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New solitary wave and computational solitons for Kundu–Eckhaus equation

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ABSTRACT

The goal of this research is to find novel optical solutions to the Kundu–Eckhaus equation, which possess crucial roles in the field of nonlinear optics. A collective variable (CV) strategy is adopted to solve governing equation including the Raman effect and quintic nonlinearity. This method is a suitable to deal with both conservative and non-conservative systems by exposing a set of equations of motion regardless of nonlinearities or dissipative components. The parameters employed in this approach are chirp, temporal position, phase, amplitude, frequency and width, namely, collective variables. The fourth order Runge–Kutta technique is a well-known numerical scheme that aims towards the solution of the resulting system of ordinary differential equations representing the variables involved in the pulse ansatz. This technique presents the evolution of pulse parameters with regard to propagation variables. The graphical profiles at suitable values of pulse parameters are also provided. The unified technique is also applied to find soliton solutions. The obtained solution is a periodic solitary wave, showed graphically. The results developed in this article are found to be new in the literature and the approach utilized, can be applied to solve a variety of nonlinear problems in the mathematical sciences.

Introduction

Nonlinear wave equations are studied extensively in the domains of applied mathematics and physics, as well as biosciences and engineering. These equations show how dynamical systems can exhibit a wide range of processes [1–3]. Nonlinear waves can be found in both physical and natural structures, atmospheric and water waves, nonlinear optics and disturbance in plasmas and hydrodynamics are all relevant applications. The propagation of light in nonlinear optical fibers and planar waveguides is modeled as nonlinear Schrödinger equation (NLSE) [4–8].

The NLSE is an eminently integrable system with soliton and breather solutions. Optical solitons are the data transmitters in the innovative optical fiber communication system because they travel over long distances without distortion of profile as well as conserve energy [9–12]. Because of the widespread social need for high bit rates in optical fiber transmission and developments in laser technology, it is both required and practical to produce shorter, high-frequency pulses in fiber by enhancing the intensity of the incident light. This substantially aids the scrutiny of higher order nonlinear phenomena in optics, which are frequently simulated using the NLSE with cubic-quintic components.

The Kundu–Eckhaus equation (KEE) is a nonlinear partial differential equation belonging to the nonlinear Schrödinger class that was independently proposed for the propagation of waves in dispersive medium by Anjan Kundu and Wiktor Eckhaus [13,14]. In quantum and nonlinear optics, the KEE can reliably predict the path of ultrashort pulses and it is used to analyze the optical properties of femtosecond lasers. In mechanics, KEE can look at the sustainability of Stokes waves in weakly nonlinear diffusive media. In plasma physics, it can be used to represent ion-acoustic waves. Some analytical such as self-localized KEE solutions, are found using approaches such as Backlünd and Darboux transformations, the first integral and the exp-function techniques. While single and dual self-localized solitons and N-soliton solutions have also been discovered [15–21].

This model has been studied by many researchers by employing different techniques due to its applications in different fields of science and engineering. Some of them are discussed here. Nisar et al. has obtained the complete spectrum of soliton solutions for the new time-fractional perturbed Boussinesq-like equation in 2022 [22]. In 2022, Akinyemi et al. worked on four different forms of generalized (2+1)-dimensional Boussinesq–Kadomtsev–Petviashvili equation to obtain numerous exact solutions [23]. Ntiamoah et al. obtained soliton, breather and approximate solutions of the higher-order modified

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Korteweg–de Vries equation [24]. Yépez-Martinez et al. investigated the soliton solutions of perturbed NLSE [25]. Gomez et al. investigated the generalized Chen-Lee-Liu model with higher order nonlinearity and achieved the optical solitons [26].

In this manuscript, we use the Ritz-like methodology combined with He's variational principle to find the KEE solitons. Regardless of dissipative components or nonlinearities, the collective variable technique presents equations of motion for both non-conservative and conservative models. Furthermore, the method pioneered by Boesch et al. [27] combines an analytical process with a computational methodology or semi-analytical process to examine the model under discussion. For analytical approximation, the Runge–Kutta technique of order four is utilized to integrate the developed system of differential equations. This approach separates the suggested solution of considered equation into two parts: soliton and residual. The number of Gaussian pulse parameters that may be implemented is computed by the governing physical model. The governing field equation is converted into CVs when a mapping is applied. Furthermore, components like amplitude, phase, width, frequency, chirp and temporal position affect soliton solutions. This approach is efficient, powerful, and up-to-date for identifying solutions to diverse nonlinear models [28–31].

The manuscript is formatted as follows: Section "Mathematical methodology" contains the governing model. The methodology is described in SEction "Soliton parameter dynamics". Soliton solutions were constructed and their graphical illustrations were supplied in Sections "Graphical Representation" and "Results and discussion". Section "Exact solution using Unified Method" contains the exact solutions by unified method along the graphical representation. Finally, in Section "Conclusion", there are some closing observations.

Governing model

In this article, the Kundu–Eckhaus equation to be considered is

$$\imath q_t + q_{xx} + \beta|q|^2 q + 4\delta^2|q|^4 q + \imath\delta^2(|q|^2)_x q = 0. \tag{1}$$

For the aforementioned model, The dependent variable $q(x,t)$ denotes a complex-valued wave profile, whereas t and x denote temporal and spatial variables, respectively.

Mathematical methodology

To demonstrate the proposed approach, the Kundu–Eckhaus solution should be divided into two parts: residual and soliton components. Solitons rely on CVs to characterize amplitude, pulse, chirp, width, temporal location, frequency, and other factors, according to the premise. The phase space of pulses and the dynamical arrangement of the soliton boundary continue to increase when CVs are initiated. The solution's residue is nearly reduced to zero.

Moreover, the soliton field $q(x,t)$ is augmented to include $\rho(x,t)$ for the soliton portion and $\eta(x,t)$ for the residual portion, with x representing the spatial component and t indicating the temporal component. Thus,

$$q(x,t) = \rho(x,t) + \eta(x,t), \tag{2}$$

$$q(x,t) = \rho(u_1(x), u_2(x), \dots, u_K(x), t) + \eta(x,t). \tag{3}$$

The residual energy is calculated as follows

$$E = \int_{-\infty}^{\infty} |\eta|^2 dt = \int_{-\infty}^{\infty} |q - \rho(u_1(x), u_2(x), \dots, u_K(x), t)|^2 dt, \tag{4}$$

$$\begin{aligned} N_j = \frac{\partial E}{\partial u_j} &= \frac{\partial}{\partial u_j}\int_{-\infty}^{\infty} \eta\eta^* dt, \\ &= \int_{-\infty}^{\infty} \frac{\partial}{\partial u_j}\eta\eta^* dt, \\ &= \int_{-\infty}^{\infty} \left(\eta^* \frac{\partial \eta}{\partial u_j} + \frac{\partial \eta^*}{\partial u_j}\eta\right) dt. \end{aligned} \tag{5}$$

By defining the inner product as follows,

$$\langle \varrho_1, \varrho_2 \rangle = \int_{-\infty}^{\infty} \varrho_1(t)\varrho_2(t) dt. \tag{6}$$

The inner product $\langle \mathbf{p}, \mathbf{q} \rangle$ that is the generalized form of dot product has the property

$\langle \mathbf{p}, \mathbf{q} \rangle = \langle \mathbf{p}, \mathbf{q} \rangle^*$. Therefore, we write Eq. (5) using the given definition

$$N_j = \langle \eta^*, \frac{\partial \eta}{\partial u_j} \rangle + \langle \frac{\partial \eta^*}{\partial u_j}, \eta \rangle$$

$$= \langle \frac{\partial \eta^*}{\partial u_j}, \eta \rangle + \langle \frac{\partial \eta^*}{\partial u_j}, \eta \rangle = 2Re\left(\langle \frac{\partial \eta^*}{\partial u_j}, \eta \rangle\right) \tag{7}$$

$$= 2Re\left(\langle \frac{\partial (q(x,t) - \rho(u_1, u_2, \dots, u_K, t))^*}{\partial u_j}, \eta \rangle\right) \tag{8}$$

$$= 2Re\left(\langle \frac{\partial q^*(x,t)}{\partial u_j} - \frac{\partial \rho^*(u_1, u_2, \dots, u_K, t)}{\partial u_j}, \eta \rangle\right)$$

$$= -2Re\left(\langle \frac{\partial \rho^*}{\partial u_j}, \eta \rangle\right) = -2Re\left(\int_{-\infty}^{\infty} \frac{\partial \rho^*}{\partial u_j}\eta dt\right).$$

Re denotes the real part and $\frac{\partial q^*(x,t)}{\partial u_j} = 0$. We may now write $\eta(x,t)$ using Eq. (3) as

$$\eta(x,t) = q(x,t) - \rho(u_1, u_2, \dots, u_K, t), \tag{9}$$

which gives the following relation

$$\frac{\partial \eta}{\partial u_j} = -\frac{\partial \rho}{\partial u_j},$$
$$\frac{\partial \eta^*}{\partial u_j} = -\frac{\partial \rho^*}{\partial u_j}.$$

Inserting the above equation in Eq. (5) yields

$$\begin{aligned} N_j = \frac{\partial E}{\partial u_j} &= \int_{-\infty}^{\infty} \left(-\frac{\partial \rho}{\partial u_j}\eta^* - \frac{\partial \rho^*}{\partial u_j}\eta\right) dt, \\ &= -\int_{-\infty}^{\infty} \left(\frac{\partial \rho}{\partial u_j}\eta^* + \frac{\partial \rho^*}{\partial u_j}\eta\right) dt, \end{aligned} \tag{10}$$

or rather

$$N_j = -2Re\int_{-\infty}^{\infty} \left(\frac{\partial \rho^*}{\partial u_j}\eta\right) dt. \tag{11}$$

Again, we also get

$$\begin{aligned} \dot{N}_j = \frac{dN_j}{dx} &= -2Re\int_{-\infty}^{\infty} \frac{d}{dx}\left(\frac{\partial \rho^*}{\partial u_j}\eta\right) dt, \\ &= -2Re\int_{-\infty}^{\infty} \left(\frac{\partial \rho^*}{\partial u_j}\frac{\partial \eta}{\partial x} + \eta\left(\frac{\partial}{\partial x}\frac{\partial \rho^*}{\partial u_j}\right)\right) dt, \end{aligned} \tag{12}$$

and

$$\frac{\partial \rho^*}{\partial x} = \sum_{n=1}^{K} \frac{\partial \rho^*}{\partial u_n}\frac{\partial u_n}{\partial x}. \tag{13}$$

Then by inserting Eq. (13) into Eq. (12), we gain

$$\begin{aligned} \dot{N}_j &= -2Re\int_{-\infty}^{\infty} \left(\frac{\partial \rho^*}{\partial u_j}\frac{\partial \eta}{\partial x} + \eta\frac{\partial}{\partial u_j}\left(\sum_{n=1}^{K} \frac{\partial \rho^*}{\partial u_n}\frac{\partial u_n}{\partial x}\right)\right) dt, \\ &= -2Re\left(\int_{-\infty}^{\infty} \frac{\partial \rho^*}{\partial u_j}\frac{\partial \eta}{\partial x} dt + \sum_{n=1}^{K}\int_{-\infty}^{\infty}\left(\frac{\partial^2 \rho^*}{\partial u_j \partial u_n}\frac{\partial u_n}{\partial x}\eta\right) dt\right). \end{aligned} \tag{14}$$

Importantly, Dirac's principal states that a function is approximately zero when changes in components are not equivalent to zero. Hence, the system will evolve such that N_j are a minimum and the equations of the constraints are obtained as

$$N_j \approx 0, \tag{15}$$

$$\dot{N}_j \approx 0. \tag{16}$$

Then by Ref. [32], we obtain the following equation by inserting Eq. (2) into Eq. (1).

$$(\rho+\eta)_x = \imath(\rho+\eta)_{tt} + \imath\beta|(\rho+\eta)|^2(\rho+\eta) + 4\imath\delta^2|(\rho+\eta)|^4(\rho+\eta) - \delta^2\left(|\rho+\eta|^2\right)_t(\rho+\eta). \tag{17}$$

Moreover, from Eq. (3), we have,

$$q_x = \frac{\partial\rho(u_1(x),u_2(x),\dots,u_K(x),t)}{\partial x} + \frac{\partial\eta(x,t)}{\partial x}. \tag{18}$$

By merging Eqs. (17) and (18), we obtain

$$\frac{\partial\eta}{\partial x} = -\sum_{n=1}^{K}\frac{\partial\rho}{\partial u_n}\frac{du_n}{dx} + \varsigma, \tag{19}$$

with

$$\varsigma = \imath(\rho+\eta)_{tt} + \imath\beta|(\rho+\eta)|^2(\rho+\eta) + 4\imath\delta^2|(\rho+\eta)|^4(\rho+\eta) - \delta^2\left(|\rho+\eta|^2\right)_t(\rho+\eta). \tag{20}$$

Likewise, we observe that

$$\dot N_j = -2Re\int_{-\infty}^{\infty}\left(\frac{\partial\rho^*}{\partial u_j}\frac{\partial\eta}{\partial x}\right)dt + \sum_{n=1}^{K}\int_{-\infty}^{\infty}\left(\frac{\partial^2\rho^*}{\partial u_j\partial u_n}\frac{du_n}{dx}\right)dt, \tag{21}$$

$$\dot N_j = 2Re\sum_{n=1}^{K}\int_{-\infty}^{\infty}\left(\frac{\partial\rho^*}{\partial u_j}\frac{\partial\rho}{\partial u_n} - \frac{\partial^2\rho^*}{\partial u_j\partial u_n}\eta\right)dt\frac{du_n}{dx} - 2Re\int_{-\infty}^{\infty}\frac{\partial\rho^*}{\partial u_j}\varsigma dt. \tag{22}$$

It may be written in a more compact form as

$$\dot N_j = \frac{\partial N}{\partial u}\dot U + \mathbf{B}, \tag{23}$$

$$\dot{\mathbf{U}} = -\left[\frac{\partial\mathbf{N}}{\partial\mathbf{u}}\right]^{-1}[\mathbf{B}], \tag{24}$$

where

$$\frac{\partial N}{\partial u} = \begin{bmatrix} \frac{\partial N_1}{\partial u_1} & \frac{\partial N_1}{\partial u_2} & \frac{\partial N_1}{\partial u_3} & \frac{\partial N_1}{\partial u_4} & \frac{\partial N_1}{\partial u_5} & \frac{\partial N_1}{\partial u_6} \\ \frac{\partial N_2}{\partial u_1} & \frac{\partial N_2}{\partial u_2} & \frac{\partial N_2}{\partial u_3} & \frac{\partial N_2}{\partial u_4} & \frac{\partial N_2}{\partial u_5} & \frac{\partial N_2}{\partial u_6} \\ \frac{\partial N_3}{\partial u_1} & \frac{\partial N_3}{\partial u_2} & \frac{\partial N_3}{\partial u_3} & \frac{\partial N_3}{\partial u_4} & \frac{\partial N_3}{\partial u_5} & \frac{\partial N_3}{\partial u_6} \\ \frac{\partial N_4}{\partial u_1} & \frac{\partial N_4}{\partial u_2} & \frac{\partial N_4}{\partial u_3} & \frac{\partial N_4}{\partial u_4} & \frac{\partial N_4}{\partial u_5} & \frac{\partial N_4}{\partial u_6} \\ \frac{\partial N_5}{\partial u_1} & \frac{\partial N_5}{\partial u_2} & \frac{\partial N_5}{\partial s_3} & \frac{\partial N_5}{\partial u_4} & \frac{\partial N_5}{\partial u_5} & \frac{\partial N_5}{\partial u_6} \\ \frac{\partial N_6}{\partial u_1} & \frac{\partial N_6}{\partial u_2} & \frac{\partial N_6}{\partial u_3} & \frac{\partial N_6}{\partial u_4} & \frac{\partial N_6}{\partial u_5} & \frac{\partial N_6}{\partial u_6} \end{bmatrix},$$

while

$$\dot{\mathbf{U}} = \begin{bmatrix}\dot u_1\\ \dot u_2\\ \dot u_3\\ \dot u_4\\ \dot u_5\\ \dot u_6\end{bmatrix} \quad\text{and}\quad \mathbf{B} = \begin{bmatrix}B_1\\ B_2\\ B_3\\ B_4\\ B_5\\ B_6\end{bmatrix}.$$

Finally, we have B_j, which is as follows,

$$\begin{aligned} B_j &= -2Re\int_{-\infty}^{\infty}\frac{\partial\rho^*}{\partial u_j}\varsigma dt,\\ &= -2Re\int_{-\infty}^{\infty}\imath\rho_{tt}\frac{\partial\rho^*}{\partial u_j}dt - 2Re\int_{-\infty}^{\infty}\imath\eta_{tt}\frac{\partial\rho^*}{\partial u_j}dt\\ &- 2Re\int_{-\infty}^{\infty}\imath\beta\rho|\rho|^2\frac{\partial\rho^*}{\partial u_j}dt - 2Re\int_{-\infty}^{\infty}\imath\beta\eta|\eta|^2\frac{\partial\rho^*}{\partial u_j}dt\\ &- 2Re\int_{-\infty}^{\infty}4\imath\delta^2\rho|\rho|^4\frac{\partial\rho^*}{\partial u_j}dt - 2Re\int_{-\infty}^{\infty}4\imath\delta^2\eta|\eta|^4\frac{\partial\rho^*}{\partial u_j}dt\\ &+ 2Re\int_{-\infty}^{\infty}\delta^2\rho\left(|\rho|^2\right)_t\frac{\partial\rho^*}{\partial u_j}dt + 2Re\int_{-\infty}^{\infty}\delta^2\eta\left(|\eta|^2\right)_t\frac{\partial\rho^*}{\partial u_j}dt, \end{aligned} \tag{25}$$

where $j = 1,2,\dots,K$.

Soliton parameter dynamics

To derive the set of equations for the CVs, we apply the constraint equations formerly generated by producing the function $\rho(u_1,u_2,\dots,u_n)$. However, we assume that the soliton function ρ is built up of six CVs in the following form by using the Gaussian ansatz.

$$\rho = u_1\exp\left(\frac{\imath}{2}u_4(t-u_2)^2 - \frac{(t-u_2)^2}{u_3^2} + \imath u_5(t-u_2) + \imath u_6\right), \tag{26}$$

where $u_p(p = 1,2,3,\dots,6)$ representing the center position, phase, amplitude, width, frequency and chirp. The primary approximation to construct differential equations for CVs, termed the lowest order CV assumption, is utilized. This concept reduces the residual component to zero, $(\eta(x,t)=0)$, $(\frac{\partial N}{\partial u})$ is as below (see Box I).
while

$$B_1 = 0, \tag{28}$$

$$B_2 = \sqrt{2\pi}u_1^2\left[\frac{4\sqrt{3}}{3}u_1^4u_5u_3 - \frac{3}{4}u_3^3u_4^2u_5 + \frac{\beta}{\sqrt{2}}u_1^2u_3u_5 - u_3u_5^2 - \frac{\delta^2u_1^2}{\sqrt{2}u_3} - \frac{3u_5}{u_3}\right], \tag{29}$$

$$B_3 = -\sqrt{2\pi}u_1^2u_4, \tag{30}$$

$$B_4 = \sqrt{2\pi}u_1^2u_3\left[\frac{-\sqrt{3}}{18}\delta_2u_1^4u_3^2 + \frac{3}{32}u_3^4u_4^2 - \frac{1}{16\sqrt{2}}\beta u_1^2u_3^3 + \frac{1}{8}u_3^2u_5^2 - \frac{1}{8}\right], \tag{31}$$

$$B_5 = \frac{\sqrt{2\pi}u_1^2u_3^3u_4u_5}{2}, \tag{32}$$

$$B_6 = \sqrt{2\pi}u_1^2\left[\frac{4\sqrt{3}}{3}\delta^2u_1^4u_3 + u_3u_5^2 + \frac{1}{4}u_3^3u_4^2 - \frac{\beta u_1^2u_3}{\sqrt{2}} + \frac{1}{u_3}\right]. \tag{33}$$

Likewise, by inserting Eqs. (27) and (28)–(33) into Eq. (24), we acquire

$$\begin{aligned} \dot u_1 &= -u_1u_4,\\ \dot u_2 &= -u_3\left(-\frac{4\sqrt{3}}{3}u_1^4u_3u_5 + \frac{\beta\sqrt{2}}{2}u_1^2u_3 - u_3u_5^3 - \frac{3}{4}u_3^3u_4^2u_5 - \frac{\sqrt{2}\delta^2u_1^2}{2u_3} - \frac{3u_5}{u_3}\right)\\ &- \frac{1}{2}u_4^2u_3^4u_5\\ &- u_3u_5\left(\frac{1}{4}u_3^3 - \frac{1}{2}\beta\sqrt{2}u_1^2u_3 + u_3u_5^2 - \frac{4\sqrt{3}}{3}\delta^2u_1^4u_3 + \frac{1}{u_3}\right),\\ \dot u_3 &= 2u_3u_4,\\ \dot u_4 &= -\frac{32}{u_3^4}\left(\frac{-\sqrt{3}}{18}\delta^2u_1^4u_3^2 - \frac{1}{32}\beta\sqrt{2}u_1^2u_3^2 + \frac{1}{8}u_3^2u_5^2 - \frac{1}{8} + \frac{3}{32}u_3^4u_4^2\right)\\ &+ \frac{4}{u_3^3}\left(\frac{1}{4}u_3^3u_4^2 - \frac{1}{2}\beta\sqrt{2}u_1^2u_3 + u_3u_5^2 - \frac{4\sqrt{3}}{3}\delta^2u_1^4u_3 + \frac{1}{u3}\right),\\ \dot u_5 &= -u_4u_3\left(\frac{4\sqrt{3}}{3}u_1^4u_3u_5 + \frac{1}{2}\beta\sqrt{2}u_1^2u_3u_5 - u_3u_5^3\right.\\ &\left.-\frac{3}{4}u_3^3u_5u_4^2 - \frac{1}{2u_3}\delta^2u_1^2 - \frac{3u_5}{u_3}\right)\\ &- u_3u_4u_5\left(\frac{1}{4}u_3^3u_4^2 - \frac{1}{2}\beta\sqrt{2}u_1^2u_3 + u_3u_5^2 - \frac{4\sqrt{3}}{3}\delta^2u_1^4u_3 + \frac{1}{u3}\right)\\ &- \frac{2\left(\frac{1}{4}u_4^2u_3^9 + u_3^5\right)u_4u_5}{u_3^5},\\ \dot u_6 &= -u_5u_3\left(\frac{4\sqrt{3}}{3}u_1^4u_3u_5 + \frac{1}{2}\beta\sqrt{2}u_1^2u_3u_5 - u_3u_5^3\right. \end{aligned}$$

$$\left[\frac{\partial N}{\partial u}\right]^{-1} = \begin{bmatrix} \frac{3}{2\sqrt{2\pi}u_3} & 0 & \frac{-1}{\sqrt{2\pi}u_1} & 0 & 0 & 0 \\ 0 & \frac{u_3}{\sqrt{2\pi}u_1^2} & 0 & 0 & \frac{u_3u_4}{\sqrt{2\pi}u_1^2} & \frac{u_3u_5}{\sqrt{2\pi}u_1^2} \\ \frac{-1}{\sqrt{2\pi}u_1} & 0 & \frac{\sqrt{\frac{2}{\pi}}u_3}{u_1^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{16\sqrt{\frac{2}{\pi}}}{u_1^2u_3^5} & 0 & \frac{-2\sqrt{\frac{2}{\pi}}}{u_1^2u_3^3} \\ 0 & \frac{u_3u_4}{\sqrt{2\pi}}u_1^2 & 0 & 0 & \frac{\frac{32\pi^{\frac{5}{2}}u_1^8}{8\sqrt{2}}\left(\frac{u_4^2u_3^9}{4}+u_3^5\right)}{\pi^3 s_1^{10}s_3^8} & \frac{u_3u_4u_5}{\sqrt{2\pi}u_1^2} \\ 0 & \frac{u_3u_5}{\sqrt{2\pi}u_1^2} & 0 & \frac{-2\sqrt{\frac{2}{\pi}}}{u_1^2u_3^3} & \frac{u_3u_4u_5}{\sqrt{2\pi}u_1^2} & \frac{\frac{32\pi^{\frac{5}{2}}u_1^8}{32\sqrt{2}}\left(u_5^2u_3^9+\frac{3u_3^7}{2}\right)}{\pi^3u_1^{10}u_3^8} \end{bmatrix}, \tag{27}$$

Box I.

$$-\frac{3}{4}u_3^3u_5u_4^2 - \frac{1}{2u_3}\sqrt{2}\delta^2u_1^2 - \frac{3u_5}{u_3}\Bigg)$$
$$+\frac{4}{u_3^2}\left(\frac{-\sqrt{3}}{18}\delta^2u_3^2u_1^4 - \frac{\beta}{32}\sqrt{2}u_1^2u_3^2 + \frac{1}{8}u_3^2u_5^2 - \frac{1}{8} + \frac{3}{32}u_3^4u_4^2\right)$$
$$-\frac{1}{2}u_4^2u_3^4u_5^2$$
$$-\frac{1}{u_3^8}\left(u_5^2u_3^9 + 3\frac{u_3^7}{2}\right)\Bigg(\frac{1}{4}u_3^3u_4^2 - \frac{1}{2}\beta\sqrt{2}u_1^2u_3 + u_3u_5^2$$
$$-\frac{4\sqrt{3}}{3}\delta^2u_1^4u_3 + \frac{1}{u\,3}\Bigg).$$

Graphical representation

Non chirping effect

Chirping effect

Results and discussion

In this article, we exhibit several computational simulations of the propagation of pulse variables to verify the effectiveness of the CV approach used on the Kundu–Eckhaus equation. The explicit Runge–Kutta technique of order four was used to get numerical solutions for the resulting equations of motion in the form of a set of six differential equations. In addition, we provide the graphical representations of the solitons features in Figs. 1 and 2 with the chirping and non-chirping effects for the corresponding parameter values: $\beta = 1, \ \ \delta = \frac{1}{4}$.

The amplitude, width, frequency, temporal position, chirp and phase of pulse are represented by the CVs $u_1, u_2, u_3, u_4, u_5, u_6$. It is worthy to note that when the pulse propagates, CVs oscillate periodically with uniform wavelengths. Furthermore, the free energy was evaluated using u_1, u_3, u_4, and u_5, which stand for amplitude, width, chirp, and pulse phase, respectively. The CV method simplified the study of the resulting equations and clarified the influence of variables in the KE model. Further, Unified Method (UM) has been applied in order to extract new solitary solution, of the governing model, in form of polynomial.

Remark. To the best of our knowledge, the obtained results are found to be new by comparing with the existing literature [33–38].

Exact solution using unified method

In this portion, the UM method is applied to extract solitary solution to Eq. (1). We take initial with the traveling wave solution of Eq. (1) as follow

$$q(x,t) = q(\alpha)\, e^{\iota\Psi}, \quad \psi = \mu x + vt, \quad \alpha = \iota\xi(x - 2\mu t), \tag{34}$$

where μ shows the wave number of soliton and v is frequency of soliton. Therefore, by utilizing

Eq. (34) in (1), we obtain [39]

$$-4\delta^2q^5 + 2\delta^2q^2q'\xi + qv - \beta q^3 + q\mu^2 + q''\xi^2 = 0, \tag{35}$$

that is

$$-(v+\mu^2)q - \xi^2q'' + \beta q^3 + 4\delta^2q^5 - 2\delta^2\xi q^2q' = 0. \tag{36}$$

Now by taking in consideration the homogeneous balance principle between the highest order derivative and non linear terms appearing in Eq. (36), we get $n = 1/2$. We will utilize the following transformation in order to obtain closed form solution:

$$q(\alpha) = u^{1/2}, \tag{37}$$

then it will transform Eq. (36) into the following ODE

$$-4(v+\mu^2)u^2 + \xi^2u'^2 - 2\xi^2uu'' + 4\beta u^3 + 16\delta^2u^4 - 4\delta^2\xi u^2u' = 0, \tag{38}$$

applying balancing criteria again, we get the balancing number $n = 1$.

Solitary solution

Analyzing Eq. (38) has the polynomial solution [40] as

$$u(\alpha) = \sum_{m=0}^{n} q_m\theta^m(\alpha), \quad q_m \neq 0. \tag{39}$$

In aforementioned equation, q_m are unknown parameters and function $u(\alpha)$ has obtained by solving the auxiliary equation:

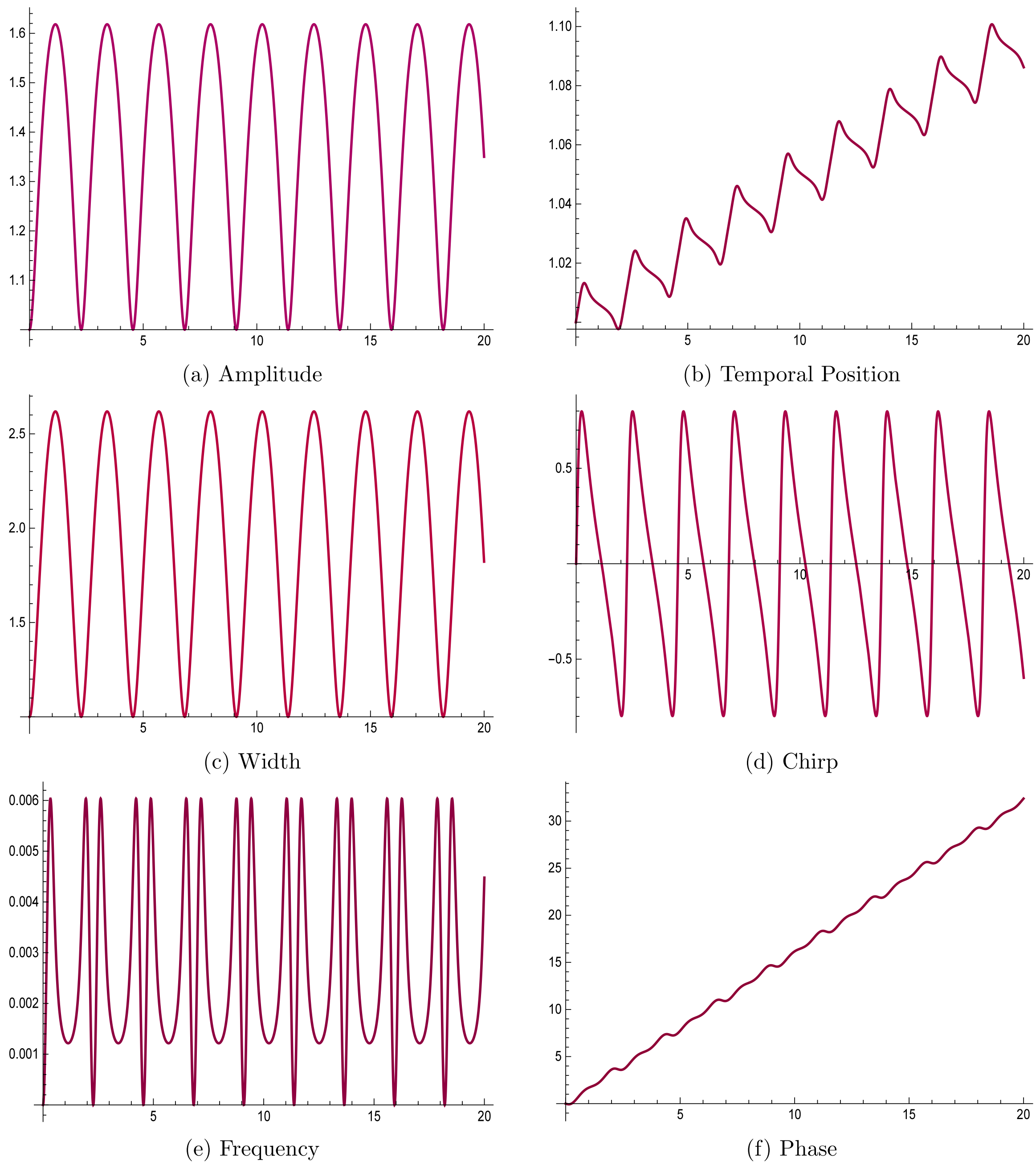
$$(\theta'(\alpha))^\sigma = \sum_{m=0}^{\sigma\upsilon} c_m\theta^m(\alpha), \quad \sigma = 1,2, \tag{40}$$

where c_m are arbitrary constants. The homogeneous balancing condition must be inserted between the highest derivative and the highest nonlinear term in Eq. (2) in order to define numerical values of n in terms of υ, and υ must be determined using the consistency criterion. The relation $n = \upsilon - 1$, where $\upsilon > 1$, has been obtained, by applying balancing criteria on (38). Here we look at the solutions when $\upsilon = 2$ with $\sigma = 1$. Therefore we obtain

$$u(\alpha) = q_0 + q_1\theta(\alpha), \tag{41}$$
$$\theta'(\alpha) = c_0 + c_1\theta(\alpha) + c_2\theta^2(\alpha).$$

A system of non-linear equations is obtained by substituting Eq. (41) into Eq. (38).

$$-4\delta^2\xi c_2{q_1}^3 + 16\delta^2{q_1}^4 - 3\xi^2{c_2}^2{q_1}^2 = 0, \tag{42}$$
$$-4\,\delta^2\xi c_1{q_1}^3 - 8\,\delta^2\xi c_2q_0{q_1}^2 + 64\,\delta^2q_0{q_1}^3 - 4\,\xi^2c_1c_2{q_1}^2$$



(a) Amplitude (b) Temporal Position (c) Width (d) Chirp (e) Frequency (f) Phase

Fig. 1. Analyzing collective variables $u_p, (p = 1, 2, 3, \dots, 6)$ versus propagation distance x with $u_1 = u_2 = u_3 = 1$, $u_4 = u_5 = u_6 = 0$.

$$-4\,\xi^2 {c_2}^2 q_0 q_1 + 4\,\beta {q_1}^3 = 0, \tag{43}$$

$$-4\,(\alpha^2+\beta)\,{q_1}^2 - \xi^2 {q_1}^2\left(2\,c_0 c_2 + {c_1}^2\right) - 6\,\xi^2 q_0 q_1 c_1 c_2 + 12\,\beta q_0 {q_1}^2 + 96\,\delta^2 {q_0}^2 {q_1}^2 - 4\,\delta^2 \xi {q_1}^3 c_0 - 8\,\delta^2 \xi q_0 {q_1}^2 c_1 - 4\,\delta^2 \xi {q_0}^2 q_1 c_2 = 0, \tag{44}$$

$$-8\,(\alpha^2+\beta)\,q_0 q_1 - 2\,\xi^2 q_0 q_1\left(2\,c_0 c_2 + {c_1}^2\right) + 12\,\beta {q_0}^2 q_1 + 64\,\delta^2 {q_0}^3 q_1 - 8\,\delta^2 \xi q_0 {q_1}^2 c_0 - 4\,\delta^2 \xi {q_0}^2 q_1 c_1 = 0, \tag{45}$$

$$-4\,(\alpha^2+\beta)\,{q_0}^2 + \xi^2 {q_1}^2 {c_0}^2 - 2\,\xi^2 q_0 q_1 c_0 c_1 + 4\,\beta {q_0}^3 + 16\,\delta^2 {q_0}^4 - 4\,\delta^2 \xi {q_0}^2 q_1 c_0 = 0. \tag{46}$$

This system will be further solve by using softwares like maple or mathematica. The following outcomes are obtained

$$v = -\frac{2\,\mu^2\delta^4 - 8\,\mu^2\delta^2 + 8\,\mu^2 - \beta^2}{2\left(\delta^2-2\right)^2}, \quad \xi = \frac{1}{4},$$
$$c_0 = \frac{4q_0(4\delta^2 q_0 + \beta - 8q_0)}{q_1(\delta^2-2)}, \quad c_1 = \frac{4(8\delta^2 q_0 - 16q_0)}{(\delta^2-2)}, \quad c_2 = 16q_1. \tag{47}$$

By solving the auxiliary equation $\theta'(\alpha) = c_0 + c_1\theta(\alpha) + c_2\theta^2(\alpha)$ and substituting together with

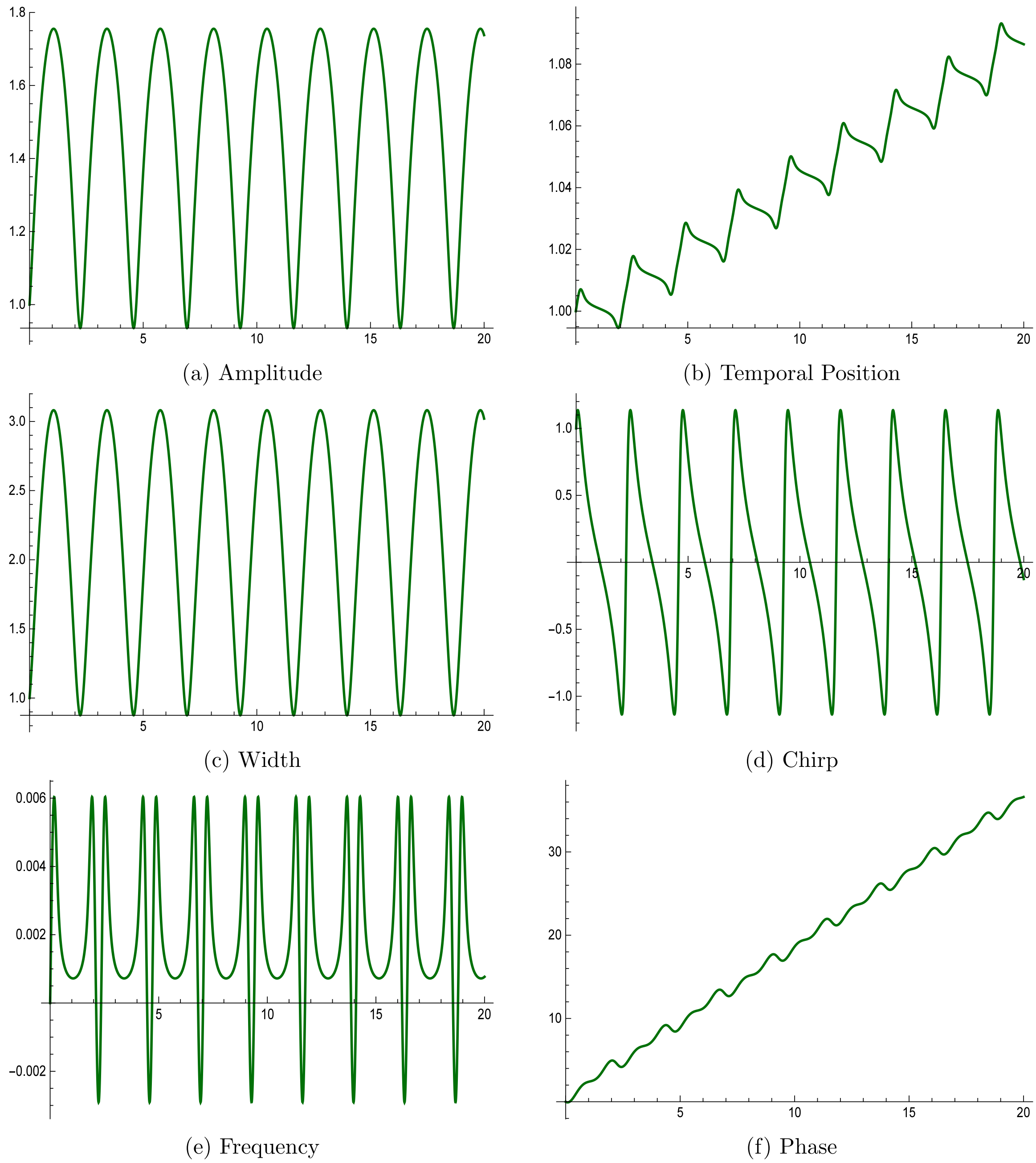


Fig. 2. Analyzing collective variables $u_p, (p = 1, 2, \ldots, 6)$ versus propagation distance x with $u_1 = u_2 = u_3 = u_4 = 1,\ u_5 = u_6 = 0$.

Eq. (47), we find that, Eq. (39) has the solitary solution.

$$u = \frac{(-\delta^2 + 2)\sqrt{\frac{\beta}{(\delta^2-2)^2}}\tanh(2\alpha\frac{\beta}{(\delta^2-2)^2}) - \beta}{8\delta^2 - 16}. \tag{48}$$

Substituting Eq. (48) in Eq. (38), we obtain

$$q(\alpha) = \sqrt{\frac{(-\delta^2 + 2)\sqrt{\frac{\beta}{(\delta^2-2)^2}}\tanh(2\alpha\frac{\beta}{(\delta^2-2)^2}) - \beta}{8\delta^2 - 16}}. \tag{49}$$

Hence, we get the following solution

$$q(x,t) = \sqrt{\frac{(-\delta^2 + 2)\sqrt{\frac{\beta}{(\delta^2-2)^2}}\tanh(2\alpha\frac{\beta}{(\delta^2-2)^2}) - \beta}{8\delta^2 - 16}}\, e^{\imath(\mu x + \nu t)}, \tag{50}$$

where $\alpha = \imath\xi(x - 2\mu t)$.

The obtained solitary solution is illustrated in 3D and 2D graphs in Fig. 3 with parameter values $\beta = -0.75,\ \delta = 1,\ \mu = 1.5$.

Conclusion

This paper effectively investigates the solutions of Kundu–Eckhaus utilizing the CV approach. A set of ordinary differential equations are determined by the CV technique for pulse dispersion in optical fibers. This study uncovers solitons in the sense of computational impacts, leading to a discrete domain of pulse behavior for a wide range of parameter values. The CV approach's core idea has been based on the application of a condition, $N_j \approx 0$, to the CVs $u_p(p = 1, 2, \ldots, k)$. Examining the graphs of the derived solutions can teach us a lot about the Kundu–Eckhaus problem's dynamics. The unified method has also been applied to find soliton solutions. The achieved solution is a periodic solitary wave, shown in Fig. 3. These methods might be beneficial for the fractional differential equation in the long run to better comprehend the model's inherent characteristics. The solutions achieved in this paper are known to be new in the existing literature and the approaches

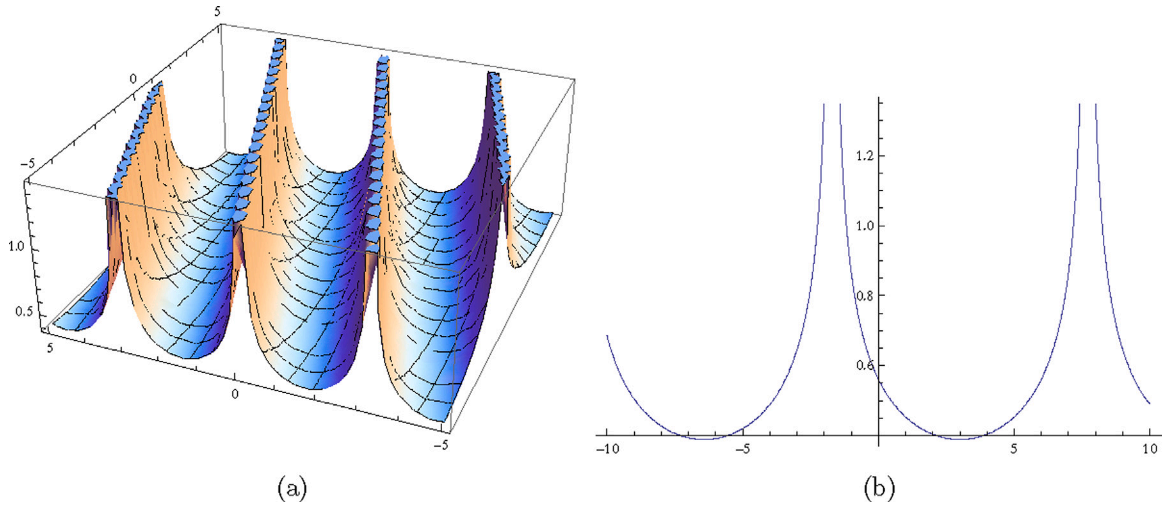


Fig. 3. Solitary solution of Eq. (50).

utilized, can be applied to solve a variety of nonlinear problems in mathematical sciences and might be valuable in identifying the physical importance of this model for mathematicians, engineers and physicists.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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