of the DHWMA control chart based on this $E(DH_t)$ and $\operatorname{Var}(DH_t)$ are defined as follows:

$$\left.\begin{aligned} LCL_{(DH\,WMA)t} &= \begin{cases} \mu_0 - L\sqrt{\left(\lambda^4\sigma_0^2/n\right)}, & \text{if } t=1 \\ \mu_0 - L\sqrt{\left(\lambda^4\sigma_0^2/n\right) + (1-\lambda)^2(1+\lambda)^2\left(\sigma_0^2/n(t-1)\right)}, & \text{if } t>1 \end{cases} \\ CL_{(DH\,WMA)t} &= \mu_0 \\ UCL_{(DH\,WMA)t} &= \begin{cases} \mu_0 + L\sqrt{\left(\lambda^4\sigma_0^2/n\right)}, & \text{if } t=1 \\ \mu_0 + L\sqrt{\left(\lambda^4\sigma_0^2/n\right) + (1-\lambda)^2(1+\lambda)^2\left(\sigma_0^2/n(t-1)\right)}, & \text{if } t>1 \end{cases} \end{aligned}\right\}. \tag{4}$$

The process remain is IC if $LCL_{(DH\,WMA)t} < DH_t < UCL_{(DHWMA)t}$; otherwise, it goes OOC.

2.3. Proposed NPDHWMA$_{RSS}$ Control Chart. Various researchers like Kim and Kim [23], Abid et al. [24], and Abbas et al. [4] used the following RSS-based Wilcoxon signed-rank statistic to monitor shifts in process location:

$$SR_{RSS_t} = \sum_{j=1}^{n}\sum_{h=1}^{m}\operatorname{sign}\left(X_{tj(h)} - \theta_0\right)R^{+}_{tj(h)}, \tag{5}$$

where θ_0 symbolizes the process median, and t, j, and h denote the number of samples, observations, and cycles used in the RSS approach, respectively. The mean and variance of SR_{RSS_t} statistic are $E(SR_{RSS_t}) = 0$ and $\operatorname{Var}(SR_{RSS_t}) = (r(r+1)(2r+1)/6)\varpi_0^2$, respectively, where r denotes the number of replications and can be defined as $r = nm$. The quantity ϖ_0^2 is used to improve the efficiency of the control chart and can be defined as $\varpi_0^2 = 1 - (4/n)\sum_{j=1}^{n}(F_k(0) - (1/2))^2$. The values of $F_k(0)$ can be obtained by solving the following mathematical expression:

$$F_k(0) = \frac{r!}{(j-1)!(r-j)!}\int_{-\infty}^{0} F(t)^{j-1}(1-F(t))^{r-j} f(t)\mathrm{d}t. \tag{6}$$

Abid et al. [24] have more information on the RSS approach and related terms. The methodology of the proposed NPDHWMA$_{RSS}$ control chart is defined as follows:

$$\left.\begin{aligned} NPH_t &= \lambda SR_{RSS_t} + (1-\lambda)\overline{SR}_{(RSS)t-1} \\ NPDH_t &= \lambda NPH_t + (1-\lambda)\overline{SR}_{(RSS)t-1} \end{aligned}\right\}, \tag{7}$$

where $\overline{SR}_{(RSS)t-1}$ is the mean of $SR_{(RSS)}$ of $t-1$ samples. The simplified form of the plotting statistic $NP\,DH_t$ is

$$\begin{aligned} NPDH_t &= \lambda^2 SR_{RSS_t} + \left(1-\lambda^2\right)\overline{SR}_{(RSS)t-1}, \\ NPDH_0 &= 0. \end{aligned} \tag{8}$$

The control limits of the proposed NPDHWMA$_{RSS}$ control chart are

$$\left.\begin{aligned} LCL_{(\mathrm{NPDHWMA_{RSS}})t} &= \begin{cases} \mu_0 - L\sqrt{\lambda^4 (r(r+1)(2r+1)/6)\varpi_0^2}, & \text{if } t=1 \\ \mu_0 - L\sqrt{\left(\lambda^4 + \left((1-\lambda)^2(1+\lambda)^2/t-1\right)\right)((r(r+1)(2r+1)/6))\varpi_0^2}, & \text{if } t>1 \end{cases} \\ CL_{(\mathrm{NPDHWMA_{RSS}})t} &= \mu_0 \\ UCL_{(\mathrm{NPDHWMA_{RSS}})t} &= \begin{cases} \mu_0 + L\sqrt{\lambda^4 ((r(r+1)(2r+1)/6))\varpi_0^2}, & \text{if } t=1 \\ \mu_0 + L\sqrt{\left(\lambda^4 + \left((1-\lambda)^2(1+\lambda)^2/t-1\right)\right)((r(r+1)(2r+1)/6))\varpi_0^2}, & \text{if } t>1 \end{cases} \end{aligned}\right\}. \quad (9)$$

If $\mathrm{NPDH}_t > \mathrm{UCLD}_{(\mathrm{NPDHWMA_{RSS}})t}$ or $\mathrm{NPDH}_t < \mathrm{LCLD}_{(\mathrm{NPDHWMA_{RSS}})t}$, the underlying process is OOC; else, it is IC.

3. Implementation of the Proposed Control Chart

This section investigates performance metrics, the IC, and the OOC performances of the proposed $\mathrm{NPDHWMA_{RSS}}$ control chart for monitoring shifts in process location. Subsection 3.1 provides the performance metrics of the proposed $\mathrm{NPDHWMA_{RSS}}$ control chart. Likewise, the proposed control chart's robustness, IC, and OOC performance are presented in Subsection 3.2.

3.1. Performance Metrics. ARL is widely used to evaluate the performance of the control chart. The ARL is the expected number of sample points before the first OOC signal from the control chart. The ARL is categorized as IC ARL (ARL_0) and out-of-control ARL (ARL_1). If a process is functioning in an in-control state, the ARL_0 needed to be large enough to avoid frequent false alarms. However, the ARL_1 should be small enough; it quickly detects the shift. It is necessary for better performance of a control chart; it should have a smaller ARL_1 as compared to other control charts at the fixed value of ARL_0. In this study, we set ARL_0 to 370 and 500, with sample sizes (n) of 5 and 10. Monte Carlo simulations with 50,000 simulations in the R program are used to determine the RL characteristics. To examine the performance behavior of the proposed $\mathrm{NPDHWMA_{RSS}}$ control chart, various values of $\lambda \in (0.05, 0.10, 0.25, 0.50)$ and $\delta \in (0.025, 0.05, 0.075, 0.10, 0.25, 0.50, 0.75, 1.00, 1.50, 2.00, 2.50, 3.00, 5.00)$ are used. The following algorithm is used for simulations:

(i) To create samples from considered distributions, a finite loop is used.
(ii) Specify the process parameters (λ and L).
(iii) Draw a sample from a distribution used in this study.
(iv) Determine the NPDH_t plotting statistics using equation (7).
(v) Find $LCL_{(\mathrm{NPDHWMA_{RSS}})t}$ and $UCL_{(\mathrm{NPDHWMA_{RSS}})t}$ from equation (8).
(vi) Plot the plotting statistic $NPDH_t$ against $LCL_{(\mathrm{NPDHWMA_{RSS}})t}$ and $UCL_{(\mathrm{NPDHWMA_{RSS}})t}$ over t.
(vii) If $NPDH_t > UCL_{(\mathrm{NPDHWMA_{RSS}})t}$ or $NPDH_t < LCL_{(\mathrm{NPDHWMA_{RSS}})t}$, note this sample of $NPDH_t$ statistic as an RL. For instance, at $t = 105$, if $NPDH_t > UCL_{(\mathrm{NPDHWMA_{RSS}})t}$ or $NPDH_t < LCL_{(\mathrm{NPDHWMA_{RSS}})t}$, record 105 as a first RL.
(viii) Repeat steps (ii) through (vi) for 50,000 times and record RLs.
(ix) Calculate ARL_0 from 50,000 and recorded RLs.
(x) $\mathrm{ARL}_0 = 100$; if not, adjust constant accordingly in step (ii) and repeat from (ii) to (ix) steps to obtain $\mathrm{ARL}_0 = 100$.
(xi) To acquire ARL_1 values, draw a shifted sample from the considered distribution again, and repeat steps (ii) to (ix).

3.2. Robustness, IC, and OOC Performances of the $NPDHWMA_{RSS}$ Control Chart. This subsection highlights the proposed $\mathrm{NPDHWMA_{RSS}}$ control chart's robustness, IC, and OOC behavior when a process location is shifted. Table 1 shows the RL characteristics of the proposed $\mathrm{NPDHWMA_{RSS}}$ control chart for location shift. These characteristics are assessed using normal and non-normal continuous symmetric distributions. The distributions used for this study are standard normal distribution, that is, $N(0, 1)$; double exponential or Laplace distribution, that is, $DE(0, (1/\sqrt{2}))$; heavy tail student's t distribution, that is, $t(v)$; logistic distribution, that is, $(0, (\sqrt{3}/\pi))$; and contaminated normal (CN) distribution, which is the mixture of $N(0, \sigma_0^2)$ and $N(0, \sigma_1^2)$. All these distributions were reparametrized with zero mean/median and unit variance for comparison purposes. For all symmetric continuous distributions, the IC RL characteristics of the NP control chart remain constant [4].

For comparison purposes, the same parameters are used as reported in numerous relevant articles. The ARL measures are used to compare the proposed and competing control charts. Based on the research findings and sensitivity analysis, the following observations have been made.

(i) The proposed control chart's IC RL distribution looks remarkably similar for all distributions examined in this study. For example, at $\lambda = (0.05, 0.10, 0.25, 0.50)$ and $n = 5$, 10, the $\mathrm{ARL}_0 = 370$, 500 for all investigated distributions (see Table 1).
(ii) As the smoothing parameter reduces, the proposed control chart becomes more effective in detecting

Table 1: RL characteristics of the proposed $NPDHWMA_{RSS}$ control chart under different distributions with nominal $ARL_0 = 500$ and $n = 10$.

(λ, L)	Distr.	Metrics	δ 0	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
(0.05, 1.064)	Normal	ARL	499.49	31.74	13.25	7.96	5.65	1.94	1.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	12.00	9.00	7.00	5.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	1,851.30	55.84	17.04	8.31	5.23	1.33	0.31	0.03	0.00	0.00	0.00	0.00	0.00	0.00
	CN	ARL	503.08	34.36	13.86	8.41	5.84	2.02	1.07	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	12.00	9.00	7.00	5.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	1,890.28	61.32	18.38	9.16	5.56	1.40	0.37	0.05	0.00	0.00	0.00	0.00	0.00	0.00
	$t(4)$	ARL	498.70	40.62	16.39	9.83	6.64	2.30	1.14	1.01	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	11.00	10.00	7.00	6.00	5.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	1,850.07	74.61	22.42	11.14	6.56	1.63	0.53	0.13	0.03	0.00	0.00	0.00	0.00	0.00
	$t(8)$	ARL	499.22	36.60	14.81	8.75	6.19	2.11	1.09	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	12.00	10.00	7.00	5.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	1,852.41	68.10	19.91	9.73	5.93	1.48	0.43	0.06	0.00	0.00	0.00	0.00	0.00	0.00
	Laplace	ARL	498.40	24.06	9.46	5.83	4.23	1.55	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	13.00	9.00	6.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	1,856.38	37.19	10.66	5.45	3.50	1.01	0.19	0.02	0.00	0.00	0.00	0.00	0.00	0.00
	Logistic	ARL	498.55	30.34	12.03	7.21	5.21	1.82	1.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	12.00	9.00	6.00	5.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	1,869.47	50.49	14.94	7.38	4.64	1.23	0.27	0.03	0.00	0.00	0.00	0.00	0.00	0.00
(0.10, 1.306)	Normal	ARL	499.48	61.76	21.72	11.82	7.79	2.46	1.12	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	81.00	28.00	13.00	8.00	6.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	781.09	80.85	23.95	11.14	6.59	1.52	0.45	0.05	0.00	0.00	0.00	0.00	0.00	0.00
	CN	ARL	499.16	65.72	23.07	12.44	8.17	2.55	1.15	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	78.00	29.00	14.00	8.00	6.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	772.97	86.04	25.58	11.96	6.97	1.58	0.51	0.08	0.01	0.00	0.00	0.00	0.00	0.00
	$t(4)$	ARL	498.28	79.24	27.20	14.72	9.64	2.92	1.31	1.03	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	83.00	34.00	16.00	10.00	7.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	779.49	105.11	30.95	14.68	8.63	1.83	0.72	0.22	0.06	0.00	0.00	0.00	0.00	0.00
	$t(8)$	ARL	499.07	70.30	24.31	12.99	8.66	2.67	1.20	1.01	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	87.00	30.00	14.00	9.00	6.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	781.02	94.06	27.71	12.61	7.60	1.65	0.58	0.12	0.02	0.00	0.00	0.00	0.00	0.00
	Laplace	ARL	499.21	42.49	14.75	8.18	5.54	1.90	1.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	81.00	22.00	10.00	6.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	780.79	51.83	14.91	6.94	4.26	1.18	0.30	0.04	0.00	0.00	0.00	0.00	0.00	0.00
	Logistic	ARL	498.88	57.01	19.44	10.56	7.02	2.26	1.09	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	82.00	27.50	12.00	7.00	5.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	771.67	72.06	21.01	9.77	5.65	1.40	0.38	0.05	0.00	0.00	0.00	0.00	0.00	0.00
(0.25, 2.113)	Normal	ARL	502.19	132.32	49.74	26.33	16.57	4.07	1.58	1.04	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	435.00	112.00	44.00	23.00	15.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	369.43	96.29	33.30	16.99	10.30	1.98	0.91	0.25	0.03	0.00	0.00	0.00	0.00	0.00
	CN	ARL	500.19	137.93	52.65	27.93	17.65	4.27	1.68	1.06	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	432.00	118.00	46.00	25.00	16.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	368.45	100.15	35.14	18.15	10.96	2.08	0.96	0.33	0.06	0.00	0.00	0.00	0.00	0.00
	t(4)	ARL	499.91	160.20	61.99	33.15	21.17	4.96	2.02	1.22	1.03	1.00	1.00	1.00	1.00	1.00
		MDRL	434.00	136.00	54.00	30.00	19.00	5.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	365.58	115.71	42.60	21.67	13.36	2.51	1.12	0.60	0.24	0.04	0.01	0.00	0.00	0.00
	t(8)	ARL	498.30	145.75	55.91	29.76	18.82	4.52	1.82	1.10	1.01	1.00	1.00	1.00	1.00	1.00
		MDRL	428.00	123.00	49.00	26.00	17.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	366.98	107.89	37.87	19.27	11.84	2.27	1.02	0.41	0.09	0.00	0.00	0.00	0.00	0.00
	Laplace	ARL	501.05	92.85	33.30	17.52	10.99	3.07	1.33	1.03	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	435.00	80.00	30.00	16.00	10.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	368.41	64.68	21.87	10.98	6.61	1.53	0.72	0.24	0.09	0.02	0.00	0.00	0.00	0.00
	Logistic	ARL	502.04	119.77	44.62	23.70	14.70	3.75	1.47	1.04	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	439.00	103.00	39.00	21.00	13.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	364.68	85.82	29.84	14.94	9.23	1.78	0.83	0.25	0.05	0.00	0.00	0.00	0.00	0.00
(0.50, 2.376)	Normal	ARL	498.71	201.01	70.69	34.34	20.17	4.26	1.67	1.08	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	355.00	147.00	55.00	28.00	17.00	4.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	482.39	184.95	58.24	25.76	13.71	2.05	0.76	0.28	0.05	0.00	0.00	0.00	0.00	0.00

Table 1: Continued.

(λ, L)	Distr.	Metrics	δ													
			0	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
	CN	ARL	497.95	210.08	75.05	36.35	21.56	4.49	1.76	1.12	1.01	1.00	1.00	1.00	1.00	1.00
		MDRL	349.00	155.00	58.00	30.00	18.00	4.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	486.93	191.48	62.24	27.33	14.78	2.20	0.80	0.33	0.09	0.00	0.00	0.00	0.00	0.00
	$t(4)$	ARL	499.23	234.12	89.78	44.68	26.26	5.37	2.08	1.29	1.06	1.00	1.00	1.00	1.00	1.00
		MDRL	353.00	172.00	68.00	36.00	22.00	5.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	483.45	216.30	76.45	34.47	18.70	2.80	0.93	0.53	0.25	0.05	0.01	0.01	0.00	0.00
	$t(8)$	ARL	501.84	220.42	79.56	39.20	23.38	4.77	1.88	1.16	1.02	1.00	1.00	1.00	1.00	1.00
		MDRL	355.50	160.50	60.00	31.00	20.00	4.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	490.89	206.08	67.97	29.98	16.20	2.36	0.85	0.40	0.13	0.02	0.00	0.00	0.00	0.00
	Laplace	ARL	498.36	142.19	44.47	21.30	12.97	3.14	1.39	1.06	1.01	1.00	1.00	1.00	1.00	1.00
		MDRL	351.00	106.00	36.00	18.00	11.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	489.74	126.58	33.90	14.60	8.05	1.41	0.62	0.24	0.07	0.01	0.00	0.00	0.00	0.00
	Logistic	ARL	499.28	186.35	62.61	29.79	17.68	3.84	1.56	1.07	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	358.00	138.00	49.00	25.00	15.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	479.40	170.70	50.14	21.40	12.02	1.76	0.71	0.26	0.07	0.00	0.00	0.00	0.00	0.00

shifts. This illustrates that the proposed $NPDHWMA_{RSS}$ control chart is more sensitive to small smoothing parameters (see Figure 1).

(iii) As the sample size increases, the proposed $NPDHWMA_{RSS}$ control chart's shift detection ability for process location improves (see Figure 2).

(iv) The Laplace distribution outperforms the other distributions in terms of OOC RL performance (see Figures 3).

(v) The proposed $NPDHWMA_{RSS}$ control chart's ARL_1 values increase as λ increases at a certain size of the shift. For instance, under normal distribution at $\lambda = 0.25, n = 10$, and $\delta = 0.025$, the $ARL_1 = 132.32$, whereas when $\lambda = 0.50, n = 10$, and $\delta = 0.025$, the $ARL_1 = 201.01$ (see Table 1).

(vi) The ARL_1 of the proposed $NPDHWMA_{RSS}$ control chart are smaller than those in the competing control charts with different shift sizes in process location (see Figure 4).

(vii) The distribution of RL values is positively skewed, that is, ARL > MRDL (see Table 1).

4. Comparative Study

This section provides a comparative performance study in terms of the ARL values of the proposed $NPDHWMA_{RSS}$ control chart for process location shifts. The proposed $NPDHWMA_{RSS}$ control chart is compared to the competing control charts, including DEWMA-$\overline{X}$, NPDEWMA-SR, NPRDEWMA-SR, DHWMA, and $NPHWMA_{RSS}$.

4.1. Proposed versus DEWMA-$\overline{X}$ Control Chart. The ARL profile demonstrates that the proposed $NPDHWMA_{RSS}$ control chart outperforms the DEWMA-$\overline{X}$ control chart. For example, under normal distribution, at $= 0.05, n = 10$, and $\delta = 0.025, 0.05, 0.075, 0.10, 0.25, 0.50, 0.75$, the ARL_1 values of the proposed $NPDHWMA_{RSS}$ control chart are $31.74, 13.25, 7.96, 5.65, 1.94, 1.05, 1.00$, whereas the ARL_1 values of the DEWMA-$\overline{X}$ control charts are $284.02, 127.00, 69.34, 43.71, 9.68, 2.93, 1.57$ (see Tables 1 and 2). Similarly, when we consider Laplace distribution for comparison, we observed the same behavior in the proposed $NPDHWMA_{RSS}$ control chart. For instance, at $\lambda = 0.05, n = 10$, and $\delta = 0.025, 0.05, 0.075, 0.10, 0.25, 0.50, 0.75$, the ARL_1 values of the proposed $NPDHWMA_{RSS}$ control chart are $24.06, 9.46, 5.83, 4.23, 1.55, 1.02, 1.00$, while the ARL_1 values of DEWMA-$\overline{X}$ control chart are $287.40, 129.08, 68.32, 44.01, 9.73, 2.91, 1.57$ (see Figure 4 and Tables 1 and 2).

4.2. Proposed versus NPDEWMA-SR Control Chart. The proposed $NPDHWMA_{RSS}$ control chart is more sensitive than the NPDEWMA-SR control chart for all combinations of δ and λ. For instance, in case of logistic distribution, when $n = 10, \lambda = 0.05$, and $\delta = 0.025$, 0.05, 0.075, 0.10, 0.25, 0.50, 0.75, the ARL_1 values of the proposed $NPDHWMA_{RSS}$ and NPDEWMA-SR control charts are 30.34, 12.03, 7.21, 5.21, 1.82, 1.04, and 1.00 and 280.87, 123.83, 66.59, 42.46, 9.40, 3.08, and 1.734, respectively (see Tables 1 and 3). The supremacy of the proposed $NPDHWMA_{RSS}$ control chart to the NPDEWMA-SR can also be seen in Figure 4. Likewise, in the scenario of $t_{(8)}$ distribution comparison, we noted the same behavior of the proposed $NPDHWMA_{RSS}$ control chart. As an illustration, at $\lambda = 0.05, n = 10$, and $\delta = 0.05, 0.10, 0.50$, the ARL_1 values of the proposed $NPDHWMA_{RSS}$ control chart are $14.81, 6.19, 1.09$, respectively, while the ARL_1 values of the NPDEWMA-SR control chart are $123.93, 42.69, 3.05$ (see Tables 1 and 3).

4.3. Proposed versus NPRDEWMA-SR Control Chart. The proposed $NPDHWMA_{RSS}$ control chart performs better than the NPRDEWMA-SR control chart. For instance, using the $t_{(4)}$ distribution with $n = 10, \lambda = 0.05$, and $\delta = 0.05, 0.50$, the ARL_1 values of the proposed $NPDHWMA_{RSS}$ and the NPRDEWMA-SR control charts are 16.39, 1.14, and 49.00, 1.28, respectively (see Tables 1 and 4). The proposed $NPDHWMA_{RSS}$ charts' efficiency may also be observed in the case of CN distribution. For example, at

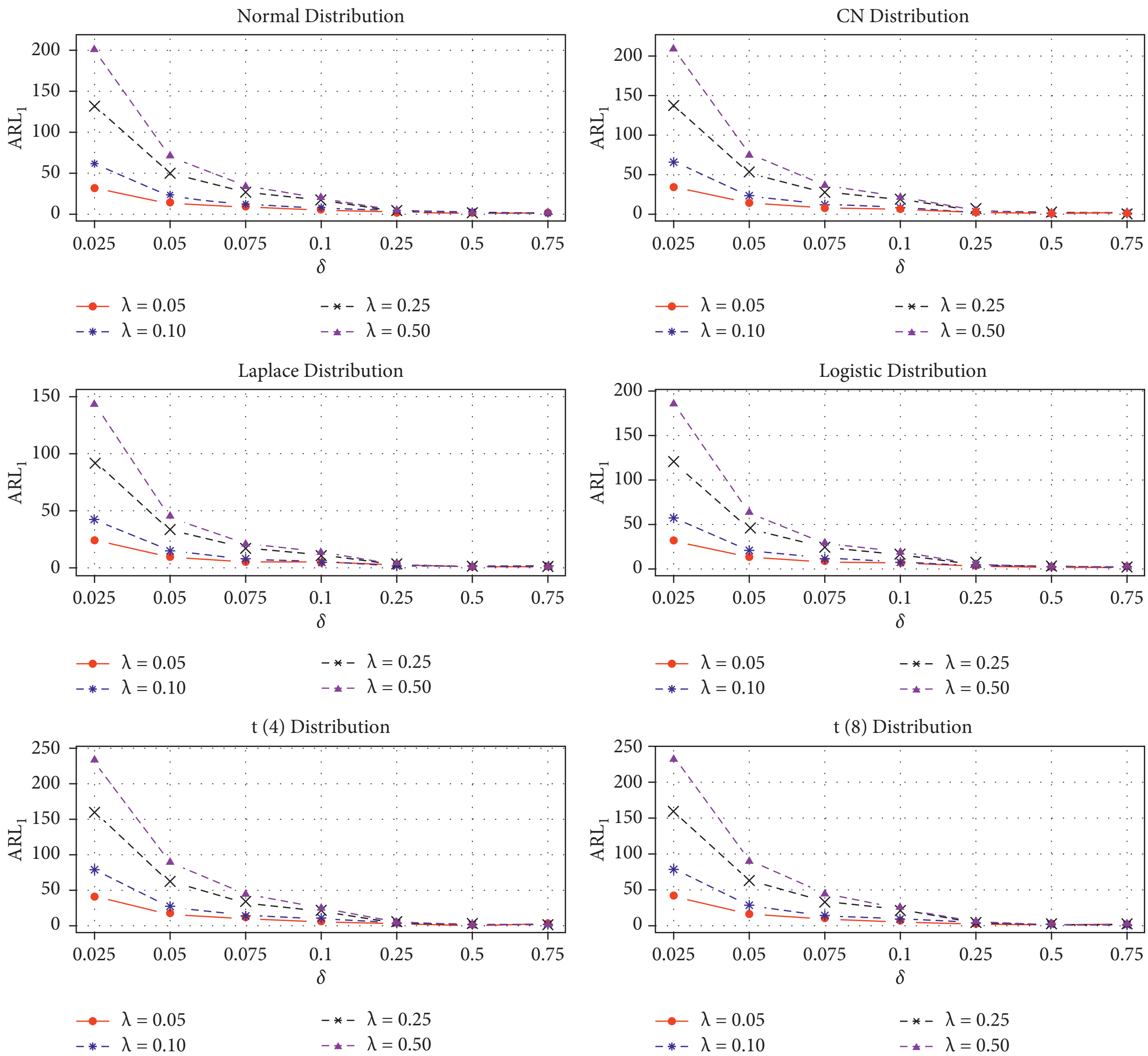

Figure 1: ARL characteristics of the proposed $\text{NPDHWMA}_{\text{RSS}}$ control chart for different values of λ when $n = 10$ and $\text{ARL}_0 = 500$.

$n = 10$, $\lambda = 0.50$ and $\delta = 0.025, 0.05, 0.075, 0.10$, the ARL_1 values for the proposed $\text{NPDHWMA}_{\text{RSS}}$ control chart are $34.36, 13.86, 8.41, 5.84$, whereas the ARL_1 values for the NPRDEWMA-SR control chart are $121.25, 40.58, 21.37$, and 13.11 (see Figure 4 and Tables 1 and 4).

4.4. Proposed versus DHWMA Control Chart. The ARL study reveals that the proposed $\text{NPDHWMA}_{\text{RSS}}$ control chart outperforms the DHWMA control chart for all combinations of η and δ (see Tables 1 and 5). As an illustration, for normal distribution, at $\lambda = 0.05$, $n = 10$, and $\delta = 0.075, 0.10, 0.25$, the ARL_1 values of the $\text{NPDHWMA}_{\text{RSS}}$ and DHWMA control charts are $7.96, 5.65, 1.94$ and $21.24, 14.81, 4.63$, respectively (see Tables 1 and 5). Figure 4 also shows the superiority of the $\text{NPDHWMA}_{\text{RSS}}$ control chart over the DHWMA control chart. The results show that the $\text{NPDHWMA}_{\text{RSS}}$ control chart is superior to the DHWMA control chart for monitoring process location shifts.

4.5. Proposed versus $\text{NPHWMA}_{\text{RSS}}$ Control Chart. The findings show that the proposed $\text{NPDHWMA}_{\text{RSS}}$ control chart is better than the $\text{NPHWMA}_{\text{RSS}}$ control chart in terms of OOC performance. For example, when we examine the $t_{(8)}$ distribution at $n = 10, \lambda = 0.10$ and $\delta = 0.025, 0.10, 0.25$, the ARL_1 values of the proposed $\text{NPDHWMA}_{\text{RSS}}$ and $\text{NPHWMA}_{\text{RSS}}$ control charts are $70.30, 8.66, 2.67$, and $153.55, 20.27, 4.77$, respectively (see Figure 4 and Tables 1 and 6). Likewise, under Laplace distribution, when $n = 10, \lambda = 0.05$ and $\delta = 0.05, 0.50$, the ARL_1 values of the proposed $\text{NPDHWMA}_{\text{RSS}}$ control chart are $9.46, 1.02$, while the ARL_1 values for $\text{NPHWMA}_{\text{RSS}}$ control chart are $31.42, 1.31$, respectively (see Tables 1 and 6). The statistics show that the proposed structure works effectively for all

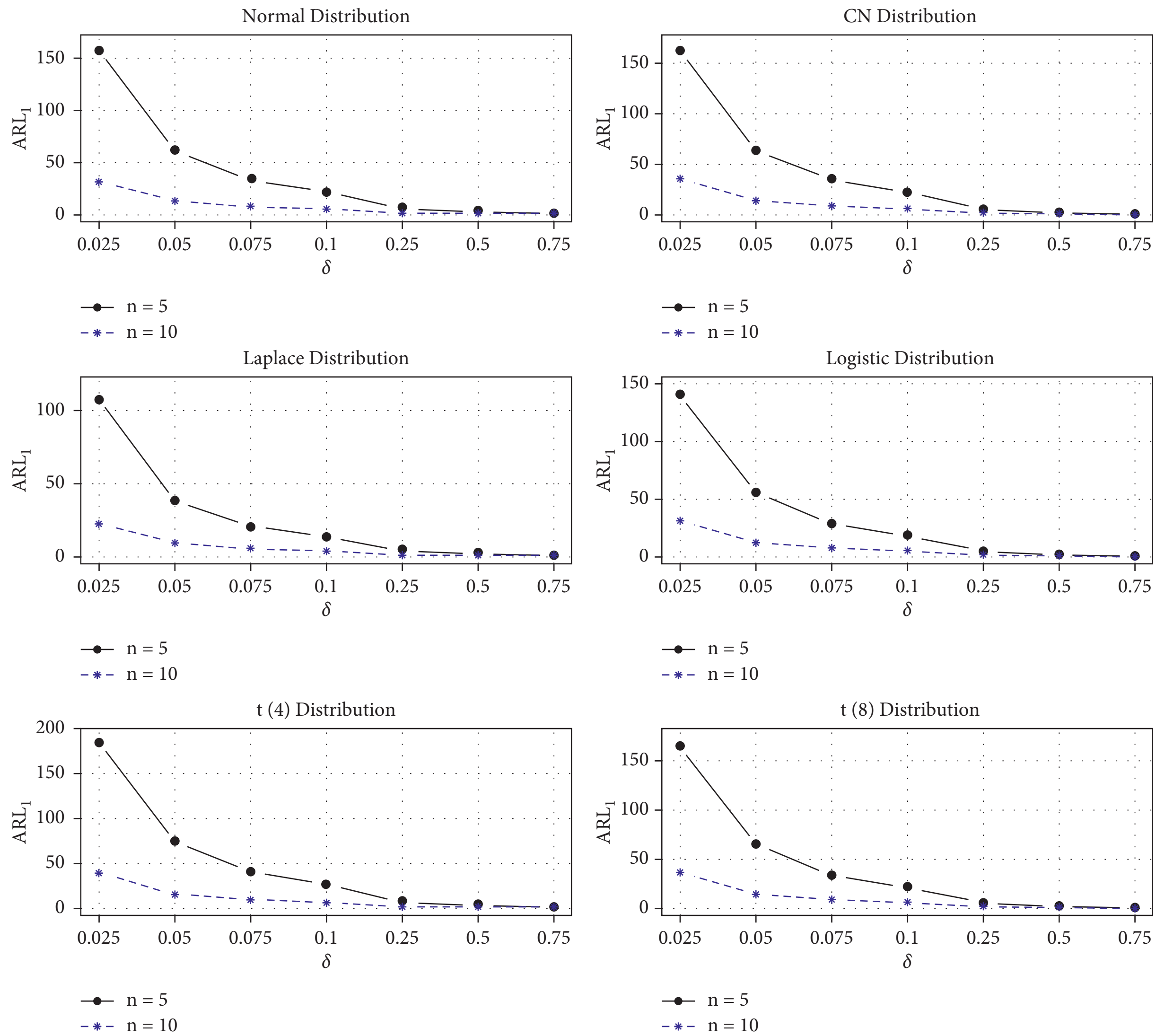

Figure 2: ARL characteristics of the proposed $\text{NPDHWMA}_{\text{RSS}}$ control chart when $n = 5, 10$ and $\text{ARL}_0 = 500$.

distributions when compared to the $\text{NPHWMA}_{\text{RSS}}$ control chart

5. Illustrative Example

The proposed charts' implementation is generally associated with industrial processes and finished products, and it can be adapted to different of many other fields such as medicine, planning, financial reporting, neutrosophic statistics, and so on. This section provides a real-life application of the non-isothermal continuous stirred tank reactor (CSTR) process to demonstrate the applicability of the proposed $\text{NPDHWMA}_{\text{RSS}}$ control chart. This real-life data was originally proposed by Marlin and Marlin [25], and has since been widely used as a standard in fault diagnosis, for instance, Xiangrong et al. [26], Ridwan et al. [27], Adegoke et al. [20], and many more. The CSTR process has nine different variables, one of which we choose as the variable of interest (X) represents the output temperature.

The data initially consists of 1,000 observations, with the first 600 occurring when the process was in an IC condition. The phase I sample's parameters are as follows: $\mu_Y = 368.2328, \mu_X = 369.8789, \sigma_Y^2 = 0.2185915, \sigma_X^2 = 0.3180327$, and $\rho_{YX} = 0.08974039$. We used the RSS approach to generate 40 paired observations of size $n = 5$ from a normal distribution. After the 24th sample, a shift in the process location is introduced following Anwar et al. [28]. The parameters of the proposed and NPRDHWMA-SR control charts used for real-life analysis are $L = 1.535, \lambda = 0.10, ARL_0 = 500$, and $L = 2.117, \lambda = 0.10, ARL_0 = 500$, respectively. Figure 5 indicates that the proposed $\text{NPDHWMA}_{\text{RSS}}$ control chart triggers the first OOC signal at sample number 25, while the NPRDEWMA-SR control chart detects the first OOC point at sample number 29. Similarly, the proposed $\text{NPDHWMA}_{\text{RSS}}$ control chart detects overall 16 OOC points, whereas the NPRDEWMA-SR control chart detects 12 OOC points (see Table 7 and Figure 5).

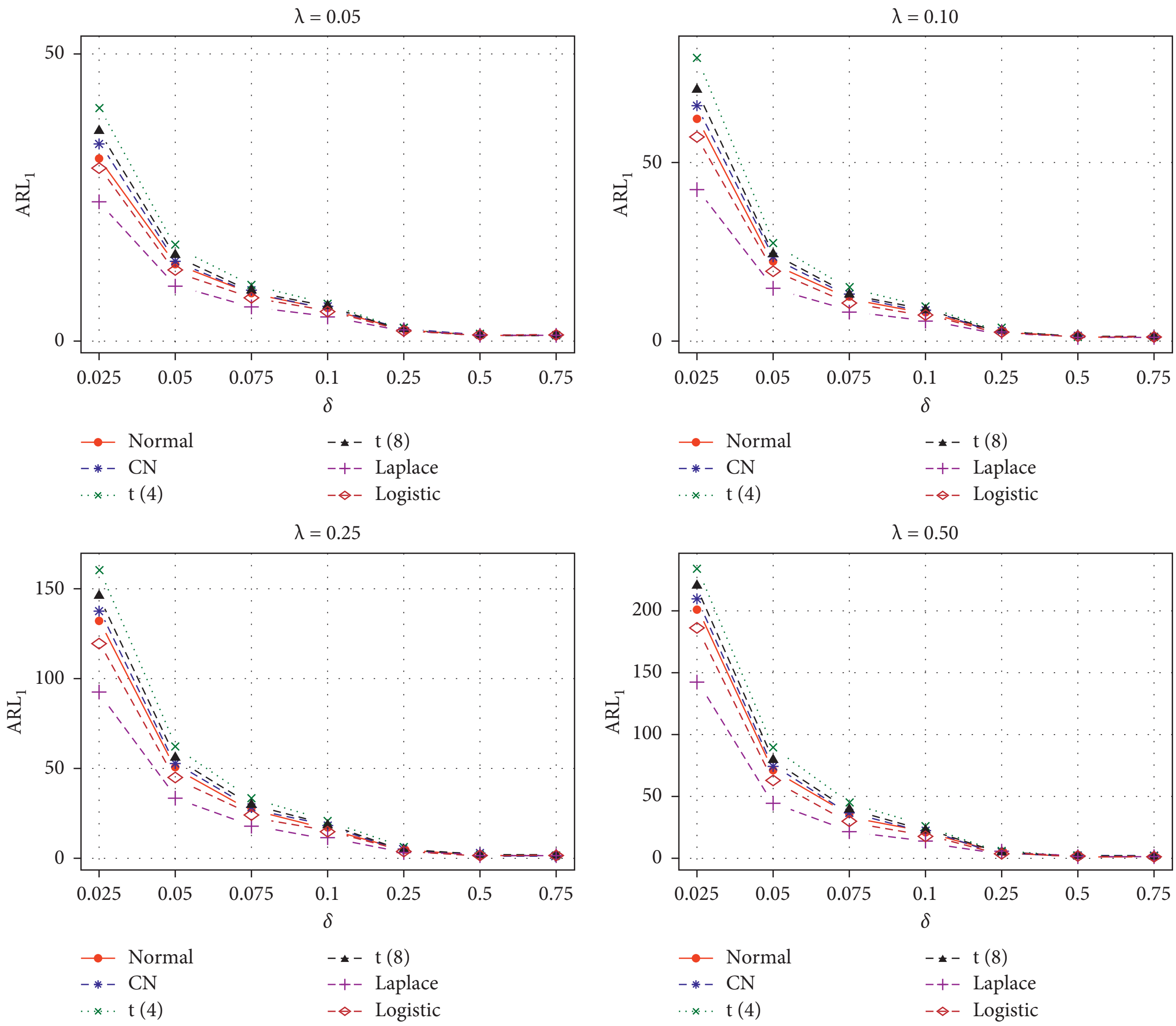

Figure 3: ARL characteristics of the proposed $\text{NPDHWMA}_{\text{RSS}}$ control chart under various distributions when $n = 10$ and $\text{ARL}_0 = 500$.

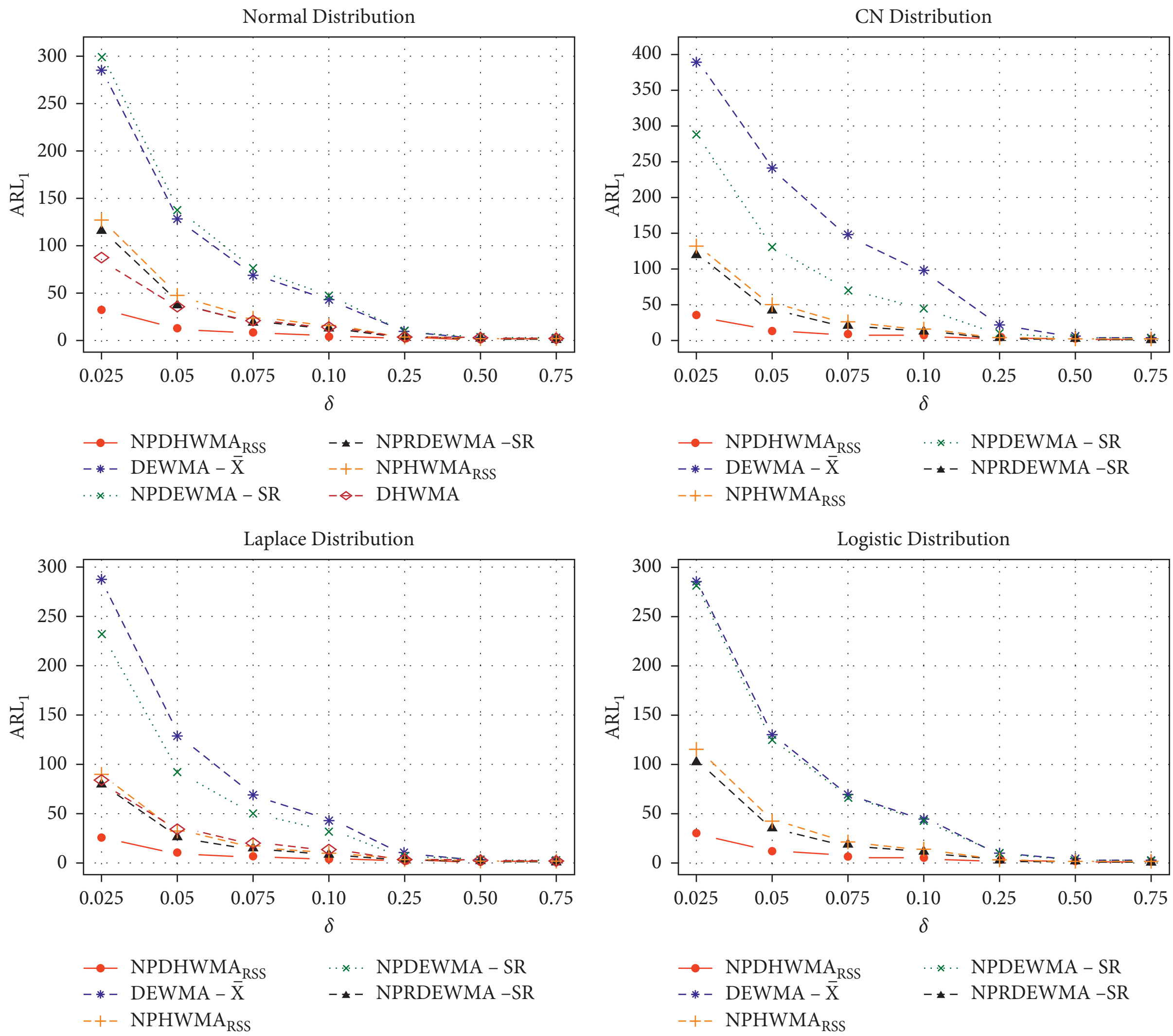

FIGURE 4: ARL characteristics of the proposed NPDHWMA$_{RSS}$ and competing control charts under various distributions when n = 10 and ARL_0 = 500.

TABLE 2: RL characteristic of the DEWMA – $\overline{X}$ control chart under different distributions with ARL_0 = 500 and n = 10.

Distr.	λ, L	Metric	δ													
			0	0.025	0.03	0.05	0.075	0.1	0.15	0.2	0.25	0.5	0.75	1	1.5	2
Normal	0.05, 2.081	ARL	500.04	284.02	239.41	127.00	69.34	43.71	23.02	14.10	9.68	2.93	1.57	1.16	1.00	1.00
$t(4)$	0.05, 2.08	ARL	498.01	287.13	241.49	127.40	69.35	43.26	22.62	14.13	9.71	2.91	1.55	1.14	1.02	1.00
CN	0.05, 3.204	ARL	498.11	390.49	359.60	240.51	146.55	96.54	51.03	32.07	22.33	6.90	3.39	2.09	1.22	1.01
Laplace	0.05, 2.083	ARL	498.67	287.40	245.01	129.08	68.32	44.01	22.48	14.10	9.73	2.91	1.57	1.16	1.01	1.00
Logistic	0.05, 2.083	ARL	503.92	287.07	241.11	129.30	69.45	44.14	22.61	14.01	9.59	2.93	1.56	1.14	1.01	1.00

Table 3: RL characteristics of the SRS-based NP double EWMA SR (NPDEWMA-SR) control chart under different distributions with nominal ARL_0 = 500 and n = 10.

(λ, L)	Distr.	Metrics	δ													
			0	0.025	0.03	0.05	0.075	0.1	0.15	0.2	0.25	0.5	0.75	1	1.5	2
(0.05, 6.57)	Normal	ARL	499.98	299.203	248.03	138.03	75.74	48.00	25.01	15.56	10.61	3.34	1.83	1.32	1.04	1.00
		SDRL	541.40	328.41	263.05	139.82	70.76	41.34	20.34	12.27	8.212	2.33	1.06	0.62	0.18	0.04
	t(4)	ARL	501.45	247.01	203.44	101.38	53.60	34.72	18.00	11.22	7.65	2.59	1.59	1.26	1.05	1.01
		SDRL	546.82	262.02	212.45	98.27	47.57	28.83	14.12	8.69	5.84	1.71	0.81	0.55	0.25	0.19
	t(8)	ARL	500.89	276.24	233.07	123.93	67.04	42.69	22.11	13.80	9.63	3.05	1.75	1.31	1.03	1.00
		SDRL	538.51	294.92	249.16	125.19	62.10	37.00	17.45	10.71	7.34	2.06	0.95	0.61	0.25	0.11
	CN	ARL	498.95	287.54	241.30	131.10	70.67	45.13	23.52	14.58	10.10	3.21	1.79	1.28	1.05	1.00
		SDRL	545.49	310.49	257.85	131.20	65.12	39.27	18.58	11.42	7.71	2.22	1.00	0.58	0.23	0.09
	Laplace	ARL	499.36	231.49	188.36	92.09	49.45	31.50	16.38	10.37	7.19	2.58	1.64	1.29	1.08	1.01
		SDRL	535.19	247.33	197.44	90.20	42.65	26.11	13.01	8. 00	5.46	1.71	0.89	0.59	0.29	0.15
	Logistic	ARL	499.45	280.87	236.702	123.83	66.59	42.46	21.79	13.52	9.40	3.08	1.734	1.30	1.03	1.00
		SDRL	549.41	300.01	250.82	124.01	61.03	36.55	17.47	10.57	7.32	2.12	1.00	0.56	0.32	0.05

Table 4: RL characteristics of the RSS-based NP double EWMA SR (NPRDEWMA-SR) control chart under different distributions with nominal ARL_0 = 500 and n = 10.

(λ, L)	Distr.	Metrics	δ													
			0	0.025	0.03	0.05	0.075	0.1	0.15	0.2	0.25	0.5	0.75	1	1.5	2
(0.05, 2.975)	Normal	ARL	501.58	117.13	87.78	38.34	20.01	12.54	6.29	3.82	2.65	1.13	1.00	1.00	1.00	1.00
		SDRL	562.31	119.81	85.90	32.90	16.02	9.69	4.80	2.88	1.80	0.40	0.07	0.00	0.00	0.00
	t(4)	ARL	499.96	141.02	111.21	49.00	25.81	15.87	8.23	4.86	3.40	1.28	1.03	1.00	1.00	1.00
		SDRL	557.27	138.50	109.76	44.58	20.93	13.20	6.39	3.70	2.49	0.58	0.20	0.06	0.00	0.00
	t(8)	ARL	500.13	129.01	97.70	44.00	22.64	14.15	7.04	4.32	2.95	1.19	1.01	1.00	1.00	1.00
		SDRL	555.79	131.18	96.12	39.73	18.28	12.03	6.25	3.23	2.13	0.49	0.11	0.01	0.00	0.00
	CN	ARL	500.89	121.25	94.89	40.58	21.37	13.11	6.80	4.04	2.81	1.16	1.00	1.00	1.00	1.00
		SDRL	553.38	122.51	92.44	38.10	17.30	11.36	5.20	2.98	1.96	0.43	0.08	0.01	0.01	0.00
	Laplace	ARL	498.94	79.00	57.70	25.40	13.63	8.20	4.18	2.60	1.92	1.06	1.00	1.00	1.00	1.00
		SDRL	565.29	73.18	52.35	20.65	11.00	7.02	3.07	1.79	1.19	0.25	0.06	0.00	0.01	0.00
	Logistic	ARL	499.71	103.12	78.02	34.36	18.01	11.07	5.48	3.32	2.36	1.10	1.00	1.00	1.00	1.00
		SDRL	552.61	100.70	72.60	29.77	15.77	9.11	4.17	2.46	1.62	0.35	0.03	0.00	0.00	0.00

Table 5: RL characteristics of the DHWMA control chart under normal distribution with nominal ARL_0 = 500 and n = 10.

λ, L		δ													
		0	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
0.05, 1.39	ARL	501.14	87.08	34.97	21.24	14.81	4.63	2.10	1.35	1.08	1.00	1.00	1.00	1.00	1.00
	SDRL	1,930.36	198.03	63.17	32.33	19.69	3.91	1.46	0.81	0.39	0.04	0.00	0.00	0.00	0.00
0.10, 1.7105	ARL	502.29	167.38	67.17	37.15	24.27	6.19	2.56	1.54	1.15	1.00	1.00	1.00	1.00	1.00
	SDRL	805.27	248.79	88.87	45.54	27.40	4.95	1.65	0.97	0.53	0.06	0.00	0.00	0.00	0.00
0.25, 2.7422	ARL	498.17	294.65	141.71	83.10	54.34	12.59	4.29	2.48	1.64	1.03	1.00	1.00	1.00	1.00
	SDRL	373.80	227.63	105.02	57.95	37.24	7.67	2.13	1.28	0.93	0.22	0.00	0.00	0.00	0.00
0.50, 3.075	ARL	502.05	371.48	210.92	121.28	76.50	14.66	4.54	2.49	1.65	1.05	1.00	1.00	1.00	1.00
	SDRL	489.97	355.66	195.21	107.13	64.30	9.40	2.32	1.14	0.79	0.22	0.02	0.00	0.00	0.00

Table 6: RL characteristics of the $NPHWMA_{RSS}$ control chart under various distributions with nominal $ARL_0 = 500$ and $n = 10$.

(λ, L)	Distr.	Metrics	δ													
			0	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
(0.05, 2.011)	Normal	ARL	502.18	126.47	47.44	24.83	15.55	3.86	1.56	1.04	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	450.00	108.00	41.00	22.00	14.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	369.20	93.57	32.86	16.66	10.06	1.91	0.87	0.24	0.03	0.00	0.00	0.00	0.00	0.00
	CN	ARL	498.95	131.67	49.92	26.01	16.50	4.06	1.65	1.06	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	447.00	114.00	44.00	23.00	15.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	354.98	96.62	35.19	17.65	10.85	2.00	0.93	0.31	0.04	0.00	0.00	0.00	0.00	0.00
	Laplace	ARL	503.02	89.97	31.42	16.34	10.24	2.93	1.31	1.03	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	447.00	78.00	28.00	14.00	9.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	357.84	66.10	21.28	10.81	6.45	1.46	0.69	0.20	0.07	0.01	0.00	0.00	0.00	0.00
	Logistic	ARL	502.13	115.25	42.13	22.17	13.63	3.57	1.45	1.03	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	447.00	99.00	36.00	19.00	12.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	357.84	83.49	29.56	14.54	8.81	1.75	0.81	0.23	0.03	0.00	0.00	0.00	0.00	0.00
	$t(4)$	ARL	503.11	153.20	59.65	31.34	19.66	4.77	1.97	1.22	1.03	1.00	1.00	1.00	1.00	1.00
		MDRL	449.00	130.00	52.00	27.00	17.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	361.52	115.04	42.01	21.93	13.15	2.47	1.07	0.59	0.23	0.02	0.03	0.00	0.00	0.00
	$t(8)$	ARL	500.36	138.20	53.18	28.35	17.56	4.24	1.74	1.10	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	437.00	115.50	46.00	25.00	16.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	369.84	105.14	36.77	19.05	11.45	2.15	0.98	0.40	0.08	0.00	0.00	0.00	0.00	0.00
(0.10, 2.268)	Normal	ARL	501.08	141.18	53.78	28.36	17.91	4.33	1.69	1.06	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	416.00	116.00	47.00	25.00	16.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	409.43	104.61	35.48	17.45	10.47	2.03	0.92	0.30	0.04	0.00	0.00	0.00	0.00	0.00
	CN	ARL	498.89	188.39	84.69	47.51	31.26	7.05	2.63	1.35	1.10	1.02	1.00	1.00	1.00	1.00
		MDRL	401.00	123.00	49.00	27.00	17.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	392.85	106.58	37.05	18.45	11.47	2.12	0.97	0.36	0.06	0.00	0.00	0.00	0.00	0.00
	Laplace	ARL	499.36	100.25	35.70	18.93	11.97	3.22	1.42	1.05	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	409.00	84.00	32.00	17.00	11.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	401.09	72.33	22.59	11.48	6.81	1.50	0.76	0.27	0.08	0.00	0.00	0.00	0.00	0.00
	Logistic	ARL	500.14	129.12	47.66	25.23	15.93	3.92	1.58	1.05	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	409.00	108.00	41.00	22.00	14.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	401.09	94.77	31.64	15.46	9.24	1.78	0.86	0.25	0.06	0.00	0.00	0.00	0.00	0.00
	t(4)	ARL	498.01	169.35	66.27	36.09	22.57	5.34	2.19	1.28	1.05	1.00	1.00	1.00	1.00	1.00
		MDRL	398.50	141.00	56.00	32.00	20.00	5.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	395.19	124.85	45.22	22.77	13.46	2.65	1.12	0.63	0.27	0.03	0.01	0.00	0.00	0.00
	$t(8)$	ARL	499.21	153.55	60.16	32.02	20.27	4.77	1.92	1.15	1.01	1.00	1.00	1.00	1.00	1.00
		MDRL	400.50	125.00	52.00	29.00	18.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	400.94	119.02	39.91	19.71	11.99	2.23	1.02	0.46	0.14	0.00	0.00	0.00	0.00	0.00

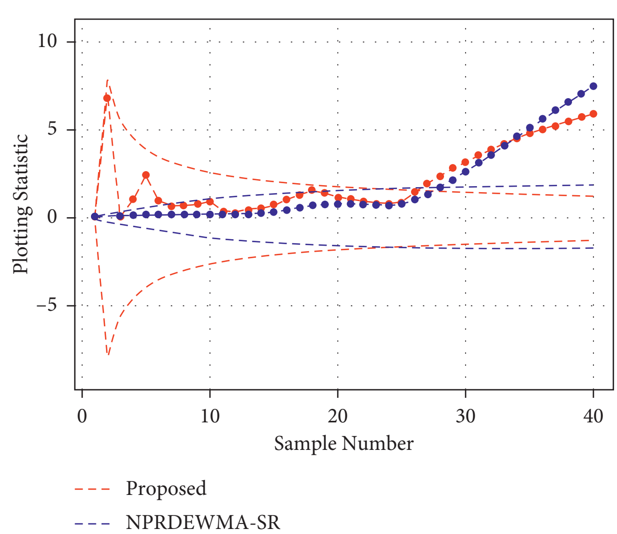

Figure 5: Real-life application of the proposed $NPDHWMA_{RSS}$ and the NPRDEWMA-SR control charts.

Table 7: Application of the proposed versus NPRDEWMA-SR control charts.

Sample #	Proposed			NPRDEWMA-SR		
	Statistic	UCL	LCL	Statistic	UCL	LCL
1	0.07	0.08	−0.08	0.07	0.11	−0.11
2	6.86	7.89	−7.89	0.06	0.23	−0.23
3	0.03	5.58	−5.58	0.07	0.35	−0.35
4	1.06	4.56	−4.56	0.16	0.47	−0.47
5	2.43	3.95	−3.95	0.17	0.60	−0.60
6	0.98	3.53	−3.53	0.18	0.71	−0.71
7	0.67	3.22	−3.22	0.19	0.82	−0.82
8	0.72	2.98	−2.98	0.20	0.92	−0.92
9	0.77	2.79	−2.79	0.24	1.02	−1.02
10	0.94	2.63	−2.63	0.22	1.10	−1.10
11	0.39	2.50	−2.50	0.19	1.18	−1.18
12	0.30	2.38	−2.38	0.20	1.25	−1.25
13	0.51	2.28	−2.28	0.21	1.31	−1.31
14	0.56	2.19	−2.19	0.25	1.37	−1.37
15	0.76	2.11	−2.11	0.33	1.42	−1.42
16	1.04	2.04	−2.04	0.44	1.47	−1.47
17	1.31	1.97	−1.97	0.59	1.51	−1.51
18	1.56	1.92	−1.92	0.70	1.54	−1.54
19	1.40	1.86	−1.86	0.75	1.57	−1.57
20	1.19	1.81	−1.81	0.78	1.60	−1.60
21	1.08	1.77	−1.77	0.78	1.63	−1.63
22	0.96	1.72	−1.72	0.74	1.65	−1.65
23	0.82	1.68	−1.68	0.72	1.67	−1.67
24	0.81	1.65	−1.65	0.68	1.68	−1.68
25	0.89	1.61	−1.61	0.79	1.70	−1.70
26	1.46	1.58	−1.58	1.02	1.71	−1.71
27	1.96	1.55	−1.55	1.33	1.72	−1.72
28	2.39	1.52	−1.52	1.72	1.73	−1.73
29	2.82	1.49	−1.49	2.14	1.74	−1.74
30	3.19	1.47	−1.47	2.62	1.74	−1.74
31	3.56	1.44	−1.44	3.10	1.75	−1.75
32	3.89	1.42	−1.42	3.62	1.76	−1.76
33	4.21	1.40	−1.40	4.13	1.76	−1.76
34	4.50	1.38	−1.38	4.65	1.76	−1.76
35	4.81	1.36	−1.36	5.18	1.77	−1.77
36	5.04	1.34	−1.34	5.64	1.77	−1.77
37	5.21	1.32	−1.32	6.11	1.77	−1.77
38	5.47	1.30	−1.30	6.58	1.77	−1.77
39	5.73	1.28	−1.28	7.04	1.78	−1.78
40	5.90	1.27	−1.27	7.44	1.78	−1.78

6. Summary, Conclusions, and Future Recommendations

Usually, control charts are used to monitor the process parameters (location and/or dispersion) when the quality characteristic follows the specific distribution. When this assumption is not fulfilled, the nonparametric (NP) control charts are used to handle this situation. On the other hand, the double homogeneously weighted moving average (DHWMA) is the advanced version of the double exponentially weighted moving average (DEWMA) control chart for process location monitoring. Similarly, the ranked set sampling (RSS) technique is more efficient than simple random sampling (SRS). So this study combines the NP DHWMA control chart and RSS scheme and presents an NPDHWMA Wilcoxon signed-rank control chart under the RSS technique (denoted by $\text{NPDHWMA}_{\text{RSS}}$) for enhanced monitoring of process location shifts. The performance of the proposed control chart is investigated in terms of ARL, MDRL, and SDRL. The results revealed that the proposed control chart performs better than the competing control charts such as DEWMA-$\overline{X}$, NPDEWMA-SR, NPRDEWMA-SR, DHWMA, and $\text{NPHWMA}_{\text{RSS}}$. Moreover, a real-life application is also offered to show the proposed control chart's applicability in practice. This study is carried out where the process variable follows the univariate distributions. However, the proposed $\text{NPDHWMA}_{\text{RSS}}$ charting scheme can be used to enhance the monitoring of high-quality processes [29, 30], time-between-events [31], multivariate processes [32], and neutrosophic statistics [33, 34] scenarios.

Data Availability

The data used in the real-life application can be obtained from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] W. A. Shewhart, D. Van Nostrand, *Economic Control of Quality Manufactured Product*American Society for Quality Control in 1980, Milwaukee, WI, 1931.
[2] E. S. Page, "Continuous inspection schemes," *Biometrika*, vol. 41, no. 1-2, pp. 100–115, 1954.
[3] S. W. Roberts, "Control chart tests based on geometric moving averages," *Technometrics*, vol. 1, no. 3, pp. 239–250, 1959.
[4] Z. Abbas, H. Z. Nazir, M. Abid, N. Akhtar, and M. Riaz, "Enhanced nonparametric control charts under simple and ranked set sampling schemes," *Transactions of the Institute of Measurement and Control*, vol. 42, no. 14, pp. 2744–2759, 2020.
[5] A. Haq, J. Brown, E. Moltchanova, and A. I. Al-Omari, "Effect of measurement error on exponentially weighted moving average control charts under ranked set sampling schemes," *Journal of Statistical Computation and Simulation*, vol. 85, no. 6, pp. 1224–1246, 2015.
[6] M. Noor-ul-Amin and M. Tayyab, "Enhancing the performance of exponential weighted moving average control chart using paired double ranked set sampling," *Journal of Statistical Computation and Simulation*, vol. 90, no. 6, pp. 1118–1130, 2020.
[7] R. W. Amin and A. J. Searcy, "A nonparametric exponentially weighted moving average control scheme," *Communications in Statistics - Simulation and Computation*, vol. 20, no. 4, pp. 1049–1072, 1991.
[8] S. T. Bakir, "A distribution-free Shewhart quality control chart based on signed-ranks," *Quality Engineering*, vol. 16, no. 4, pp. 613–623, 2004.
[9] S.-F. Yang, J.-S. Lin, and S. W. Cheng, "A new nonparametric EWMA sign control chart," *Expert Systems with Applications*, vol. 38, no. 5, pp. 6239–6243, 2011.
[10] M. Graham, S. Chakraborti, and S. Human, "A nonparametric EWMA sign chart for location based on individual measurements," *Quality Engineering*, vol. 23, pp. 227–241, 2011.
[11] M. Graham, S. Chakraborti, and S. Human, "A nonparametric exponentially weighted moving average signed-rank chart for monitoring location," *Computational Statistics & Data Analysis*, vol. 55, pp. 2490–2503, 2011.
[12] S.-L. Lu, "An extended nonparametric exponentially weighted moving average sign control chart," *Quality and Reliability Engineering International*, vol. 31, no. 1, pp. 3–13, 2015.
[13] C. F. Tsai, S. L. Lu, and C. J. Huang, "Design OF an extended nonparametric EWMA sign chart," *International Journal of Industrial Engineering: Theory, Applications and Practice*, vol. 22, no. 6, pp. 705–716, 2015.
[14] N. Chakraborty, S. Chakraborti, S. W. Human, and N. Balakrishnan, "A generally weighted moving average signed-rank control chart," *Quality and Reliability Engineering International*, vol. 32, 2016.
[15] S.-L. Lu, "Non parametric double generally weighted moving average sign charts based on process proportion," *Communications in Statistics - Theory and Methods*, vol. 47, no. 11, pp. 2684–2700, 2018.
[16] S. Chakraborti and M. Graham, "Nonparametric (distribution-free) control charts: an updated overview and some results," *Quality Engineering*, vol. 31, pp. 1–22, 2019.
[17] Z. Rasheed, H. Zhang, S. M. Anwar, and B. Zaman, "Homogeneously mixed memory charts with application in the substrate production process," *Mathematical Problems in Engineering*, vol. 2021, Article ID 2582210, 2021.
[18] Z. Rasheed, H. Zhang, M. Arslan et al., "An efficient robust nonparametric triple EWMA Wilcoxon signed-rank control chart for process location," *Mathematical Problems in Engineering*, vol. 2021, Article ID 2570198, 2021.
[19] N. Abbas, "Homogeneously weighted moving average control chart with an application in substrate manufacturing process," *Computers & Industrial Engineering*, vol. 120, pp. 460–470, 2018.
[20] N. A. Adegoke, A. N. H. Smith, M. J. Anderson, R. A. Sanusi, and M. D. M. Pawley, "Efficient homogeneously weighted moving average chart for monitoring process mean using an auxiliary variable," *IEEE Access*, vol. 7, pp. 94021–94032, 2019.
[21] S. M. Anwar, M. Aslam, B. Zaman, and M. Riaz, "An enhanced double homogeneously weighted moving average control chart to monitor process location with application in automobile field," *Quality and Reliability Engineering International*, vol. 38, 2021.
[22] M. Abid, A. Shabbir, H. Z. Nazir, R. A. K. Sherwani, and M. Riaz, "A double homogeneously weighted moving average control chart for monitoring of the process mean," *Quality and Reliability Engineering International*, vol. 38, 2020.
[23] D. H. Kim and Y. C. Kim, "Wilcoxon signed rank test using ranked-set sample," *Korean Journal of Computational & Applied Mathematics*, vol. 3, no. 2, pp. 235–243, 1996.
[24] M. Abid, H. Z. Nazir, M. Riaz, and Z. Lin, "An efficient nonparametric EWMA Wilcoxon signed-rank chart for monitoring location," *Quality and Reliability Engineering International*, vol. 33, no. 3, pp. 669–685, 2017.
[25] T. E. Marlin and T. Marlin, *Process Control: Controling processes and control systems for dynamic performance*, MC Master University, Canada, 2000.
[26] S. Xiangrong, L. Yan, F. ZhengShun, and L. Jun, "A multivariable statistical process monitoring method based on multiscale analysis and principal curves," *Int. J. Innov. Comput. Inf. Control.*vol. 9, pp. 1781–1800, 2013.
[27] A. S. Ridwan, R. Muhammad, A. A. Nurudeen, and X. Min, "An EWMA monitoring scheme with a single auxiliary variable for industrial processes," *Computers & Industrial Engineering*, vol. 114, pp. 1–10, 2017.
[28] S. M. Anwar, M. Aslam, B. Zaman, and M. Riaz, "Mixed memory control chart based on auxiliary information for simultaneously monitoring of process parameters: an application in glass field," *Computers & Industrial Engineering*, vol. 156, Article ID 107284, 2021.
[29] T. Mahmood and M. Xie, "Models and monitoring of zero-inflated processes: the past and current trends," *Quality and Reliability Engineering International*, vol. 35, no. 8, pp. 2540–2557, 2019.
[30] T. Mahmood, "Generalized linear model based monitoring methods for high-yield processes," *Quality and Reliability Engineering International*, vol. 36, no. 5, pp. 1570–1591, 2020.
[31] I. M. Zwetsloot, T. Mahmood, and W. H. Woodall, "Multivariate time-between-events monitoring: an overview and some overlooked underlying complexities," *Quality Engineering*, vol. 33, 2020.

[32] J. O. Ajadi, K. Hung, M. Riaz, N. A. Ajadi, and T. Mahmood, "On the multivariate progressive control chart for effective monitoring of covariance matrix," *Journal of Quality Reliability Engineering International*, 2021.
[33] M. Aslam, O. H. Arif, and R. A. K. Sherwani, "New diagnosis test under the neutrosophic statistics: an application to diabetic patients," *BioMed Research International*, vol. 2020, pp. 1–7, Article ID 2086185, 2020.
[34] M. Albassam, N. Khan, and M. Aslam, "Neutrosophic D'agostino test of normality: an application to water data," *Journal of Mathematics*, vol. 2021, Article ID 5582102, 2021.