Modelling the dynamic pile-soil-pile interaction in a multi-layered half-space

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Summary

Within the context of railway ground-borne vibration, the dynamic pile-soil-pile interaction remains an area that has not been sufficiently investigated. Whilst a number of researchers have scrutinised the vibration response of piled-foundations, their approaches exhibit a compromise between computation time and solution accuracy. In this paper, two models of piled-foundations in a multi-layered half-space are presented; one is an efficient semi-analytical model and the other is a fully-coupled boundary element model. The pile is simulated, in both models, by an elastic bar for axial loading and an Euler-Bernoulli beam for transverse loading. A set of comparisons has been performed, including the driving point response of a single pile, the interaction between two piles and the 'far-field' response due to axial and transverse loading on the pile head. The comparisons reveal that at most frequencies the semi-analytical model predicts the driving point response and the dynamic interaction with acceptable accuracy and computational efficiency. It is highlighted, however, that the semi-analytical model with its present assumptions does not accurately approximate the 'far-field' response.

PACS no. 43.10.-a, 43.10.Ce

1. Introduction

Ground-borne vibration generated by sources such as earthquakes, railways, roads and construction activities often causes disturbance to nearby buildings that may lead to significant social and economic impacts. Usually, structural foundations, such as piled-foundations, act as a transmission path for ground-borne vibration into buildings. In some cases, however, piled-foundations can serve as wave barriers to isolate vibration when arranged in an appropriate configuration. As such, over the past four decades dynamic pile-soil-pile interaction (PSPI) has received extensive research effort. Different techniques for modelling the dynamic PSPI have been formulated; most of them were recently reviewed by Kuo and Hunt [1].

The research interest in the dynamic PSPI was sparked by the pioneering work of Poulos [2, 3] on the static behaviour of pile groups. In his work, Poulos introduced the concept of interaction factors, which express the displacement of a pile group as a function of the motion of the loaded, adjacent pile. A number of researchers have later applied this concept to study the dynamic behaviour of pile groups, which was observed to be strongly frequency-dependent [4, 5]. These findings, along with increasing interest in soil-structure interaction, have led researchers to develop analytical methods for modelling the dynamic behaviour of pile groups [6, 7]. However, despite offering considerable reduction in computation time, the analytical models only provide approximate solutions for PSPI due to ignoring the presence of neighbouring piles when calculating the interaction between two piles. The existence of neighbouring piles has two effects: the soil-stiffening effect, which dominates at low frequencies (wavelengths greater than pile spacing), and the wave scattering effect, which dominates at high frequencies (wavelengths less than the diameter of the piles) [8]. These shortcomings limit the application of such models in studying ground-borne vibration, and hence an efficient, yet rigorous analytical model is required.

The boundary element (BE) method, on the other hand, is reputed to be a reliable approach for modelling the dynamic PSPI. This is because wave radiations are inherently accounted for resulting in accurate solutions within the interior of the domain. For instance, Sen *et al.* [9] presented a BE model of a pile group, in which the piles are modelled as an elastic bar/beam and inserted into a homogeneous half-space with multiple cavities. Talbot and Hunt [8] avoided adding extra cavities in the soil by adopting instead periodic structure theory to model an infinitely long row of piles embedded in a homogeneous half-space. Maeso *et al.* [10] developed a pile group model by simulating the piles as continuum elastic solids and the soil as an isotropic homogeneous fluid-filled poroelastic material. More recently, Millán and Domínguez [11] devised a simplified BE model for pile groups in viscoelastic and poroelastic soils. All these previous BE models are a good solution for PSPI and capable of considering different types of motion of pile heads. BE models of piles, in general, can serve as a benchmark to scrutinise the effects of simplifying assumptions inherited in the analytical models. However, BE models are computationally inefficient and thus, from a practising Engineer's viewpoint, they may not be a suitable tool for design. This in turn urges the need for efficient models that account for essential aspects of the dynamic PSPI.

This paper presents two approaches to modelling piled-foundations in a multi-layered half-space. The first is an efficient semi-analytical model that simulates the soil as a horizontally layered semi-infinite system. The second is a fully-coupled model that utilises the BE method to model the soil. The pile is modelled, in both approaches, by an elastic bar for axial loading and an Euler-Bernoulli beam for transverse loading. In Section 2 the approaches adopted for modelling the ground are briefly discussed. The pile model and the coupling techniques are described in Section 3. The simulation parameters and results are presented in Section 4, and finally some findings and conclusions are highlighted in Section 5.

2. Ground modelling

To model the dynamic soil-structure interaction appropriately, it is essential to have acceptable representation of the soil. Given the low strain amplitudes of ground-borne vibration, it is plausible to assume that the soil behaves in a linear-elastic manner. In the following sections, the two techniques adopted to model the ground, which both treat the soil as a linear-elastic medium, are described.

2.1. Semi-analytical approach

Here, the soil is modelled as a horizontally layered semi-infinite system, and the response field is calculated using a hybrid formulation of the thin-layer and direct stiffness methods. For soil along the length of the pile, where there are multiple traction sources, the thin-layer method is used since it offers high density of discretization. The direct stiffness method is utilised to model the soil layers and the half-space below the tip of the pile, where there are no traction sources.

Applying this hybrid formulation, the soil response $\mathbf{u}_s(r)$ due to an area load $\mathbf{g}$ at a distance r is given by

$$\mathbf{u}_s(r) = \mathbf{H}_h(r)\mathbf{g}, \tag{1}$$

where $\mathbf{H}_h(r)$ is the frequency response function (FRF) matrix relating displacements and tractions at the frequency of interest due to an area load, which is applied as a circular load. It should be noted that in this formulation it is not possible to introduce a cavity in the soil, which leads to neglecting the volume of the piles.

2.2. Boundary element approach

In the second approach, the soil is modelled using the BE method, based on 3D Green's functions for a multi-layered half-space obtained with the aid of the ElastoDynamics Toolbox (EDT) [12]. These fundamental solutions are calculated numerically based on the direct stiffness and thin-layer methods of modelling wave propagation in layered media.

A BE mesh is generated at the soil-structure interface to simulate a vertical cylindrical cavity. The mesh consists of a number of elements with nodal collocation, at which there are three values of displacements and tractions. These variables are related at each of the N_s collocation points by

$$\mathbf{Hu} = \mathbf{Gp}, \tag{2}$$

where $\mathbf{H}$ and $\mathbf{G}$ are $3N_s \times 3N_s$ matrices describing the behaviour of the soil in terms of its density (ρ), shear modulus (μ), Poisson's ratio (ν), damping ratio (η), shear wave speed (c_s), compression wave speed (c_p) and frequency of interest (f). The $3N_s \times 1$ vectors $\mathbf{u}$ and $\mathbf{p}$ are assembled from the complex displacement and traction amplitudes of each node as

$$\begin{aligned} \mathbf{u} &= \{u_x^1\ u_y^1\ u_z^1\ u_x^2\ u_y^2\ u_z^2 \cdots u_x^{N_s}\ u_y^{N_s}\ u_z^{N_s}\}^T \\ \mathbf{p} &= \{p_x^1\ p_y^1\ p_z^1\ p_x^2\ p_y^2\ p_z^2 \cdots p_x^{N_s}\ p_y^{N_s}\ p_z^{N_s}\}^T \end{aligned}, \tag{3}$$

where $\mathbf{u}^j$ and $\mathbf{p}^j$ are the displacement and traction vectors of node j.

Equation (2) is rearranged as,

$$\begin{aligned} \mathbf{u} &= \mathbf{H}^{-1}\mathbf{Gp} \\ \mathbf{u} &= \mathbf{H}_s\mathbf{p} \end{aligned}, \tag{4}$$

in which $\mathbf{H}_s$ is the FRF matrix relating displacements and tractions at the frequency of interest. To couple the soil cavity to the pile, this FRF matrix is modified to relate displacements to forces instead of tractions by dividing it by the area of the elements in the BE mesh. Throughout the BE analysis, it is ensured that there are more than six elements for each wavelength to satisfy Domínguez recommendations [13] and achieve convergence.

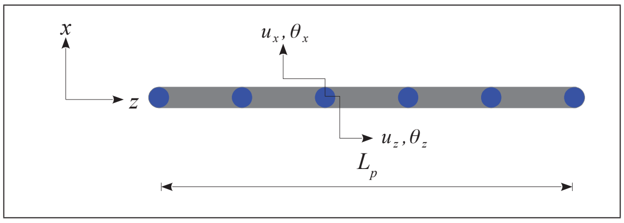

Figure 1. Pile centroid (drawn horizontally). The circles represent the nodes at which the forces are applied and the responses are calculated. Only the $x-z$ plane is shown.

3. Pile modelling and coupling

The pile model is identical in both modelling approaches, and has been used before in [14]. The pile is modelled as an elastic bar for axial loading and an Euler-Bernoulli beam for transverse loading. It is represented by its centroid axis, which has N_l equally spaced nodes (Figure 1). At each node, there are six degrees of freedom representing displacements and rotations in the three directions. The pile is assumed to be constraint-free at its ends and any local deformation of the cross-section is neglected. It must be noted that only the responses due to unit harmonic axial and transverse loads are given in this paper.

3.1. Pile response

The response of the pile to a unit harmonic force with angular frequency (ω) applied in the vertical direction (z) at node j is calculated as

$$\begin{array}{l} u_z^I(z,\omega) = A^I \cos\alpha z + B^I \sin\alpha z \text{ for } 0 \leqslant z < z^j \\ u_z^{II}(z,\omega) = A^{II} \cos\alpha z + B^{II} \sin\alpha z \text{ for } z^j \leqslant z < L_p \end{array}, \quad (5)$$

where $\alpha = \omega\sqrt{\frac{\rho_p}{E_p}}$, the superscripts I and II indicate the sections above and beneath the node j and coefficients A^I, B^I, A^{II}, B^{II} are found from the boundary conditions. L_p is the length, ρ_p is the density and E_p is Young's modulus.

The general response of the pile to a unit harmonic force with angular frequency (ω) applied in the transverse directions (x, y) at node j reads

$$\begin{array}{l} u_{x,y}^I(z,\omega) = A^I \exp(\beta z) + B^I \exp(i\beta z) + C^I \exp(-\beta z) \\ \qquad +D^I \exp(-i\beta z) \text{ for } 0 \leqslant z < z^j \\ u_{x,y}^{II}(z,\omega) = A^{II} \exp(\beta z) + B^{II} \exp(i\beta z) + C^{II} \exp(-\beta z) \\ \qquad +D^{II} \exp(-i\beta z) \text{ for } z^j \leqslant z < L_p \end{array}, \quad (6)$$

where $\beta = \left(\frac{\rho_p A_p \omega^2}{E_p I_p}\right)^{\frac{1}{4}}$ and the coefficients $A^I - D^I$, $A^{II} - D^{II}$ are found from the boundary conditions. A_p is the cross-section area and I_p is the second moment of area.

The FRF matrix of the pile centroid ($\mathbf{H}_l$) is assembled from the general responses in equations (5) and (6). This FRF matrix is transformed, for the case of the BE approach, to give the FRF matrix ($\mathbf{H}_p$) of the nodes around the pile circumference as,

$$\mathbf{H}_p = \mathbf{T}_r' \mathbf{H}_l \mathbf{T}_r, \quad (7)$$

in which $\mathbf{T}_r$ is the transformation matrix, and the size of $\mathbf{H}_p$ is $3N_s \times 3N_s$.

3.2. Coupling technique

In the semi-analytical approach, two assumptions are adopted to perform the coupling between the soil and pile. First, compatibility of displacements, i.e. soil displacement $\mathbf{u}_s(r)$ equals pile displacement ($\mathbf{u}_l$), and equilibrium of forces are assumed at the soil-pile interface. Second, the loads transmitted by the pile to the ground are distributed over a circular area of a radius equal to the pile radius.

Considering these assumptions, the coupling is achieved by combining equation (1) with

$$\mathbf{u}_l = \mathbf{H}_l (\mathbf{f}_a - \mathbf{g}), \quad (8)$$

where $\mathbf{f}_a$ is the force applied to the pile centroid, and $\mathbf{g}$ is given by

$$\mathbf{g} = [\mathbf{H}_h(0) + \mathbf{H}_l]^{-1} \mathbf{H}_l \mathbf{f}_a. \quad (9)$$

Knowing $\mathbf{g}$, the response at any point in the soil can be calculated using equation (1). For the case of multiple piles, the soil response is given as a superposition of the displacement fields due to the tractions $\mathbf{g}_i$ of each individual pile i as,

$$\mathbf{u}_s = \sum_{i=1}^{N} \mathbf{H}_h(r_i)\mathbf{g}_i \delta_{ij}, \quad (10)$$

where r_i is the distance between the receiver and i^{th} pile, and δ_{ij} is the Kronecker delta. For known applied forces $\mathbf{f}_{a_i}$, the soil forces $\mathbf{g}_i$ are given by

$$\mathbf{H}_h(0)\mathbf{g}_i - \mathbf{H}_l \mathbf{g}_i + \sum_{i=1}^{N} \mathbf{H}_h(r_{ij})\mathbf{g}_j \delta_{ij} = \mathbf{H}_l \mathbf{f}_{a_i}. \quad (11)$$

In the BE approach, the coupling is performed by applying at the interface compatibility of displacements, i.e. soil displacement ($\mathbf{U}_s$) equals pile displacement ($\mathbf{U}_p$), and equilibrium of forces. This is obtained by combining the systems in equations (4) and (7) as,

$$\begin{array}{l} \mathbf{U}_s = \mathbf{H}_s \mathbf{F}_s \\ \mathbf{U}_p = \mathbf{H}_p (\mathbf{F}_a - \mathbf{F}_s) \\ \mathbf{F}_s = (\mathbf{H}_s + \mathbf{H}_p)^{-1} \mathbf{H}_p \mathbf{F}_a \end{array}, \quad (12)$$

where $\mathbf{F}_a$ is the force applied to the pile circumference and $\mathbf{F}_s$ is the resulting soil force, from which the response at any point in the soil can be calculated.

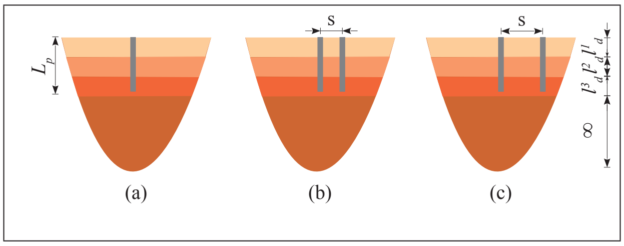

Figure 2. Pile in a multi-layered half-space: (a) single pile, (b) two piles with s = $4a$, and (c) two piles with s = $8a$, where s is the separation distance and a is the pile radius.

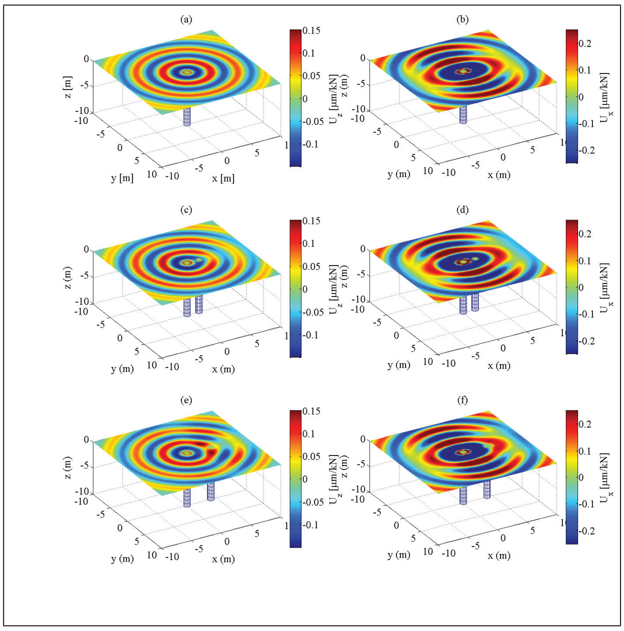

Figure 3. Free-surface real part displacement field predicted by the BE model due to a unit harmonic load on the pile head at a frequency 50Hz. (a), (c), (e) vertical response due to axial load and (b, d, f) horizontal response due transverse load for (a)-(b) single pile, (c)-(d) two piles with s = 2m, and (e)-(f) two piles with s = 4m.

4. Simulation parameters and results

To compare both modelling approaches, three cases are considered as shown in Figure 2. The pile has radius a = 0.5m, length L_p = 10m and is made of concrete with Young's modulus E_p = 50GPa, Poisson's ratio ν_p = 0.3, damping loss factor η_p = 0.01, and density ρ_p = 2500kg/m^3. The soil consists of four layers of increasing stiffness; the top three are 2m, 4m and 6m in depth, while the fourth represents the half-space. All layers have density ρ_s = 2000kg/m^3 and damping loss factor η_s = 0.06 associated with both Lamé constants. The layers have shear wave velocities c_1 = [185, 228, 260, 309]m/s and compression wave velocities c_2 = [277, 373, 485, 944]m/s respectively.

4.1. Free-surface displacement field

The first set of results shows the predictions of the BE model for the displacement field at the free surface for the cases in Figure 2 when one pile head is subject to a unit harmonic axial or transverse load at a frequency of 50Hz. In Figure 3(a), the vertical displacement field depicts concentric circular wavefronts, which is expected when a single pile is subject to axial load on its head. However, these circular wavefronts diffract when a second pile is inserted at 2m in Figure 3(c) and at 4m in Figure 3(e). This diffraction is more apparent in Figure 3(e) as the wavelength ($\sim$ 3.7m) is of the order of the separation distance.

When the pile is subject to transverse load as in Figure 3(b), the horizontal displacement field does not show cylindrical wavefronts due to the nature of the source and the dynamic interaction between the soil and the pile. These wavefronts are not much influenced by the insertion of the pile at a distance 2m as in Figure 3(d) where they slightly diffract. Yet, when a second pile is added at a distance 4m the diffraction is more pronounced as shown in Figure 3(f).

4.2. Comparisons between the models

To compare the models the driving point response of a single pile, the interaction between two piles and the free-surface response at a distance from the pile are considered. The first is used to ensure that the semi-analytical model is able to simulate the dynamic pile-soil interaction, whereas the other measures are utilised to scrutinise the dynamic PSPI and whether the semi-analytical model can predict the 'far-field' response. Since the BE model has been validated in [14] for a single pile, it is used to benchmark the semi-analytical model.

Figure 4 presents the driving point response of a single pile predicted by both models for axial and transverse loading. In general, the agreement between the semi-analytical model and the BE model is seen to be very good. The FRF due to axial loading (Figure 4(a)) of both models compares well up to 40Hz, above which almost a constant difference of less than 1dB remains. In the FRF due to transverse load (Figure 4(b)), however, this difference begins at very low frequencies and continues to increase until it reaches about 2dB at frequencies higher than 70Hz. In both loading cases, it is seen that the FRF of the semi-analytical model is higher than that of the BE model, indicating that the semi-analytical model is softer although it does not have a cavity as the BE model does. This is believed to be due to the way of calculating the soil response in the semi-analytical model by applying a circular load instead of a rigid disk load. The absence of the cavity in the semi-analytical model is believed to be another reason for these differences, which may lead to inertia effects. This is apparent in the phase plots, where the differences reach about 5^o at high frequencies particularly for axial loading.

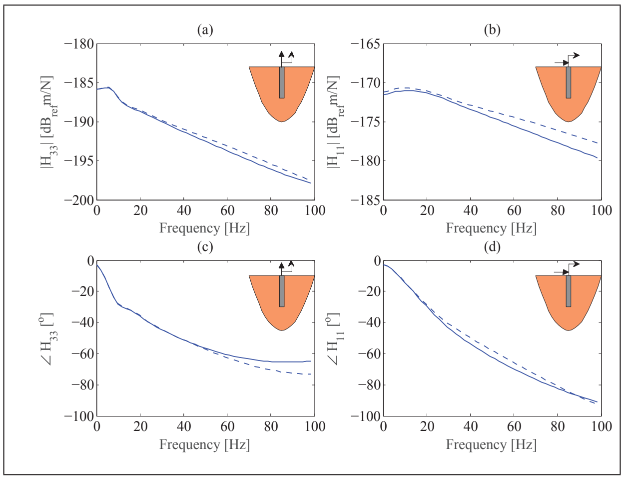

Figure 4. Driving point response of a single pile in a multi-layered half-space due to (a)-(c) axial load and (b)-(d) transverse load predicted by BE model (solid) and semi-analytical model (dashed).

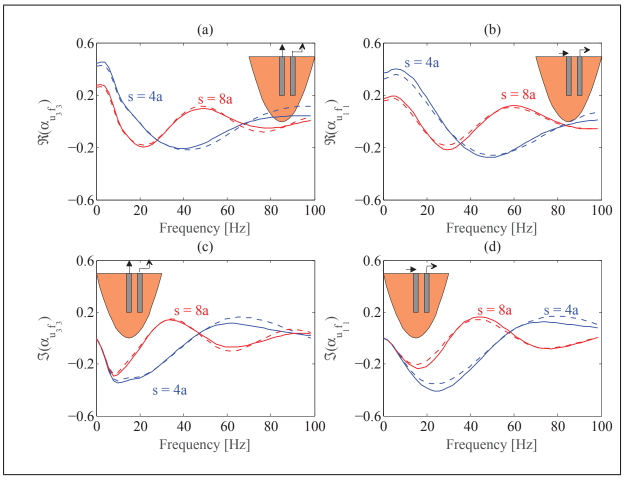

Figure 5. Dynamic interaction between two piles in a multi-layered half-space due to (a)-(c) axial load and (b)-(d) transverse load predicted by BE model (solid) and semi-analytical model (dashed). The piles separation distances are $4a$ and $8a$, where a is the pile radius.

The interaction between two piles, which is calculated at the pile head, is presented by means of the dynamic interaction factor as

$$\alpha_{u_i f_i} = \frac{u_{i_b}, f_{i_a}}{u_{i_a}, f_{i_a}}, \tag{13}$$

where u_{i_b}, f_{i_a} is the displacement i of pile b due to load i applied on pile a, and u_{i_a}, f_{i_a} is the static displacement i of pile a due to load i applied on its head.

Figure 5 shows the dynamic interaction factors between two piles at 2m and 4m separation distances. Both real (top) and imaginary (bottom) parts of the dynamic interaction factors are given for axial and transverse loading. The predictions of the semi-analytical model are generally in good agreement with those of the BE model, in particular when the separation distance is 4m. For axial loading, discrepancies appear at frequencies higher than 50Hz, while discrepancies appear at very low frequencies for transverse loading, especially for the real part (Figure 5(b)). The reason for these discrepancies is believed to be twofold: one is due to the absence of the cavity and the other is because of the procedure followed in calculating the soil response.

To this end, the semi-analytical model has shown its capability of predicting the driving point response of a single pile and the dynamic PSPI with acceptable accuracy. Previous models of piled-foundations, which mainly were concerned with seismic loading, scrutinised only the driving point responses as in Figures 4 and 5, but did not investigate the 'far-field' response – a measure that is of particular interest to ground-borne vibration. In this paper, the response at a distance from the pile is presented in terms of the insertion gain (IG) between the displacements for two piles and a single pile as

$$\mathrm{IG} = 20 \log_{10} \left(\frac{u_{i,\,\mathrm{two}}}{u_{i,\,\mathrm{single}}} \right), \tag{14}$$

where $u_{i,\,\mathrm{two}}$ is the displacement of the two piles and $u_{i,\,\mathrm{single}}$ is the displacement of the single pile at a distance i.

Figure 6 illustrates the vertical and horizontal displacement IG due to the insertion of a second pile at distances 2m and 4m. When the piles are separated by 2m, the displacement is calculated at a distance of 4m (top plots), i.e. beyond the second pile. It can be seen in Figure 6(a) that the insertion of a second pile reduces the vertical displacement due to an axial load on one pile head, and this reduction increases with frequency. The semi-analytical model qualitatively captures this behaviour, and it compares reasonably well with the BE model up to a frequency of 35Hz. Yet, above this frequency the difference between the models reaches about 10dB at some frequencies. For the case of transverse load (Figure 6(b)) a different behaviour is observed, where the insertion of a second pile slightly increases the horizontal displacement at low frequencies. The displacement then reduces at mid-frequencies, where the wavelength is of the order of the spacing between the piles (4m), and starts to increase again at frequencies higher than 70Hz. In general, the semi-analytical model captures this behaviour at frequencies below 70Hz, above which the difference reaches about 3dB.

When the piles are separated by 4m in Figure 6, the displacement is calculated at a distance of 2m (bottom plots), i.e. in between the piles. The semi-analytical model for both axial and transverse loading compares well with the BE model up to a frequency of about 40Hz. Differences of more than 4dB are observed between the models at some frequencies, particularly for axial loading. It is worth noting that in Figure 6(c) the influence of the second pile is not consistent, where the IG notably fluctuates.

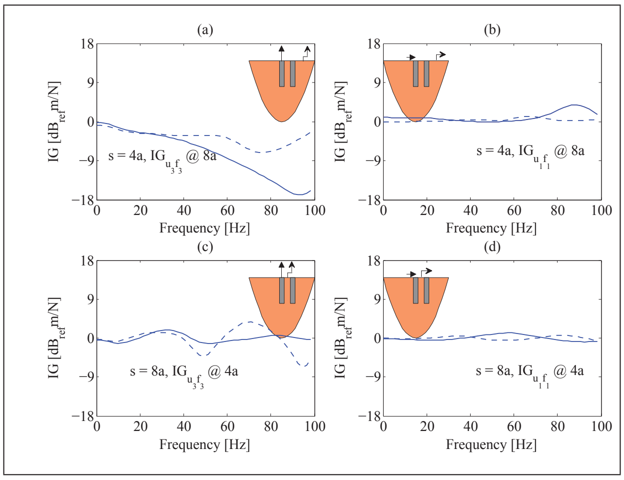

Figure 6. IG for 'far-field' displacement due to (a)-(c) axial load and (b)-(d) transverse load predicted by BE model (solid) and semi-analytical model (dashed). The IG is calculated at (a)-(b) $8a$ for separation of $4a$ and (c)-(d) $4a$ for separation of $8a$, where a is the pile radius.

It can be said from the comparisons shown in Figure 6 that the semi-analytical model with its present assumptions does not predict the 'far-field' response particularly well especially at high frequencies. However, the model has a potential to give better predictions once the problems outlined earlier, namely the absence of the cavity and calculation of the soil response, are alleviated. Given its computational efficiency, the semi-analytical model can be a reliable tool for design and prediction of the dynamic PSPI.

5. Conclusions

Investigating the dynamic interaction between piled-foundations and soil is of great importance to predictions of ground-borne vibration. Such investigation requires comprehensive yet efficient models that consider essential aspects of the problem. This paper has presented an efficient semi-analytical model and a BE model to simulate the dynamic pile-soil-pile interaction in a multi-layered half-space. The models are compared together for predictions of the driving point response of a single pile subject to axial and transverse loading, the dynamic interaction between two piles and the 'far-field' response. It has been demonstrated that the semi-analytical model captures the driving point response and the dynamic interaction between piles reasonably well, with some differences at high frequencies. However, its predictions for the 'far-field' response show some differences compared with those of the BE model, mainly at high frequencies. It is believed that the differences are due to the absence of cavities in the semi-analytical model, which may provoke more inertia effects and to the procedure followed in obtaining the soil response, which may result in a softer system. This is the subject of ongoing work.

Acknowledgement

The authors would like to thank the EPSRC for funding this research, which is part of the MOTIV project (www.motivproject.co.uk), grant numbers (EP/K006665/1, EP/K006002/1, EP/K005847/2). The authors also acknowledge the Katholieke Universiteit Leuven for providing the EDT and for some functions of BEMFUN used in the BE models.

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