QATAR UNIVERSITY

COLLEGE OF BUSINESS AND ECONOMICS

Property Assessment in Qatar (Case Study the Pearl Residential Apartments)

BY

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A Project Submitted to

Faculty of the Collage of Buisness and

Economics

in Partial Fulfillment

of the Requirements

for the Degree of

Master of Buisness Adminstration

January-2017

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ABSTRACT

Alkhelaifi, Mubarak, A., Masters : January : 2017, Master of Business Administration

Title: Property Assessment in Qatar (Case Study the Pearl Residential Apartments)

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The Gulf countries have been maintained as Tax Free heavens due to their strong revenue from their natural resources like Oil. However, by the end of 2014, the oil prices has dropped in the world resulting in the gulf region to have a tremendous drop in their earning; which lead to a big deficits in the gulf countries' budgets. Many analysts believe that property taxes are on the way, because governments are seeking means to diversify their income base and reduce their dependence on oil. Property taxes are one of the key determinants of property value in the west, and so will require a method to evaluate the market price of properties. This research paper will draw inferences from the limitations of traditional and contemporary real estate appraisal methods from academic research. Many of the past and contemporary methods assume vast assumptions which make them largely unreliable. A goal programming model can be made inclusive of the all variables affecting a property's value and does not make substantial assumptions. We applied a goal programming model to assess the model's effectiveness in estimating the price of a real estate property and found that it comes significantly close to the actual value price.

KEYWORDS

Real estate Qatar, property assessment, Goal Programing Model, Property Valuation.

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Acknowledgments

The completion of this report could not have been possible without the assistance of Professor Belaid Aouni, Associate Dean for Research and Graduate Studies. I am also grateful to Ibrahim Al-Othman the Chief Executive Officer (CEO) and Hussain Al Baker, Director of Marketing & Communications and other team members at United Development Company for their prolonged support.

Thank you.

Introduction

Oil and gas account for about 85 percent of Qatar’s export revenues and over 50 percent of GDP. Qatar's proven oil has the world’s third-largest oil reserve, exceeding over 25 billion barrels. Though Qatar ranks the highest in the world in terms of GDP per capita, it is now feeling the financial pressure of low oil prices. Oil prices have dropped over 70% over the least two years. After 15 consecutive years of surpluses, Qatar is now running a “minor” budget deficit of $12.8 billion, equal to about 0.7 percent of its GDP. (Reuters, 2015). The government hiked utility rates, doubled fines for wasting water and even increased rates for postal services, to finance the shortfall from the reduced revenue due to dropping Oil prices,

Many analysts today believe that Qatar’s recent economic measures have strongly indicated that it is headed in the right direction to recovery from the austerity measures (Trading Economics, 2016). Qatar’s Zero property tax has been a part of the government’s efforts to search for new ways to attract local entrepreneurs and foreign direct investments to diversify its economy (Sylva, 2015). Though there are is no property tax in Qatar, when buying property, there is a one-time ‘transfer fee’ of 0.25 percent of the total value of the property. The zero tax strategy brought in much foreign direct investment in Qatar over the last few decades. Today here is an increasing realization that in the future, Qatar may be introducing a real estate tax on both residential and commercial properties, similar to other countries in the region, like Saudi Arabia, have done to diversify their revenue base.

With changes, most likely to occur within Qatar's property tax system, it is pertinent to assess models of property valuation and determine the model likely to be more efficient in its purpose to accurately estimate property value. We will look at traditional approaches to valuation such as the income approach, the residual method, the market to sales comparative approach and mean/median transaction prices and also some of the contemporary approaches like the hedonic pricing model and various indices. The traditional, as well as the popular contemporary approaches, have been thoroughly studied in academics, and so we can learn from the inferences that have been drawn. We will look at how the traditional valuation models used today offer many limitations and undertake many broad assumptions, making them at best, not a satisfactory predictor of property value. Moreover, a research report by the Instituto Superior Tecnico in France (Dutra, 2009) conjectures that the value of a property is a compound of the asset's relevant determinants, also known as its attributes. There are many property specific determinants which affect the value of a property and taxation has been a key determinant in North America and the West (Pomykacz, 2003). The report persuades that a model which takes into account property specific attributes is a better determinant of the final value of the property.

Over the last few decades, the goal programming model has gained much attention after it has been successfully used to solve some of the most complex decision making and resource allocation problems across many disciplines. Given that there is a lack of research on a model which account for property specific attributes without having to take into accounting vest assumptions, we execute a goal programming model using properties in Qatar to evaluate its usefulness for our purpose. The model will incorporate property specific attributes of a set of properties to produce a quantified output on how each attribute affect the property's value. For our purpose, a goal programming estimation model is adopted.

This research report starts with a literature review of the historical and contemporary real estate valuation methods, the span of indices used assessing the change in property value and stating the effectiveness of these approaches. Then we will introduce the goal programming model and the model's methodology on how it estimates the value of a property. Moreover, we will apply the model to three different sets of properties and evaluate the effectiveness of the model by looking at how close it comes to predicting the property's value. Then we will shift our focus on the limitations of the goal programming model. The conclusion will take into account the observations from the literature review about the different models in evaluating real property and the results of the goal programming application to assess if goal programming is a reliable approach towards property valuation.

Literature review

In this section, we will review different valuation methods for properties. Next, we will distinguish between the goal programming model and the contemporary and historical approaches to valuation to identify the importance of the goal programming model. Then brief the origins of the goal programming model. And lastly, introduce the Pearl-Qatar Island, from which we took properties and apply the goal programming model.

2.1 Valuations Methods

The real estate market is where buyers and sellers unite to transfer their rights or obligations concerning a real estate property. A large number of real estate transactions allow a price comparison among properties and also enable the analyst to estimate the values of similar properties. The value of a property can be defined as the amount that people are willing to pay for a specific property.

On March 28, 1874, a groundbreaking paper, titled 'methods for the just and equal distribution of taxation" was presented at the social science Association of Philadelphia, USA. The paper outlined the need to develop standard procedures for valuing real estate for tax purposes (J. Wayne Moore, 2012). During that time, there were to "established procedures" to value property and people just valued real estate based on what they believed was a true value according to what they believed was right. Even Adam Smith, in the "The Wealth of Nations," described how market operated and showed that markets could not function effectively without accurate valuation (Smith, 1904).

A restatement of valuation theory has been proposed by Richard Ratcliff, the professor at Maxwell University. Ratcliff emphasized that valuation is a prediction of human behavior under uncertainty and can never be accurate. He discussed "transaction zones" which point out that depending on negotiation skills and the unique circumstances of the buyer and seller the prices can vary in a certain range which might emerge from a sale process. Later, Maurice Squirrel expanded on Ratcliff's notion of uncertainty in property prices (Lawson, 2012).

Ratcliff's argument seems to have substance if we look at what happened in the recent economic crisis. During 2008 and 2009 we say that griefs had left many real estate owners and even investment fund managers stumbling on how to value property assets because post-crisis, many properties traded at inflated values and values rapidly fell during the crisis. It is unknown if this has been the result of a lack of a standardized appraisal method, but many analysts believe the limitations to appraisal methods contributed to mispriced real estate property. After the crisis, many analysts also believed that there needs to be a "standard method" to analyze real estate.

The decades following the 1870s, saw many real estate valuation methods emerge. However, those methods, as explained below, came with significant limitations and assumptions which did not allow for a true prediction. It is noteworthy that mark-to-market valuations based on comparable properties and DFC methods are lashing down the market value of many assets even though the first fundamentals have not changed (Kummerow, 2008).

Primarily, we will look at one of the oldest approaches, the income approach to valuation, where the "income producing" ability of a property a key determinant of price. Then the residual method, which incorporates two property specific elements in its valuation. Next, the market to sales comparison approach is perhaps a more popular as it compares prices to other properties in the market. Then we look at a lesser known, the mean or median prices method, which uses basic arithmetic on prices in a particular geographic area. The following method is the residual method which is more of an all-inclusive method that can incorporate any number of variables as determinants of a property's value. And finally we will review the use of indices in property valuation. Though most indices are used to monitor price changes, the FNC index is differentiated by incorporated the hedonic pricing element in it to create a predictive model.

a. The Income Approach to Valuation

The Croatian Information Technology Society conducted a study in 2014, where it valued petrol station facilities using the income approach to valuation. The society taking into account the "income generating" capacity of the properties and the user's expenses. Here, the first step is to determine the Gross Operating Income (GOI) of the property, which is calculated as; Gross Potential Income - Vacancy and Credit Loss = Gross Operating Income. After that, we determine the operating expenses of the property. Moreover, finally, subtract the operating expenses from the GOI, and we have the net operating income (Sabina Źróbek, 2014).

The society in its research found significant variations between the market value calculated using the net operating income and actual transaction prices of the underlying properties. They concluded that the variations were the result of the assumption of continued and constantly increasing income flow from the properties. The Society also examined the 'historical valuation approach' by applying it to the castles and palaces of Poland. One may argue their sample size is subtle (419 palaces and 2021 castles), but their findings showed "significantly different prices of castles and palaces with similar or the same fundaments." The abstract of the research says that it is important to take into account "the potential of a specific property." (Sabina Źróbek, 2014). The discounted cash flow approach is a similar approach which came up in the 1960s. This method discounts expected future cash flows (income) from the property to the present and calculates its value based on that. However, predicting the value of a real estate based merely on the "anticipated" income is probably not a good choice, given that there is a broad range of factors which impact the price (Sabina Źróbek, 2014). Though there are a broad range of variables which impact a property's value, the residual method is able to take into account two factors.

b. The Residual Method

The residual method combined with a sensitivity analysis is also a popular method. It allows the evaluator to take into account certain aspects, the research paper mentions; "it is possible to take into account the potential of the property, as well as drawbacks, charges and dangers related to the property itself and the micro- and macroeconomic phenomena……”. The residual method which regarding property development calculates whether a profit can be achieved on an estate development project. This approach was tested on the castle and palace complex in Międzylesie. This method subtracts the present value of property from its future value and development cost. The study highlighted some issues with this approach by comparing values of the estates during the development phase and after the development phase. This approach failed to take into account changing variables in the market and their influences on valuation. The analysis assumed that “physically possible, appropriately justified, legally permissible financially feasible, and result in the highest value of the property undervaluation.” The sensitivity analysis in the research showed a change in the property’s value (by its present state), taking into consideration the development potential. A sensitivity analysis can only demonstrate the dependence of result on two parameters at a time, but in a valuation model, there are usually more variables, so the results were concluded to be far from accurate given noteworthy differences in the valuations and actual transactions (Sabina Źróbek, 2014). Perhaps the prices of other properties in the area are better able to explain the value of a property. The market to sales comparative does just that.

c. Market to Sales Comparative Approach

One very popular approach to valuing property is the ‘market or sales comparative approach.’ This method entails comparing like properties that have recently been in the market transaction with the present subject property. That price normally serves as a guideline for appraisers to take a better-informed decision. This approach is based on the theory that real estate value is derived from the views of the typical buyer and seller of properties. Miller and Geltner in their research report in 2005 inferred that this approach has a tendency to not effectively incorporate property specific factors in its valuation and thus can often produce unrealistic results. The extent to which a sales comparison model is reliable will depend on how completely and correctly the analyst has identified the points of difference that matter in pricing (Sabina Źróbek, 2014).

Real estate constitutes an agglomeration of fixtures, rights, the attached land, building and other aesthetics that cannot always be quantified. Real estate valuation is a function of the attached rights and the physical features and Valuation approaches like the approaches discussed above, value property using the income they generate along with other 'limited factors' leave out the unquantifiable aspects. The research later looked at "dominant Variables" which are backed by research to have significant impacts on valuation but provided evidence with some statistical techniques that these variables are not reliable because they change with unpredictable market conditions (Dutra, 2009).

House Canary Inc., a leading real estate research firm has found compelling evidence in its June 2015 study that "shifting demographics" in the US are reshaping housing demand, making some valuation methodologies not as relevant as other ones and that the valuation methodology is not constant. Although the aspirations for institutional standardization of the valuation methodology in global terms have not produced any strong results, the establishment and Popularization of normal definition standards, as well as the valuation process standards, undoubtedly contributed to bringing together the concept of value and the process of its Determination. A single, common valuation methodology will probably never be created, but the aspiration for bringing valuation methodologies together will remain topical (Sicklick, 2015). The market to sales comparative approach is similar to using mean or median transaction prices.

d. Mean or Median Transaction Prices

The mean or median transaction prices index is another widely used method at the institutional level. The Quebec Federation of real estate boards revealed in a research note that the mean or median is better than using other arithmetical methods such as the average because the latter two are not influenced by final numbers and thus prevent biased interpretations (Quebec Federation fo Real Estate Boards , 2010). This method consists of an index which simply calculates and provides each for interpreting summaries of sales activities within specific geographical areas. However, Calhoun in its study in 2001 found that this model fails to control for the different compositions of the sample and the relative quality of properties transacting period over period. As a result, it 's hard to segregate different prices that occur due to actual appreciate in the property's value and the appreciation of other characteristics attached to the property (Eke, 2014). Due to its simplicity, this method is mostly used to report prices in specific geographical areas but is less efficient of a forecasting or valuation method (International Monetary Fund, 2006).

Property update, a proprietary research firm showed in its investigations that median prices are not an effective indicator for all geographical areas. For example, it may be a good indicator for properties in suburban areas where properties are largely homogeneous and there for similar in prices. Likewise, using median prices at a city can be misleading. Different data providers also provide different median prices because they use different sample sizes which are reported in various periods. Statistics are more reliable when used in the long-term (Yardney, 2016). Perhaps a model which incorporates many different property specific attributes can be an effective predictor of price. The hedonic pricing model is able to take into account many factors as determinants of a property's price.

e. Hedonic Pricing Model

A hedonic pricing model may solve many of the problems with valuation highlighted above because the model uses a regression analysis to determine the degree to which each of the independent factors, such as land and fixtures, which constitute the property's total value, affect the property's value. The degree to which each independent variable affects the value of a property is called the regression coefficient of each of the variables. The regression analysis also produces an R-squared value, which explains how well a data fits the statistical model, how much of the deviation in the prices is explained by the variables.

The coefficient is multiplied by the 'per unit attribute' and added up to predict the property's value (Monson, 2009). We will further discuss this model in indices.

The hedonic analysis has been extensively used to predict values of properties. The model was applied to Chicago's office market where the time of sale and location near employment centers and away from employment centers, were used as the dependent variables and transaction prices as the independent variables (Peter F Colwell, 1998). The results showed that there is a substantial premium for office properties located within the certain employment centers. The adjusted R-squared for each model exceeded 80%. When the price per square foot was made the dependent variable, R-squared fell to about 40% (Peter F Colwell, 1998). These findings supported the conclusions of the earlier research and so are believed to be of value.

The hedonic model was further tested by Ronald W. Spahr and Mark A. Sunderman to value property surrounding a resort community and some agricultural properties near Jackson, Wyoming (Sunderman, 2012). The study found that attributes which affected the price of properties near a resort community are significantly different from the attributes influence the price of agricultural property. Resort properties had higher coefficients for attributes like sceneries and distance from the city while rural properties had higher coefficients for attributes like access to irrigation water. This shows that property specific attributes are significant in determining the value of assets (Sunderman, 2012).

However, there are issues with the hedonic approach as well. Building a model which includes the all the possible factors affecting the value of a real estate is a difficult and very complex task and requires in-depth research about each and every factor and its credibility. Other than that, a regression analysis has some issues.

Primarily, it does not take into account the variance. The variance is the distance of the results from the mean. It is important to take into account the dispersion as a variable. Secondly, a regression analysis assumes that the input variables are independent of each other and when this assumption is violated, the analysis can produce misleading results. Another key issue is selection bias which could be overcome with selecting a random sample of properties, but a random sampling is not applicable in real estate valuation because as we discussed earlier, properties have unique characteristics (Haurin, US commercial real estate indices: transaction-based and constant-liquidity indices , 2013). We will further discuss this method under the "hedonic index model."

Indices have been in use for a long time and are becoming more popular in real estate valuation as well. The hedonic model is combined with the index model to create a predictive model.

2.2 Valuation Using Indices

Fundamentals of statistical analysis teach us that the more data we have, the better results and valuable information we get. When valuing real estate, appraisal professionals have limited transaction prices available to them because a real estate property is not traded as often as, for example, stock on an exchange. As a result, price indices are made for classes of real estate assets; for instance, downtown Manhattan Office Buildings and Jersey City Office buildings. Price indices can be compiled either by standard formulas or regression techniques that estimate the value of a composite or standard unit of real estate (Government of Australia, 2011).

A price index can remove the effects of changes in the composition of transactions or changes in quality, to arrive at a more accurate measure of prices for comparable units of real estate. Price indices are absolute numbers which describe changes from a benchmark unit of value (usually 100) in a base period and, as such, can be compared to economies with different types of real estate (Government of Australia, 2011).

Traditionally, price indices have only been available for the commercial property based on appraised values and actual transaction prices. Indices based on prices are composed of some supporting empirical evidence about the market value of properties and their transaction prices. The challenge with indices is to control the difference between the properties that transact from period to period because the characteristics of properties also change over time. For example, the development of a major train station near an office building is likely to have a more significant impact on the value of the property nearest to the station (Haurin, US commercial real estate indices: transaction-based and constant-liquidity indices , 2008). The home price index is the most fundamental and widely used real estate index.

a. The Home Price Index

There are three fundamental methods of composing a real estate value index; the repeat sales method, hedonic pricing method and the hybrid method which combines the two prior methods. The largest price based index in the US is composed of prices for over 17M property valuations from Freddie Mac and Fannie Mae (gov. backed mortgage issuing companies). It is called the conventional home price index (Federal Housing Agency, 2016)

Year	Quarter	Purchase-Only Index (1991Q1=100)	Seasonally-Adjusted Purchase-Only Index (1991Q1=100)	HPI % Change Over Previous 4 Quarters	HPI % Change Over Previous Quarter	Purchase-Only Index % Change Over Previous 4 Quarters	Purchase-Only Index % Change Over Previous Quarter	Seasonally-Adjusted Purchase-Only Index % Change Over Previous 4 Quarters	Seasonally-Adjusted Purchase-Only Index % Change Over Previous Quarter
1991	1	100.00	100.00	1.46%	0.84%				
1991	2	100.52	100.02	1.73%	0.67%		0.52%		0.02%
1991	3	100.79	100.18	1.29%	0.17%		0.27%		0.17%
1991	4	101.47	101.08	3.13%	1.42%		0.67%		0.89%
1992	1	102.27	102.29	3.05%	0.77%	2.27%	0.79%	2.29%	1.20%
1992	2	102.71	102.19	2.35%	-0.02%	2.18%	0.43%	2.17%	-0.09%
1992	3	103.71	103.06	3.34%	1.14%	2.90%	0.98%	2.87%	0.85%
1992	4	104.26	103.88	2.42%	0.51%	2.75%	0.53%	2.77%	0.80%
1993	1	103.88	103.92	1.67%	0.04%	1.57%	-0.36%	1.60%	0.04%
1993	2	105.51	104.96	2.66%	0.95%	2.73%	1.57%	2.71%	1.00%
1993	3	106.47	105.75	2.32%	0.81%	2.66%	0.91%	2.61%	0.75%
1993	4	107.09	106.75	2.64%	0.82%	2.72%	0.58%	2.77%	0.95%

Figure 1. Home Price Index

Source: http://www.fhfa.gov/DataTools/Downloads/Pages/House-Price-Index.aspx

The figure above is a home price index which measures the movement of single-family house prices (HPI). The HPI is a weighted, repeat-sales index. This repeat sales index measures the average price changes in repeat sales on the same properties. The seasonally adjusted column adjusts the index value to smooth out the seasonal effects on the price. A base of 100 is used to track the change in prices. The rest of the columns show the changes over the underlying time periods (Federal Housing Finance Agency, 2014). Though the repeat sales methods have proved useful, it is not without problems.

The most prominent issue is that those single sales are excluded, which reduces the sample size significantly. Also for there to be repeat sales, we must have some sales, to begin with (Chaitra H. Nagaraja, 2010). As we can see the home price index is a basic index. Other indices, like Laspeyres indices, are more comprehensive in using the prices and properties as components of the index.

b. Laspeyres price index

The Laspeyres price index is a popular index which computes the weighted average change in prices over a period for a given basket of properties drawn from a base period. It then compares the total cost of buying a specific quantity and mix of properties in the base period with the total cost of buying the same amount and mix in another period. After that, an index is built.

Table 9.1. Quantities and Prices of Real Estate

Type of Property K_i	Base Period Quantity q_0	Base Period Price p_o	Current Period Price p_t
A ($i = 1$)	160	50	60
B ($i = 2$)	30	70	90
C ($i = 3$)	10	100	110

Figure 2. Laspeyres price index

Source: Financial Soundness Indicators, Chapter 9: Real Estate Indices, March 2006

Figure 2 shows a Laspeyres price index which compares the prices of a basket of properties in a base period P0 and the current fiscal year Pt.

The total cost of buying A, B, and C properties in the base period is (160)*(50) + (30)*(70) + (10)*(100) = $11,100. At prices prevailing in the current period, the total cost of purchasing the base period quantities is (160)*(60) + (30)*(90) + (10)*(110) = $13,400. The Laspeyres index for the current period is therefore 100*(13,400/11,100) = 120.7. This means that prices in the current period are 20.7% higher than what they were in the base period.

The purpose of the index is to calculate a price of the properties in the market using information from transactions over a period and from the appraised values of the underlying properties (International Monetary Fund, 2006). One key disadvantage of the Laspeyres index is that it cannot be used to determine exactly how much of a total price change is correlated with another element or changing quality since there is no solid quality associated with the base year. As we discussed earlier, there are property specific items which need to be taken into account when assessing the value of real estate (Chaitra H. Nagaraja, 2010). We can see that different indices are composed in different ways. A recent development is a liquidity index which uses transaction volume as a determinant of price.

c. Liquidity Adjusted Indices

A liquidity-adjusted price index adjusts price measures to interpret the influence of changing transaction volume on prices separately. Market liquidity by definition refers to how quickly real estate transactions take place, which reflects the relative strength of market demand for housing compared to the supply. A statistically significant relationship has been found between market liquidity and transaction prices (Haurin, US commercial real estate indices: transaction-based and constant-liquidity indices , 2008). Often, prices which are correlated with liquidity, rise during periods of quick turnovers periods and fall during slow turnover periods. By taking into account information on the volume of transactions for a given period, we can estimate the intended impact on price, due to changes in the number of operations and thus derive a measure of the underlying price movements as if the number of transactions are constant (International Monetary Fund, 2006). The flow with this one is quite evident. Accounts for just one factor, the change in volume. From what we discussed earlier, the value of a real estate property is a compound of many factors, and any one factor is not likely an accurate predictor of its value.

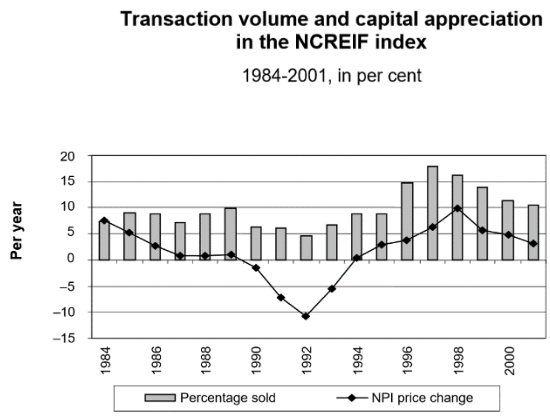

Figure 3. Liquidity Adjusted Indices

Source: BIS paper #21, US commercial real estate indices: transaction-based and constant-liquidity indices, Donald R Haurin

The graph above shows the relationship between transaction volume and appreciation in the value of property. A visual analysis would reveal that an operation volume translates into higher prices. The index can be constructed by taking the correlation coefficient of transaction prices of a set of properties with different transaction volumes. This would reveal the significance of volume on price. The predictive model can predict the value of properties in particular geographical areas with a particular transaction volume.

Often models are combined to produce better results. The FNC index, does just that, by combining an index model with a hedonic pricing model.

d. The FNC Index: A hybrid between a traditional index and the Hedonic model

The FNC (unable to find what the abbreviation is for) was one of the first few companies to combine index data with blended data from appraisals and take into account property specific attributes. Now this seems like a great solution to the problem with indices we have highlighted above. The method of the FNC index is explained in Dorsey, Hua, Mayer, and Wang (2010) “Hedonic versus repeat-sales housing price indexes for measuring the recent boom-bust cycle” Journal of Housing Economics 19 (2010) 87–105. The hedonic method was described earlier, and we have discussed the process of making an index (FNC, 2014).

As we have discussed earlier, the hedonic model embraces a regression analysis to determine the degree to which each of the independent factors, such as land and fixtures, contribute to the property’s value as a whole. So the first step is to do a regression analysis. A regression analysis shows the degree to which one set of variables (the independent variables) affect the value of a set of dependent variables. In our case, the independent variables are the individual attributes of the properties and the dependent variables are the mean prices at which the houses are sold. A regression analysis can be done in Excel, but the methodology is beyond the scope of our research.

The results of a regression analysis reveal the degree to which each of the attributes affects the price of the property; or we can say, the per unit contribution to price.

This index is constructed by first taking the mean sales prices of properties in a particular area and taking particular attributes, followed by the average and standard deviation of prices and the special attributes. This approach allows all of the properties to be included in the index which then provides a stable broad-based index for each period (FNC, 2014).

The hedonic approach involves recording the prices of houses with a detailed set of quality characteristics to form a constant quality index. Note that the value of a property depends on its features and the features of other properties around it. So to hold the value of a property constant we have to hold other constables constant.

This model employs mathematics with which the changes in the characteristics of a house change the expected price of the house, directly and indirectly, their impact on the normal prices of other houses. The coefficients represent the portions of the percentage change in the expected price of a house per unit increase in a private holding the expected prices of other houses constant. The coefficient can be obtained by doing a regression analysis (Robert E Dorsey, 2010).

The final index is constructed by multiplying the coefficients in Table 4 by the mean values of the attributes shown in Table 2 and then adding them all up. So for example, for the age attribute, we would do -0.00100*41.92 = -0.04192. The final figure represents impact a property aged 41.92 on the price. The final figure is the estimated price using the model.

The table above is a sample hedonic index which shows the change % change in price over a period (this is a hypothetical table). The last row, census_1, shows the estimated price using the model (Robert E Dorsey, 2010). It looks like any other index. The only difference is that the hedonic model produces the final figures.

Even though price indices are available for the various classes of real estate investments, the problem is comparing assets within a class. So for example, is one downtown building the same as another? Moreover, how do we control the various factors in the value of a property? Moreover, particularly, what about the location of the building? (Damodaran, 2014). Appraisal professionals can use real estate indices to get an overview of the prices of properties in a particular area and the long term trend. However, again, ceteris paribus; a past trend is not guaranteed to continue in the same direction. However, this can prove to be a good predictive model.

Judging the observations we can see that most models lack the ability to predict prices due to the vast assumptions they under take or are unable to incorporate enough property specific attributes as determinants of value. The goal programming approach has not been widely used for property valuation but has been used across many disciplines to solve many completed problems. Before understanding and applying this model, it’s important to differentiate it from models that are widely used in real estate valuation.

2.3 The Goal Programming Model VS. Historical and Contemporary approaches

The goal programming approach is not widely used in valuing real property, and there is a limited study on its utilization and application for this purpose. It is important to note that goal programming has the potential to address many of the problems we have discussed earlier. The model has been applied by Aouni and Martel (2004) for Property assessment when the information about the selling price of the properties are. They found the model to significantly use full in its ability to allow the appraiser to have quantified control for the variables which impact the property's value and further assess the results with a satisfaction function.

Primarily, the goal programming model leaves out some of the assumptions with other models such as the net operating income model which assumes consistent income flow from the property. The mean/median prices model is also very inconsistent given that it is dependent on a broad average. The sales comparison model does not take into account property specific factors and the residual model is only able factor in two variables at a time which will produce misleading results because it is not an inclusive model which takes into account all possible. With the goal programming model we are able to incorporate as many variables as we like. Even though the hedonic method takes into account any number of variables, it adopts the regression analysis method which ignores the variance. The hedonic model assumes that all variables are independent of one another and when this assumption is violated, the results can be misleading. The problem with price indices is not indifferent. At best, those provide information about the price of a specific property class and does not incorporate at all, or just one, property specific factor.

In contrast, the goal programming model can take into account as many property specific factors as are available and does not have to rely on any hard assumptions necessarily. With the goal programming model, we are also able to assign weights to certain variables, which explain the degree to which the variable influences the property's price. The goal programming scenario can handle any number of constraints (Ragsdale, 2008). The goal program model has been modified to be used in different scenarios.

2.4 Goal Programming: A Brief History

In a goal programming model, the appraiser sets a specific goal on the property's value such as a maximum or minimum value. The appraiser can limit the property's value with certain constraints, for example, the amount of land attached may affect the value to a certain extent or the average price of properties with similar characteristics in the same geographical area. So using those constraints the appraiser can set the limits for each of the variables which affect the property's value and produce a value within those limit.

Goal programming was introduced by Charnes and Cooper in the early 1960s as a simple linear program and has been developing further to include hundreds of papers with dealing with an extensive variety of complex problems (Aouni & Martel). The goal programming methodology was further popularized with the applications by Lee (1972) for the forest management problem. Later in the fuzzy multi-objective programming approach for administration of the reservoir watershed (Field, 1973) and the management of solid wastes (Aouni & Martel). The application of goal programming is prevented in its suitability to address issues across many disciplines including financial resource management, human resources, and production related issues (Aouni & Martel). The application and extent of inclusive issues which can be addressed through goal programming models will continue to increase with the globalization of economies, democratization of collective activities and competitive requirements for decision model (Aouni & Kettani, 2001).

It's best to apply this model to a specific geographical area. We will be applying the model to properties in the pearl Qatar Island.

2.4 The Pearl-Qatar

The Pearl-Qatar is an artificial island covering about four million square meters. It is the first land in Qatar to be made available for foreign nationals to buy. The Pearl-Qatar has over 12,000 residents as of January 2015. The pearl is still under construction and is expected to be completed in 2018 and projected to have about 45,000 inhabitants. The pearl will add about 32 Kilometers of coastline and is supposed to have nearly 19000 dwellings.

The residential development on the island is planned to include some national and international themes, including aspects of Mediterranean, Arabic, and European culture. Ten areas make up The Pearl-Qatar Island. One of these are the Porto Arabia Towers, which are 31 one total, with a total of 4,700 apartments. The Porto Arabia was the first phase of the Pearl Qatar which offered apartments for sale. The tower apartments vary in floorplans and fit outs, but all show views from the open Arabia Sea, downtown, and Marina.

Then there are the Qanat Quartiers, which are 977 residential apartments in 31 buildings. There are currently about 1,756 apartments on sale on the island. There are 28 Viva Behria towers some which are completed and some of which are still under construction. These have an elegant residence with one to three bedrooms, luxury penthouses and some townhouses around them. The townhouses are located on the beach. The Qanat Quartiles are designed magnificently with colorful Venetian design around sophisticated canals and large retail plazas.

Property rates on the island had been increasing for many years, until last year, prices have stopped climbing and remain flat. According to DTZ Research, Vacancy rates on the island have started to tiptoe up to 5 to 10%. However, occupancy in the long term is expected to keep growing. Many of the properties, such as the Alfardans units are already fully occupied. This will probably prove true as builders are now focused on building more affordable homes making the properties available to a larger population for even faster development (Kovessy, 2015). With a little more Vacancy than a few years ago, rents have dropped a little on the Island, which is now starting to draw more people towards to the island.

Methodology

The goal programming model can be better illustrated through a mathematical model. The model can illuminate the different components of goal programming. Followed by the illustration, the model is applied to the select properties.

3.1 Goal Programming Model

Goal programming is a branch of multi-objective optimization which can handle multiple and normally conflicting objective measures. Each of the measures is assigned a goal or objective which has to be achieved. Though there can be multiple objectives in a goal programming situation, in our analysis there will be only one. An objective can be, let's say, to maximize the profit on a product, find the maximum value of a real estate property or even estimate the value of a property. We will be doing the later, by adopting a model to estimate the prices of real estate property.

Whatever the purpose of the model is, the goal programming scenario is made up of three elements. The objective function is one of the three elements in a goal programming scenario, which represents final output.

We will use the formula produced by Charles and Cooper (1977); pioneers of the goal programming model:

$$Min_{\beta} \sum_{i=1}^{n} |y_i - \sum_{j=1}^{m} \beta_j \ x_{ij}|$$

$$\beta \in \mathcal{B}$$

The mathematical model to be utilized to estimate the property values is as follow:

$$Min\ z = \sum_{i=1}^{n} (\delta_i^+ + \delta_i^-)$$

Subject to:

$$\beta_0 + \sum_{j=1}^{m} \beta_j\ x_{ij} + \delta_i^- - \delta_i^+ = y_i \quad For\ i = 1,2, \ldots n$$

$$\beta_0, \beta_j\ are\ free\ (unrestricted\ in\ sign)\ (for\ j - 1,2 \ldots .m)$$

$$\delta_i^+, \delta_i^- \geq 0\ , \quad (for\ i = 1,2 \ldots , n)$$

The objective function, might be for example, to minimize a set of values in order to optimize a given set of variables. The objective function can be expressed mathematically as follows:

$$Min\ z = \sum_{i=1}^{n} (\delta_i^+ + \delta_i^-)$$

Z is the objective function; which is the sum of deviations.

The other two components of a goal programming model are the constraints and the variables. The constraints represent the restrictions on the variables (Ragsdale, 2008). The variables; also known as the decision variables are what are “varied” or changed as a result of the model’s output. In a real estate pricing model, the variables can be all the different factors which impact the property’s value.

The variables are represented as follows. Where,

$$\beta_0, \beta_j \; are \; free \; (unrestricted \; in \; sign) \; (for \; j - 1,2 \ldots . m)$$

$$\delta_i^+, \delta_i^- \;\geq 0 \, , \quad (for \; i = 1,2 \ldots , n)$$

Variables are normally constrained within some limits, also called the constraints. For example, in a real estate pricing model, the estimated price of the property cannot assume a negative number. Aouni and Martel (2004) in their research on using an imprecise model to value real estate used an upper limit and a lower limit on the price of the individual properties.

Sometimes constraints are underachieved or even overachieved. For example, the optimal solution, might marginally over state a property attribute, such as the area of the backyard of a house in order to meet a specific price. We can represent the over and underachievement, or over achievement of a goal or constraint with; δ_j^-, δ_j^+ .This notation can be used to represent the under/over achievement of any variable/constraint. Often, minimizing the sum of the underachievement's and overachievement's; sum of deviations of: $\delta_j^- + \; \delta_j^+$ of each variable when minizied produces the optimum solution, or when the sum of deviations is 0. But it may not be 0 because the conflicting objectives do not always converge.

The constraints are expressed as follows;

Subject to the constraints;

$$\beta_0 + \sum_{j=1}^{m} \beta_j \; x_{ij} + \; \delta_i^- - \; \delta_i^+ = y_i \quad For \; i = 1,2, \ldots . \text{n}$$

In this model, n are the goals, β are the parameters and m are the decision variables (or attributes). The parameters β are free variables because they can take on either a positive or negative value. In our real estate scenario, it means that the parameters which we choose can either assume a positive value, meaning that they can positively contribute to the value of the property or devalue the property. The δ values are explained above. x_{ij} Is the jth variable (or attribute) and β_j is the parameter or (the coefficient) associated with the variable j in the ith goal (Charnes & Cooper, 1977).

3.2 Goal Programming Application

We can safely infer from our discussion above that a model which takes into account property specific attributes is perhaps a better predictor of the value of property. We will apply the goal programming model using 57 properties for the first half of 2015 to predict the prices of properties on the Pearl Qatar Island to assess the effectiveness of this model. The list of properties and relevant data have been provided by Untied Development Company.

Table 1. List of transactions of properties in the Pearl

No.	Precinct (Location)	Property View	Asset Type	Area (m2)	No. of Bedrooms	Balcony	Parking Space Count	Sold Price
1	Viva Bahriya	Direct Sea Side	Apartment	137.4	2	18.0	2	2,200,000
2	Porto Arabia	Partial Sea/Pool	Apartment	218.0	3	18.1	2	3,600,000
3	Porto Arabia	Direct Marina	Apartment	249.7	3	53.4	2	4,350,000
4	Viva Bahriya	Direct Sea Side	Apartment	137.4	2	18.0	2	2,200,000
5	Porto Arabia	Direct Marina	Townhouse	146.0	2	10.0	2	2,800,000
6	Viva Bahriya	Partial Sea/Entrance	Apartment	141.0	2	16.0	2	2,500,000
7	Porto Arabia	Middle	Apartment	105.0	1	10.3	1	1,750,000
8	Porto Arabia	Middle/Pool Side	Apartment	136.0	1	34.2	1	2,000,000
9	Viva Bahriya	Partial Sea/Pool	Apartment	141.4	2	16.0	2	2,550,000
10	Viva Bahriya	Direct Sea Side	Townhouse	114.5	1	17.0	1	2,550,000
11	Porto Arabia	Direct Marina	Apartment	256.1	3	54.0	2	4,350,000
12	Porto Arabia	Middle	Apartment	176.8	2	32.0	2	2,750,000
13	Porto Arabia	Partial Marina/Pool	Apartment	121.0	1	9.9	1	1,750,000
14	Porto Arabia	Direct Sea Side	Apartment	83.0	1	8.5	1	1,675,000
15	Porto Arabia	Partial Marina/Pool	Apartment	95.0	1	0.0	1	

								1,650,000
16	Porto Arabia	Direct Marina	Townhouse	474.0	3	97.0	2	9,000,000
17	Porto Arabia	Partial Marina/Entrance	Apartment	121.0	1	9.9	1	1,850,000
18	Porto Arabia	Partial Sea/Pool	Apartment	343.3	3	56.0	2	5,000,000
19	Porto Arabia	Partial Marina/Entrance	Apartment	121.0	1	9.9	1	1,950,000
20	Porto Arabia	Direct Marina/Middle Entrance	Townhouse	146.0	2	10.0	2	3,250,000
21	Porto Arabia	Partial Marina	Apartment	94.0	1	0.0	1	1,700,000
22	Porto Arabia	Middle/Entrance	Apartment	175.7	2	27.7	2	2,550,000
23	Porto Arabia	Partial Marina/Entrance	Apartment	176.8	2	32.0	2	2,950,000
24	Viva Bahriya	Direct Sea Side	Apartment	141.0	2	16.0	2	2,500,000
25	Porto Arabia	Direct Marina	Apartment	125.5	2	9.5	2	2,460,000
26	Porto Arabia	Direct Marina	Apartment	125.5	2	9.5	2	2,450,000
27	Porto Arabia	Partial Marina	Townhouse	146.0	2	10.0	2	2,400,000
28	Viva Bahriya	Direct Sea Side	Townhouse	114.5	1	17.0	1	2,575,000
29	Porto Arabia	Partial Sea Side/Entrace	Apartment	150.5	1	33.3	1	2,200,000
30	Porto Arabia	Terrace View	Townhouse	146.0	2	10.0	2	2,350,000
31	Porto Arabia	Direct Marina	Apartment	249.7	3	53.4	2	4,350,000
32	Porto Arabia	Partial Marina/Entrance	Apartment	176.2	2	27.7	2	

								2,630,000
33	Viva Bahriya	Direct Sea Side	Apartment	137.4	2	18.0	2	2,200,000
34	Porto Arabia	Partial Sea Side/Entrace	Apartment	136.0	1	38.5	1	2,060,000
35	Porto Arabia	Partial Sea/Pool	Apartment	218.0	3	18.1	2	3,600,000
36	Porto Arabia	Direct Marina	Townhouse	116.5	1	9.5	1	2,100,000
37	Porto Arabia	Direct Marina	Apartment	180.2	2	10.3	2	3,225,000
38	Porto Arabia	Partial Marina	Townhouse	146.0	2	0.0	2	2,800,000
39	Porto Arabia	Direct Marina	Townhouse	146.0	2	10.0	2	3,150,000
40	Porto Arabia	Middle	Apartment	104.7	1	0.0	1	1,600,000
41	Porto Arabia	Partial Sea Side	Apartment	170.5	2	18.1	2	2,700,000
42	Porto Arabia	Direct Marina	Apartment	312.8	3	48.0	2	4,850,000
43	Porto Arabia	Direct Marina	Apartment	312.8	3	48.0	2	4,850,000
44	Porto Arabia	Direct Marina	Apartment	249.7	3	53.4	2	4,850,000
45	Porto Arabia	Direct Marina	Apartment	249.7	2	27.7	2	2,800,000
46	Viva Bahriya	Direct Sea Side	Apartment	135.1	2	16.0	2	2,000,000
47	Porto Arabia	Direct Marina	Townhouse	116.5	1	95.0	1	2,350,000
48	viva Bahriya	Direct Sea Side	Townhouse	117.5	1	8.0	1	2,400,000
49	Porto Arabia	Direct Marina	Apartment	125.8	2	10.0	2	

								2,750,000
50	Porto Arabia	Direct Marina	Townhouse	116.5	1	9.5	2	2,400,000
51	Viva Bahriya	Direct Marina	Apartment	144.5	2	16.0	2	2,500,000
52	Viva Bahriya	Direct Marina	Apartment	141.3	2	16.0	2	2,500,000
53	Viva Bahriya	Direct Marina	Apartment	134.8	2	16.0	2	2,500,000
54	Porto Arabia	Middle Entrance	Apartment	125.1	1	10.3	1	1,950,000
55	Porto Arabia	Direct Marina	Apartment	250.0	3	53.4	2	4,000,000
56	Porto Arabia	Partial Marina/Entrance	Apartment	176.5	2	32.0	2	2,825,000
57	Porto Arabia	Partial Marina/Pool	Apartment	105.0	1	10.3	1	1,800,000

We will apply the goal programming model to predict the prices of 57 properties on the Pearl Qatar Island and assess the effectiveness of this model. Some may argue that the sample size of our analysis is too small and thus is not sufficient to produce a reliable outcome. Therefore, it is important to take into account specific categories of properties and that we are conducting three different analysis to evaluate the effectiveness of this model better as a predictor of property value. So choose to apply the model to three sets of properties because the output of a single model may not be sufficient to come to a conclusion about the effectiveness of the model.

Primarily, we applied model just on the two types of properties Apartments and townhouses together. Then we applied the model to the apartments only. Lastly, we applied the model just the townhouse. There are 44 independent apartments under consideration, 13 townhouses and 57 apartments and townhouses all combined.

a. Parameters and Variables

We will apply the goal programming model to properties on the Pearl Qatar Island. The properties undertaken for the analysis include apartments and townhouses. An apartment is defined as a suit of rooms which form one residence. Building contains some flats. A townhouse, is a narrow, but tall, traditional row house; generally with three or more floors (Oxford, 2016).

We have chosen seven key property variables (or attributes) which are significant components in property valuation (Sunderman, 2012). The attributes are represented from x_1 to x_7. β_1 to β_7 are the parameters which shows the influence of each independent variable of x_1 to x_7 respectively.

β_0 is termed as a constant which represents the discrepancy between what the actual price of the property is and what is estimated by the 7 attributes combined. In other words, it is the value that the 7 attributes do not explain.

The first element; location, is represented by x_1 . Though there are ten areas which make up the pearl island, the properties in our analysis are Porto Arabia represented by 0 and Viva Behria' represented by 1. The second variable; view, is denoted x_2 . This variable is often of key significance for properties in main city locations and islands.

View is numbered on a scale from 1, for the best view and 4 for the best view (See appendix 4). Each property is, of course, unique in its location and view. The third element, x_3 is the type of property; which is either an apartment or a townhouse. This is not applicable to the first two models, so it does have a value there. The forth element is the area of the whole property, represented by x_4 . The fifth variable is one of the key ones; the number of bedrooms; x_5 . Then we have the sixth variable; x_6 : the total area of the balcony. Moreover, lastly, 7th variable; x_7 is the number of parking spaces associated with the property.

- x_1 =Location Porto Arabia represented by 0 and Viva Behria' represented by 1.
- x_2 = View (See Table 2. View Type Level).
- x_3 = Type of property; which is either an apartment or a townhouse.
- x_4 = Area
- x_5 = Number of bedrooms
- x_6 =Balcony Area
- x_7 = Number of Parking spaces

Table 2. View Type Level

View Type	Level
Direct Sea Side	4
Direct Marina	4
Partial Marina	4
Partial Sea Side	4
Partial Sea/Pool	3
Partial Marina/Pool	3
Middle/Pool Side	3
Partial Sea/Entrance	2
Direct Marina/Middle Entrance	2
Partial Marina/Entrance	2
Middle/Entrance	1
Middle	1
Middle Entrance	1
Terrace View	1

b. Decision Variables

Below is a representation of the price equation of the first property, which is an apartment in the combined model for the townhouses and apartments. Here, the final price is the actual price of the property (See table 1).

$$\beta_0 + 1 \times \beta_1 + 4 \times \beta_2 + 0 \times \beta_3 + 2 \times \beta_4 + 137.4 \times \beta_5 + 18 \times \beta_6 + 2 \times \beta_7 + \delta_1^- - \delta_1^+ = 22{,}00{,}000$$

As we discussed above, the variables; also referred to as the decision variables in a goal programming model represent the variable inputs in the model. The seven attributes we have selected are one part of the decision variables (RagsDale, 2013).

The decision variables in our model add up all the attributes we have selected and add the positive δ_i^+ values and negative δ_i^- Values which are produced by Lindo to explain the discrepancy between the actual price of the property and the sum of the relevant attribute value. The price of the property is the price at which the property sold for.

c. The Objective Function

As we have discussed above in the goal programming section, the objective function is one of the three elements of a goal programming model. The objective function speaks for itself and outlines what the model is to achieve, or simply put; it is the single formula which describes precisely what the model has to achieve. Our objective function seeks to minimize the sum of deviations of the positive (δ_i^+) value of each property and the negative (δ_i^-) values of each of the properties.

Appling Our Model for the list of 57 properties in table 1:

$$Min\ z = \sum_{i=1}^{57} (\delta_i^+ + \delta_i^-)$$

Subject to

$$\beta_0 + 0 \times \beta_1 + 4 \times \beta_2 + 0 \times \beta_3 + 137.4 \times \beta_4 + 2 \times \beta_5 + 18 \times \beta_6 + 2 \times \beta_7 + \delta_1^- - \delta_1^+ = 2{,}200{,}000$$

$$\beta_0 + 1 \times \beta_1 + 3 \times \beta_2 + 0 \times \beta_3 + 218.0 \times \beta_4 + 3 \times \beta_5 + 18.16 \times \beta_6 + 2 \times \beta_7 + \delta_2^- - \delta_2^+ = 3{,}600{,}000$$

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$$\beta_0 + 1 \times \beta_1 + 3 \times \beta_2 + 0 \times \beta_3 + 105.0 \times \beta_4 + 1 \times \beta_5 + 10.28 \times \beta_6 + 1 \times \beta_7 + \delta_{57}^- - \delta_{57}^+ = 1{,}770{,}835$$

$$\beta_0, \beta_j\ are\ free\ (unrestricted\ in\ sign)$$

Analysis and Results

As we have discussed above, a model encompasses distinct parameters, also called 'variables'. We will first get an understanding of the variables in our model and then analyze the results of the model using a number of valuable statistical means. This model is not without limitations and those will then be highlighted.

4.1 Using Lindo

We used Lindo; a tool for optimization modeling, to execute the goal programming model. Lindo is widely employed by institutions of all sizes to assist them in decision making, such as, to reduce cost, to optimally allocate resources and more. In excel one has designed the model from scratch, while Lindo allows helpful input of variables, constraints, and the objective function. Though much of the goal programming modeling can be done in excel, except that Lindo makes it easier to set out the problem. In the appendices there are three Lindo codes for townhouses only (see Appendix E), apartments only (see Appendix C) and combined (see Appendix A).

4.2 Interpreting the Variables

The results return a value for each of the parameters from β_0 toβ_7. These values represent the contribution that each unit of the attribute makes towards the price of the property. In the apartment's model in (see appendix D) we see that the number of bedrooms is the most influential factor with a value of 775156.7 while the parking space tends to undercut the price and the type of property; whether its townhouse or not, seems to have the least contribution towards a higher price. The other variables are shown to contribute towards a higher price. The number of parking spaces available for the townhouses turned out to be the most influential factor in determining their price, while the location seems to undercut its value. The balcony space is shown to have a positive, but minimal impact (see appendix D). Moreover, lastly, in the model combined with the apartments and townhouses we see somewhat of a similar and different outcome. Here, the parking space seems to have an adverse impact as well and the location seems to be the most significant factor (See appendix B). The significance of these results can be assessed by adopting some statistical measures.

4.3 Significance of the Mean Absolute Difference

When assessing statistical results, the mean absolute difference has been moderated as a reliable approach to drawing statistical inferences about the differences in two population proportions (Black, 2012). Output is statistically significant when there is confidence that the output was not produced by chance (The mean difference (or difference in means), 2001).

The average absolute difference (MAD) is a measure of dispersion of two individual averages drawn from a probability distribution. It is one of the most common measures of how much a set of observations differ from the mean (The mean difference (or difference in means), 2001). It is calculated by taking the average (mean) of a set of numbers and subtracting from the average difference from another set of numbers and taking the absolute value of that. By absolute, means that we ignore whether the average difference is negative or positive.

The MAD is considered a much more robust static compared to other statistical measures such as the standard deviation or variance. The variance is a little difficult to interpret since we have to take the squared differences between each data point to make the numbers positive, which means that the variance is no longer in the same unit of measure as the original data set (Mean absolute deviation vs. standard deviation, 2015). The problem with standard deviation is that it has a tendency to distort the concerned values because the values are squared. Even though we take the square root of the number, later on, it is still distorted. Many academia argues that the act of squaring, before actually adding up and then taking the square root after dividing make the results seem a little strange (Gorard, 2004). The MAD, on the other hand, is a much more simplified approach, which does not manipulate the figures in any way.

We will take the mean of the actual prices and subtract it from the error; the difference between actual price and the estimated price. Then take the mean of the error as a percentage of the actual price to assess how close on average the estimated price is the real price.

b. Mean absolute difference: Apartments and Townhouses

For the combined model the average of the estimated prices of the apartments deviates about 201,033.31 QAR that is 7.1% from the average actual prices. From a generalized statistical perspective, an "estimated" answer which is over 90% close to the actual is one is an excellent predictor (Sedgwick, 2014). Note that we have the largest sample here, with 57 properties. A larger sample size is normally intended to be a better predictor, but note that it can take the results in any direction.

c. Mean absolute difference: Apartments

For the apartments model the deviation is about 133,791 that is 4.85% to the average actual prices with a sample size of 40. A Comparative analysis of the differences between the estimated prices and the actual prices reveal that the predictions came as close to being completely accurate to as far as 23% of the real price. This is a strong deviation at the individual price level, but note that it averages out pretty good.

d. Mean absolute difference: Townhouses

For the townhouses, our model came as close as 244,548.79 QAR that is 7.92% to the average actual prices with a sample size of 13 townhouses. Note that this model did predict the correct price for 7 of the properties and then there is significant deviation for the remaining properties. Note that we have the smallest sample size here. As some statistical academics may argue, this model of ours has a very subtle sample size. This argument does have substance, but it’s important to note that the same model has produced more significant results for the apartment’s model. It is important to understand that certain attributes are more significant to specific type of properties compared to others. As for the townhouses, people tend to have more of a personally perceived value for each individual townhouse, which can vary broadly from the structural attributes to other perceived attributes of value.

4.4 Testing the function

a. Townhouse

(See Table 3. Townhouse Table and calculation of MAD)

β_3 Stands for type which all types are townhouses therefore we have removed it from the analysis and kept the numbering of the parameters and notations consistent for the attributes and parameters.

The coefficient of β_0 to β_7 be as following: (See Appendix F)

- β_0 = -597876.125
- β_1 = -319291.875
- β_2 = 218865.609375
- β_4 =12059.443359
- β_5 =240513.40625
- β_6 =20659.681641
- β_7 =300000.0

$$Estimated\ Price = \beta_0 + x_1\beta_1 + x_2\beta_2 + x_4\beta_4 + x_5\beta_5 + x_6\beta_6 + x_7\beta_7$$

$$EP = -597876.125 - 319291.9x_1 + 218865.6x_2 + 12059.4x_4 + 240513.4x_5 + 20659.7x_6 + 300000x_7$$

Table 3. Townhouse Table and calculation of MAD

No.	Precinct (Location)	Property View	Asset Type	Area (m2)	No. of Bedrooms	Balcony	Parking Space Count	RESALE VALUE QAR	Estimated Value	Absolute Difference
5	1	4	NA	146.0	2	10	2	2,800,000	3,006,597	206,597
10	0	4	NA	114.5	1	17	1	2,550,000	2,550,000	0
16	1	4	NA	474.0	3	97	2	9,000,000	9,000,000	0
20	1	2	NA	146.0	2	10	2	3,250,000	2,568,866	681,134
27	1	4	NA	146.0	2	10	2	2,400,000	3,006,597	606,597
28	0	4	NA	114.5	1	17	1	2,575,000	2,550,000	25,000
30	1	1	NA	146.0	2	10	2	2,350,000	2,350,000	0
36	1	4	NA	116.5	1	9.5	1	2,100,000	2,100,000	0
38	1	4	NA	146.0	2	0	2	2,800,000	2,800,000	0
39	1	4	NA	146.0	2	10	2	3,150,000	3,006,597	143,403

47	1	4	NA	116.5	1	95	1	2,350,000	3,866,403	1,516,403
48	0	4	NA	117.5	1	8	1	2,400,000	2,400,000	0
50	1	4	NA	116.5	1	9.5	2	2,400,000	2,400,000	0
							Average=	3,086,538	MAD=	244,549
								3,086,538	Percentage=	7.92%

b. Regular Apartments

(See Table *4.* Apartments table and calculations)

β_3 Stands for type which all types are townhouses therefore we have removed it from the analysis and kept the numbering of the parameters and notations consistent for the attributes and parameters.

The coefficient of β_0 to β_7 is as following :(See Appendix D)

- $\beta_0 =$ 166996.6094
- $\beta_1 =$ 121157.6875
- $\beta_2 =$ 61277.11328
- $\beta_4 =$ 7472.114258
- $\beta_5 =$ 775156.6875
- $\beta_6 =$ 4485.793457
- $\beta_7 =$ -306993.1563

$$Estimated\ Price = \beta_0 + x_1\beta_1 + x_2\beta_2 + x_4\beta_4 + x_5\beta_5 + x_6\beta_6 + x_7\beta_7$$

$$EP = 166996.6 + 121157.7x_1 \quad + 61277.1x_2 + 7472.1x_4 \quad + 775156.7x_5 + 4485.8x_6 - 306993.2x_7$$

Table 4. Apartments table and calculations

No.	Precinct (Location)	Property View	Asset Type	Area (m2)	No. of Bedrooms	Balcony	Parking Space Count	Price Sold	Estimated Value	Absolute Difference
1	0	4	NA	137.40	2	18	2	2,200,000	2,455,845	255,845
2	1	3	NA	218.00	3	18.11	2	3,600,000	3,893,628	293,628
3	1	4	NA	249.69	3	53.40	2	4,350,000	4,350,000	0
4	0	4	NA	137.40	2	18	2	2,200,000	2,455,845	255,845
6	0	2	NA	141.01	2	16	2	2,500,000	2,351,293	148,707
7	1	1	NA	105.00	1	10.28	1	1,750,000	1,648,281	101,719
8	1	3	NA	136	1	34.20	1	2,000,000	2,109,771	109,771
9	0	3	NA	141.40	2	16	2	2,550,000	2,415,485	134,515
11	1	4	NA	256.13	3	54	2	4,350,000	4,400,812	50,812
12	1	1	NA	176.75	2	32	2	2,750,000	2,750,000	0
13	1	3	NA	121	1	9.91	1	1,750,000	1,888,729	138,729
14	1	4	NA	83.00	1	8.53	1	1,675,000	1,659,876	15,124
15	1	3	NA	95.00	1	0	1	1,650,000	1,650,000	0
17	1	2	NA	121.00	1	9.91	1	1,850,000	1,827,452	22,548
18	1	3	NA	343.32	3	56.00	2	5,000,000	5,000,000	0
19	1	2	NA	121.00	1	9.91	1	1,950,000	1,827,452	122,548
21	1	4	NA	94.00	1	0	1	1,700,000	1,703,805	3,805
22	1	1	NA	175.68	2	27.7	2	2,550,000	2,722,716	172,716
23	1	2	NA	176.75	2	32.00	2	2,950,000	2,811,277	138,723
24	0	4	NA	140.95	2	16	2	2,500,000	2,473,399	26,601
25	1	4	NA	125.50	2	9.51	2	2,460,000	2,450,000	10,000
26	1	4	NA	125.50	2	9.51	2	2,450,000	2,450,000	0
29	1	2	NA	150.54	1	33.25	1	2,200,000	2,152,877	47,123
31	1	4	NA	249.69	3	53.4	2	4,350,000	4,350,000	0
32	1	2	NA	176.19	2	27.7	2	2,630,000	2,787,804	157,804
33	0	4	NA	137.40	2	18	2	2,200,000	2,455,845	255,845
34	1	2	NA	136	1	38.52	1	2,060,000	2,067,872	7,872
35	1	3	NA	218.00	3	18.11	2	3,600,000	3,893,628	293,628

37	1	4	NA	180.20	2	10.28	2	3,225,000	2,862,179	362,821
40	1	1	NA	104.71	1	0	1	1,600,000	1,600,000	0
41	1	4	NA	170.47	2	18.11	2	2,700,000	2,824,599	124,599
42	1	4	NA	312.78	3	48	2	4,850,000	4,797,192	52,808
43	1	4	NA	312.78	3	48	2	4,850,000	4,797,192	52,808
44	1	4	NA	249.69	3	53.4	2	4,850,000	4,350,000	500,000
45	1	4	NA	249.69	2	27.70	2	2,800,000	3,459,559	659,559
46	0	4	NA	135.06	2	16	2	2,000,000	2,429,389	429,389
49	1	4	NA	125.81	2	10	2	2,750,000	2,454,514	295,486
51	0	4	NA	144.51	2	16	2	2,500,000	2,500,000	0
52	0	4	NA	141.26	2	16.00	2	2,500,000	2,475,716	24,284
53	0	4	NA	134.78	2	16	2	2,500,000	2,427,296	72,704
54	1	1	NA	125.11	1	10.28	1	1,950,000	1,798,545	151,455
55	1	4	NA	249.98	3	53.4	2	4,000,000	4,352,167	352,167
56	1	2	NA	176.49	2	32.00	2	2,825,000	2,809,334	15,666
57	1	3	NA	105.00	1	10.28	1	1,800,000	1,770,835	29,165
							Average	2,760,795	MAD=	133,791
									Percentage	4.85%

c. Combined Analysis Regular Apartments and townhouses

(See Table 5. *Estimation* and calculations of the MAD for combined Townhouses and Apartments)

β_3 Stands for type which all types are townhouses therefore we have removed it from the analysis and kept the numbering of the parameters and notations consistent for the attributes and parameters.

The coefficient of β_0 to β_7 be as following: (See Appendix B)

- $\beta_0 = 99530.71094$
- $\beta_1 = 82781.45313$
- $\beta_2 = 69474.85156$
- $\beta_3 = 525997.5625$
- β_4 =9160.628906
- β_5 =594922.6875
- β_6 =3158.273926
- β_7 =-205919.1563

$$Estimated\ Price = \beta_0 + x_1\beta_1 + x_2\beta_2 + x_3\beta_3 + x_4\beta_4 + x_5\beta_5 + x_6\beta_6 + x_7\beta_7$$

$$EP = 99530.7 + 82781.5x_1 \quad + 69474.9x_2 + 525997.6x_3 + 9160.6x_4 \quad + 594922.7x_5 + 3158.3x_6 - 205919.2x_7$$

Table 5. Estimation and calculations of the MAD for combined Townhouses and Apartments

No.	Precinct (Location)	Property View	Asset Type	Area (m2)	No. of Bedrooms	Balcony	Parking Space Count	Sold Price	Estimated Value	Absolute Difference
1	0	4	0	137.40	2	18	2	2,200,000	2,470,957	270,957
2	1	3	0	218.00	3	18.11	2	3,600,000	3,817,880	217,880
3	1	4	0	249.69	3	53.40	2	4,350,000	4,289,111	60,889
4	0	4	0	137.40	2	18.00	2	2,200,000	2,470,957	270,957
5	1	4	1	146.00	2	10	2	2,800,000	3,133,251	333,251
6	0	2	0	141.01	2	16	2	2,500,000	2,358,760	141,240
7	1	1	0	105.00	1	10.28	1	1,750,000	1,635,124	114,876
8	1	3	0	136.00	1	34.2	1	2,000,000	2,133,599	133,599
9	0	3	0	141.40	2	16	2	2,550,000	2,431,808	118,192
10	0	4	1	114.49	1	17	1	2,550,000	2,394,922	155,078
11	1	4	0	256.13	3	54	2	4,350,000	4,350,000	0
12	1	1	0	176.75	2	32	2	2,750,000	2,750,000	0
13	1	3	0	121	1	9.91	1	1,750,000	1,919,475	169,475
14	1	4	0	83.00	1	8.53	1	1,675,000		

									1,636,487	38,513
15	1	3	0	95.00	1	0	1	1,650,000	1,650,000	0
16	1	4	1	474.00	3	97	2	9,000,000	7,007,630	1,992,370
17	1	2	0	121	1	9.91	1	1,850,000	1,850,000	0
18	1	3	0	343.32	3	56	2	5,000,000	5,085,557	85,557
19	1	2	0	121	1	9.91	1	1,950,000	1,850,000	100,000
20	1	2	1	146.00	2	10	2	3,250,000	2,994,301	255,699
21	1	4	0	94.00	1	0	1	1,700,000	1,710,314	10,314
22	1	1	0	175.68	2	27.7	2	2,550,000	2,726,618	176,618
23	1	2	0	176.75	2	32	2	2,950,000	2,819,475	130,525
24	0	4	0	140.95	2	16	2	2,500,000	2,497,160	2,840
25	1	4	0	125.50	2	9.51	2	2,460,000	2,417,913	42,087
26	1	4	0	125.50	2	9.51	2	2,450,000	2,417,913	32,087
27	1	4	1	146.00	2	10	2	2,400,000	3,133,251	733,251
28	0	4	1	114.49	1	17	1	2,575,000	2,394,922	180,078
29	1	2	0	150.54	1	33.25	1	2,200,000	2,194,319	5,681
30	1	1	1	146.00	2	10	2	2,350,000		

									2,924,826	574,826
31	1	4	0	249.69	3	53.4	2	4,350,000	4,289,111	60,889
32	1	2	0	176.19	2	27.7	2	2,630,000	2,800,764	170,764
33	0	4	0	137.40	2	18	2	2,200,000	2,470,957	270,957
34	1	2	0	136.00	1	38.52	1	2,060,000	2,077,768	17,768
35	1	3	0	218.00	3	18.11	2	3,600,000	3,817,880	217,880
36	1	4	1	116.50	1	9.5	1	2,100,000	2,472,430	372,430
37	1	4	0	180.2	2	10.28	2	3,225,000	2,921,431	303,569
38	1	4	1	146.00	2	0	2	2,800,000	3,101,668	301,668
39	1	4	1	146.00	2	10	2	3,150,000	3,133,251	16,749
40	1	1	0	104.71	1	0	1	1,600,000	1,600,000	0
41	1	4	0	170.47	2	18.11	2	2,700,000	2,857,027	157,027
42	1	4	0	312.78	3	48	2	4,850,000	4,850,000	0
43	1	4	0	312.78	3	48	2	4,850,000	4,850,000	0
44	1	4	0	249.69	3	53.4	2	4,850,000	4,289,111	560,889
45	1	4	0	249.69	2	27.7	2	2,800,000	3,613,020	813,020
46	0	4	0	135.06	2	16	2	2,000,000		

									2,443,204	443,204
47	1	4	1	116.5	1	95	1	2,350,000	2,742,462	392,462
48	0	4	1	117.47	1	8	1	2,400,000	2,393,796	6,204
49	1	4	0	125.81	2	10	2	2,750,000	2,422,300	327,700
50	1	4	1	116.50	1	9.5	2	2,400,000	2,266,510	133,490
51	0	4	0	144.51	2	16	2	2,500,000	2,529,772	29,772
52	0	4	0	141.26	2	16	2	2,500,000	2,500,000	0
53	0	4	0	134.78	2	16	2	2,500,000	2,440,639	59,361
54	1	1	0	125.11	1	10.28	1	1,950,000	1,819,344	130,656
55	1	4	0	249.98	3	53.4	2	4,000,000	4,291,767	291,767
56	1	2	0	176.49	2	32	2	2,825,000	2,817,093	7,907
57	1	3	0	105.00	1	10.28	1	1,800,000	1,774,073	25,927
							Average	2,835,088	MAD	201,033
									Percentage	7.1%

In the townhouses model and the combined model with the townhouses and apartments, we tested the function which the goal programming model produced. We only used 12 of the 13 properties in the townhouses model in order to do this. The general function is as follows:

$$f(x) = \beta_0 + x_1\beta_1 + x_2\beta_2 + x_3\beta_3 + x_4\beta_4 + x_5\beta_5 + x_6\beta_6 + x_7\beta_7$$

Property Estimated Value= 482072.9+ (Location)*(-317536.531250) + (Property View)* (16666.7) + (Asset Type) 0 +(Area)*(15767.9)+ (No. Bedrooms)*(-476097.3) + (Balcony Area)*(21887.6) + (No. parking spaces)*(300000).

We applied the corresponding values of the variables from b0 to b7 to property number 47 which was omitted from the model. By applying the function, the estimated price of the 47th property deviates about 65% from the actual price while the average deviation for the remaining properties is only 4%. The function has proved to be of little value here. This is because there is significant deviation for some properties while for others it is close to no deviation.

The model was created using 52 of the 57 properties in the combined analysis. The values for the remaining the four properties were inputted using the values (b0 to b7) produced by the model. The MAD for the 5 properties is 115457 while the MAD for the remaining 52 properties is 120826. This is not such a significant difference, so we can infer that the estimated price of the 52 properties and four properties deviate about the same amount from the mean. We can draw further inferences by looking at how much the estimated price of the property deviates from the actual prices. For the 52 properties the deviation is 4.2133%, and for the remaining four properties, it is 4.3672%. So the deviation is about the same. Thus we can infer that the function is reliable.

For the apartment's model, the estimated prices from the model deviate about 4.81% while the 5 estimates produced by the function deviate about 5.19%. This shows that the function is reliable.

4.5 Slack or surplus

Slack or Surplus column in a Lindo solutions report reveals how closely we are from satisfying a constraint we incorporated in our model. A slack exists when the quantity is less than or equal to the restriction. A surplus exists when a quantity is greater than or equal to the restriction. If the constraint is fully satisfied, there is no slack or surplus and a value of 0 is returned (Lindo inc., n.d.). A slack or surplus can also be negative if the constraint is violated or has landed in an infeasible region. The apartment's model see that the slack or surplus is 0 for all 40 properties under consideration. The townhouses file also reveals a slack/surplus of 0, and same results follow the results where the townhouses and apartments are combined.

4.6 Dual Prices

There is also a dual prices figure for each constraint which represents the amount that the objective would improve on the right-hand side, or constant term if we increase the constraint by one unit. A dual price is sometimes referred to as a shadow price because it reveals the cost of using an additional resource unit, or our analysis, the price impact of an additional attribute. The dual prices value varies by each property in our analysis. It takes both negative and positive values (Lindo Inc. , n.d.).

4.6 Steps to Optimum Solution and Iteration

In the apartment's model, there were 59 iterations. In the townhouses model, the optimum solution was found at step 13. For the combined apartments and townhouses scenario there were 88 iterations (88 steps) where the excellent solution was found.

Lindo also returns an LP linear optimum value as well, which reveals in which iteration was the optimum solution was found. The optimum solution is where all constraints in the model are satisfied. Often a more complicated or inclusive model needs to run a number of ties before the optimum solutions is found. Note that the larger the sample size, the larger the iterations. Here the optimum solution is where the positive (P values) and the negative (N values) are minimized, and the maximum contribution of each attribute towards the property is found.

4.7 Objective Function

The objective function value for the apartment's model is 5338364; which is the minimized sum of the negative and positive values. For the townhouses model the objective function value is 1662731 which is much less than the value for the townhouses. This essentially means that the minimized sum of deviations of the attributes for apartments is less than that of townhouses and so the apartment's model is a better predictor of prices than the townhouses model. The value for the combined model is 10885900 which is much higher than the apartments or the townhouses, thus the attributes of the combined model are less explanatory about the property’s value.

Goal programming: limitations

Goal programming is about 30 years old. Though the goal programming model has been able to come very close to the actual prices of the property, it is not without its limitations. Other than the difficulty in incorporating all the possible factors which can influence the property's value, the model does also make some assumptions. The model assumes that values of all model parameters ($\beta_0 \ldots \beta_7$) are known with certainty, while this may not be the case. The model also assumes that the decision variables can take on fractional value and therefore continuous as opposed to an integer in nature. Also, the terms in the objective function and constraints must be additive. Perhaps the most significant drawback is that the model assumes the parameters in the model to be constant, but in reality, this may not always be the case (Harrald, 1978).

However, it is important to take into account that the model does not make any hard assumptions about the property specifics like other models. For example, it does not necessarily assume that the property will produce rent income for a certain period as the income approach. The assumptions the model makes are rather soft assumptions which do not have an immediate and direct impact on the results and do not manipulate the results. Though the popularity of the model is increasing, there is little evidence that that goal programming is being used by academics to address issues of academics. Academics only use it sparingly in a research methodology, while they do use many other methods (W. Lin & O'Leary, 1988).

Conclusion

The quest towards developing a standardized is underway but is likely not be successful given the broad range of tools and interpretation of results. The application of the goal programming model has produced prices that are very close to the actual prices or are at least statistically significant. The significance of the goal programming model is that it is able to incorporate property specific attributes and determine the contribution that each attribute made towards the total price of the property. This model allows the analyst to control the various attributes which are significant to a particular property's value (Aouni & Martel). This gives analysts the chance to apply their know-how in the model as they can control for the variables as they like. Choosing the right fit of variables is discretionary, but it is also difficult given the right quantity and quality of variables have to adopt for optimum results or consequently the model can prove dissatisfactory.

Though individual estimates have varied significantly, from being as close to predicting the actual price to as far as 23% from the actual price, we can see that it averages out pretty close to about 4.5% for the apartment buildings, 7.92% for townhouses and 7.2% for the townhouses and apartments combined. The larger sample composed of the segregated apartments produced prices closer to the actual prices and the smaller data set; the townhouses, produced worse results. We can infer, that applying the model to one particular type of property in a particular geographical area, with a substantial data set, produces better results, compared to a larger sample size composed of different properties or a smaller data set composed of one particular type of property. This seems to be the outcome of various factors affecting the two properties to varying degrees. Note that one particular attribute may be explanatory to one particular property and largely unrelated to another one, which can be the reason for varying results among the apartments and townhouses model.

The only model which speaks a similar language as our goal programming model is the hedonic pricing model. Though the hedonic model produces the contribution of each variable on the property, it is a statistical method which assumes that the variables do not affect one another and make the results misleading if this assumption is violated while the goal programming model does not make any such assumptions and subsequently,

Our results for the apartments were better than the results for townhouses. This shows that results using this model may vary depending on the type of property and the relative mix of the chosen variables. But the analyst can assess the significance of the results before practically using a model.

As we discussed in the paper, a real estate property is a function of the attached rights and quantified physical features. Socioeconomic factors and government policies are likely to impact the quantified components of property which in return affects the total value of property. Imposing, reducing or increasing the tax on property is one government policy which may affect valuation. The valuation process will not produce reliable results if the model that we use does not take into account these factors. It may be impossible to predict the transaction price accurately or to build a model which includes all the possible factors which can influence property value, from our analysis we can infer that choosing relevant variables can produce excellent results.

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Appendices

Appendix A. Lindo Code for Apartments and townhouses combined

min

n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12+n13+n14+n15+n16+n17+n18+n19+n20+n21+n22+n23+n24+n25+n26+n27+n28+n29+n30+n31+n32+n33+n34+n35+n36+n37+n38+n39+n40+n41+n42+n43+n44+n45+n46+n47+n48+n49+n50+n51+n52+p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16+p17+p18+p19+p20+p21+p22+p23+p24+p25+p26+p27+p28+p29+p30+p31+p32+p33+p34+p35+p36+p37+p38+p39+p40+p41+p42+p43+p44+p45+p46+p47+p48+p49+p50+p51+p52

st

b0+0b1+4b2+0b3+137.4b4+2b5+18b6+2b7+n1-p1=2200000

b0+1b1+3b2+0b3+218b4+3b5+18.11b6+2b7+n2-p2=3600000

b0+1b1+4b2+0b3+249.69b4+3b5+53.4b6+2b7+n3-p3=4350000

b0+0b1+4b2+0b3+137.4b4+2b5+18b6+2b7+n4-p4=2200000

b0+1b1+4b2+1b3+146b4+2b5+10b6+2b7+n5-p5=2800000

b0+0b1+2b2+0b3+141.01b4+2b5+16b6+2b7+n6-p6=2500000

b0+1b1+1b2+0b3+105b4+1b5+10.28b6+1b7+n7-p7=1750000

b0+1b1+3b2+0b3+136b4+1b5+34.2b6+1b7+n8-p8=2000000

b0+0b1+3b2+0b3+141.4b4+2b5+16b6+2b7+n9-p9=2550000

b0+0b1+4b2+1b3+114.49b4+1b5+17b6+1b7+n10-p10=2550000

b0+1b1+4b2+0b3+256.13b4+3b5+54b6+2b7+n11-p11=4350000 b0+1b1+1b2+0b3+176.75b4+2b5+32b6+2b7+n12-p12=2750000

b0+1b1+3b2+0b3+121b4+1b5+9.91b6+1b7+n13-p13=1750000

b0+1b1+4b2+0b3+83b4+1b5+8.53b6+1b7+n14-p14=1675000

b0+1b1+3b2+0b3+95b4+1b5+0b6+1b7+n15-p15=1650000

b0+1b1+4b2+1b3+474b4+3b5+97b6+2b7+n16-p16=9000000

b0+1b1+2b2+0b3+121b4+1b5+9.91b6+1b7+n17-p17=1850000

b0+1b1+3b2+0b3+343.32b4+3b5+56b6+2b7+n18-p18=5000000

b0+1b1+2b2+0b3+121b4+1b5+9.91b6+1b7+n19-p19=1950000

b0+1b1+2b2+1b3+146b4+2b5+10b6+2b7+n20-p20=3250000

b0+1b1+4b2+0b3+94b4+1b5+0b6+1b7+n21-p21=1700000

b0+1b1+1b2+0b3+175.68b4+2b5+27.7b6+2b7+n22-p22=2550000

b0+1b1+2b2+0b3+176.75b4+2b5+32b6+2b7+n23-p23=2950000

b0+0b1+4b2+0b3+140.95b4+2b5+16b6+2b7+n24-p24=2500000

b0+1b1+4b2+0b3+125.5b4+2b5+9.51b6+2b7+n25-p25=2460000

b0+1b1+4b2+0b3+125.5b4+2b5+9.51b6+2b7+n26-p26=2450000

b0+1b1+4b2+1b3+146b4+2b5+10b6+2b7+n27-p27=2400000

b0+0b1+4b2+1b3+114.49b4+1b5+17b6+1b7+n28-p28=2575000

b0+1b1+2b2+0b3+150.54b4+1b5+33.25b6+1b7+n29-p29=2200000

b0+1b1+1b2+1b3+146b4+2b5+10b6+2b7+n30-p30=2350000

b0+1b1+4b2+0b3+249.69b4+3b5+53.4b6+2b7+n31-p31=4350000

b0+1b1+2b2+0b3+176.19b4+2b5+27.7b6+2b7+n32-p32=2630000

b0+0b1+4b2+0b3+137.4b4+2b5+18b6+2b7+n33-p33=2200000

b0+1b1+2b2+0b3+136b4+1b5+38.52b6+1b7+n34-p34=2060000

b0+1b1+3b2+0b3+218b4+3b5+18.11b6+2b7+n35-p35=3600000

b0+1b0+4b2+1b3+116.5b4+1b5+9.5b6+1b7+n36-
p36=2100000

b0+1b1+4b2+0b3+180.2b4+2b5+10.28b6+2b7+n37-
p37=3225000

b0+1b0+4b2+1b3+146b4+2b5+0b6+2b7+n38-
p38=2800000

b0+1b0+4b2+1b3+146b4+2b5+10b6+2b7+n39-
p39=3150000

b0+1b1+1b2+0b3+104.71b4+1b5+0b6+1b7+n40-
p40=1600000

b0+1b1+4b2+0b3+170.47b4+2b5+18.11b6+2b7+n41-
p41=2700000

b0+1b1+4b2+0b3+312.78b4+3b5+48b6+2b7+n42-
p42=4850000

b0+1b1+4b2+0b3+312.78b4+3b5+48b6+2b7+n43-
p43=4850000

b0+1b1+4b2+0b3+249.69b4+3b5+53.4b6+2b7+n44-
p44=4850000

b0+1b1+4b2+0b3+249.69b4+2b5+27.7b6+2b7+n45-
p45=2800000

b0+0b1+4b2+0b3+135.06b4+2b5+16b6+2b7+n46-
p46=2000000

b0+1b1+4b2+1b3+116.5b4+1b5+95b6+1b7+n47-
p47=2350000

b0+0b1+4b2+1b3+117.47b4+1b5+8b6+1b7+n48-
p48=2400000

b0+1b1+4b2+0b3+125.81b4+2b5+10b6+2b7+n49-
p49=2750000

b0+0b1+4b2+0b3+144.51b4+2b5+16b6+2b7+n50-
p50=2500000

b0+0b1+4b2+0b3+141.26b4+2b5+16b6+2b7+n51-
p51=2500000

b0+0b1+4b2+0b3+134.78b4+2b5+16b6+2b7+n52-
p52=2500000

end

free b0

free b1

free b2

free b3

free b4

free b5

free b6

free b7

Appendix B. Lindo File Result for townhouses and apartments combined

LP OPTIMUM FOUND AT STEP 88

OBJECTIVE FUNCTION VALUE

1) 0.1088590E+08

VARIABLE	VALUE	REDUCED COST
N1	0.000000	2.000000
N2	0.000000	2.000000
N3	60889.417969	0.000000
N4	0.000000	2.000000
N5	0.000000	2.000000
N6	141239.859375	0.000000
N7	114876.359375	0.000000
N8	0.000000	2.000000
N9	118192.367188	0.000000
N10	478439.468750	0.000000
N11	0.000000	0.692994
N12	4944.493652	0.000000
N13	0.000000	2.000000
N14	38512.617188	0.000000
N15	0.000000	0.209786
N16	2320676.750000	0.000000
N17	0.000000	1.580427
N18	0.000000	2.000000
N19	100000.000000	0.000000
N20	588949.687500	0.000000
N21	0.000000	2.000000
N22	0.000000	2.000000
N23	135469.640625	0.000000
N24	2839.795166	0.000000
N25	47031.757812	0.000000
N26	37031.757812	0.000000
N27	0.000000	2.000000
N28	503439.468750	0.000000
N29	5680.897949	0.000000
N30	0.000000	2.000000
N31	60889.417969	0.000000

N32	0.000000	2.000000	P1	270956.531250	0.000000
N33	0.000000	2.000000	P2	217879.906250	0.000000
N34	0.000000	2.000000	P3	0.000000	2.000000
N35	0.000000	2.000000	P4	270956.531250	0.000000
N36	0.000000	2.000000	P5	0.000000	0.000000
N37	308513.468750	0.000000	P6	0.000000	2.000000
N38	0.000000	2.000000	P7	0.000000	2.000000
N39	318417.250000	0.000000	P8	133598.765625	0.000000
N40	0.000000	0.209786	P9	0.000000	2.000000
N41	0.000000	2.000000	P10	0.000000	2.000000
N42	0.000000	1.307006	P11	0.000000	1.307006
N43	0.000000	2.000000	P12	0.000000	2.000000
N44	560889.437500	0.000000	P13	169474.859375	0.000000
N45	0.000000	2.000000	P14	0.000000	2.000000
N46	0.000000	2.000000	P15	0.000000	1.790214
N47	0.000000	2.000000	P16	0.000000	2.000000
N48	329565.250000	0.000000	P17	0.000000	0.419573
N49	332644.406250	0.000000	P18	85556.953125	0.000000
N50	0.000000	2.000000	P19	0.000000	2.000000
N51	0.000000	1.000000	P20	0.000000	2.000000
N52	59360.878906	0.000000	P21	10314.225586	0.000000

P22	171673.046875	0.000000
P23	0.000000	2.000000
P24	0.000000	2.000000
P25	0.000000	2.000000
P26	0.000000	2.000000
P27	400000.000000	0.000000
P28	0.000000	2.000000
P29	0.000000	2.000000
P30	241575.437500	0.000000
P31	0.000000	2.000000
P32	165819.828125	0.000000
P33	270956.531250	0.000000
P34	17767.658203	0.000000
P35	217879.906250	0.000000
P36	75706.031250	0.000000
P37	0.000000	2.000000
P38	0.000000	0.000000
P39	0.000000	2.000000
P40	0.000000	1.790214
P41	152082.890625	0.000000
P42	0.000000	0.692994
P43	0.000000	0.000000
P44	0.000000	2.000000
P45	808075.812500	0.000000
P46	443204.093750	0.000000
P47	64155.722656	0.000000
P48	0.000000	2.000000
P49	0.000000	2.000000
P50	29772.044922	0.000000
P51	0.000000	1.000000
P52	0.000000	2.000000
B0	109419.695312	0.000000
B1	77836.953125	0.000000
B2	69474.851562	0.000000
B3	197691.312500	0.000000
B4	9160.628906	0.000000
B5	599867.125000	0.000000
B6	3158.273926	0.000000
B7	-215808.140625	0.000000

ROW SLACK OR SURPLUS DUAL PRICES

2)	0.000000	1.000000	23)	0.000000	1.000000
3)	0.000000	1.000000	24)	0.000000	-1.000000
4)	0.000000	-1.000000	25)	0.000000	-1.000000
5)	0.000000	1.000000	26)	0.000000	-1.000000
6)	0.000000	1.000000	27)	0.000000	-1.000000
7)	0.000000	-1.000000	28)	0.000000	1.000000
8)	0.000000	-1.000000	29)	0.000000	-1.000000
9)	0.000000	1.000000	30)	0.000000	-1.000000
10)	0.000000	-1.000000	31)	0.000000	1.000000
11)	0.000000	-1.000000	32)	0.000000	-1.000000
12)	0.000000	-0.307006	33)	0.000000	1.000000
13)	0.000000	-1.000000	34)	0.000000	1.000000
14)	0.000000	1.000000	35)	0.000000	1.000000
15)	0.000000	-1.000000	36)	0.000000	1.000000
16)	0.000000	-0.790214	37)	0.000000	1.000000
17)	0.000000	-1.000000	38)	0.000000	-1.000000
18)	0.000000	0.580427	39)	0.000000	1.000000
19)	0.000000	1.000000	40)	0.000000	-1.000000
20)	0.000000	-1.000000	41)	0.000000	-0.790214
21)	0.000000	-1.000000	42)	0.000000	1.000000
22)	0.000000	1.000000	43)	0.000000	0.307006

44)	0.000000	1.000000
45)	0.000000	-1.000000
46)	0.000000	1.000000
47)	0.000000	1.000000
48)	0.000000	1.000000
49)	0.000000	-1.000000
50)	0.000000	-1.000000
51)	0.000000	1.000000
52)	0.000000	0.000000
53)	0.000000	-1.000000

NO. ITERATIONS= 88

Appendix C. Lindo Code for Regular apartments only

min

n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12+n13+n14+n15+n16+n17+n18+n19+n20+n21+n22+n23+n24+n25+n26+n27+n28+n29+n30+n31+n32+n33+n34+n35+n36+n37+n38+n39+n40+p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16+p17+p18+p19+p20+p21+p22+p23+p24+p25+p26+p27+p28+p29+p30+p31+p32+p33+p34+p35+p36+p37+p38+p39 +p40

st

b0+0b1+4b2+137.4b4+2b5+18b6+2b7+n1-p1=2200000

b0+1b1+3b2+218b4+3b5+18.11b6+2b7+n2-p2=3600000

b0+1b1+4b2+249.69b4+3b5+53.4b6+2b7+n3-p3=4350000

b0+0b1+4b2+137.4b4+2b5+18b6+2b7+n4-p4=2200000

b0+0b1+2b2+141.01b4+2b5+16b6+2b7+n5-p5=2500000

b0+1b1+1b2+105b4+1b5+10.28b6+1b7+n6-p6=1750000

b0+1b1+3b2+136b4+1b5+34.2b6+1b7+n7-p7=2000000

b0+0b1+3b2+141.4b4+2b5+16b6+2b7+n8-p8=2550000

b0+1b1+4b2+256.13b4+3b5+54b6+2b7+n9-p9=4350000

b0+1b1+1b2+176.75b4+2b5+32b6+2b7+n10-p10=2750000

b0+1b1+3b2+121b4+1b5+9.91b6+1b7+n11-p11=1750000

b0+1b1+4b2+83b4+1b5+8.53b6+1b7+n12-p12=1675000

b0+1b1+3b2+95b4+1b5+0b6+1b7+n13-p13=1650000

b0+1b1+2b2+121b4+1b5+9.91b6+1b7+n14-p14=1850000

b0+1b1+3b2+343.32b4+3b5+56b6+2b7+n15-p15=5000000

b0+1b1+2b2+121b4+1b5+9.91b6+1b7+n16-p16=1950000

b0+1b1+4b2+94b4+1b5+0b6+1b7+n17-p17=1700000

b0+1b1+1b2+175.68b4+2b5+27.7b6+2b7+n18-p18=2550000 b0+1b1+2b2+176.75b4+2b5+32b6+2b7+n19-

p19=2950000

b0+0b1+4b2+140.95b4+2b5+16b6+2b7+n20-

p20=2500000

b0+1b1+4b2+125.5b4+2b5+9.51b6+2b7+n21-

p21=2460000

b0+1b1+4b2+125.5b4+2b5+9.51b6+2b7+n22-

p22=2450000

b0+1b1+2b2+150.54b4+1b5+33.25b6+1b7+n23-

p23=2200000

b0+1b1+4b2+249.69b4+3b5+53.4b6+2b7+n24-

p24=4350000

b0+1b1+2b2+176.19b4+2b5+27.7b6+2b7+n25-

p25=2630000

b0+0b1+4b2+137.4b4+2b5+18b6+2b7+n26-

p26=2200000

b0+1b1+2b2+136b4+1b5+38.52b6+1b7+n27-

p27=2060000

b0+1b1+3b2+218b4+3b5+18.11b6+2b7+n28-

p28=3600000

b0+1b1+4b2+180.2b4+2b5+10.28b6+2b7+n29-

p29=3225000

b0+1b1+1b2+104.71b4+1b5+0b6+1b7+n30-

p30=1600000

b0+1b1+4b2+170.47b4+2b5+18.11b6+2b7+n31-

p31=2700000

b0+1b1+4b2+312.78b4+3b5+48b6+2b7+n32-

p32=4850000

b0+1b1+4b2+312.78b4+3b5+48b6+2b7+n33-

p33=4850000

b0+1b1+4b2+249.69b4+3b5+53.4b6+2b7+n34-

p34=4850000

b0+1b1+4b2+249.69b4+2b5+27.7b6+2b7+n35-

p35=2800000

b0+0b1+4b2+135.06b4+2b5+16b6+2b7+n36-

p36=2000000

b0+1b1+4b2+125.81b4+2b5+10b6+2b7+n37-

p37=2750000

b0+0b1+4b2+144.51b4+2b5+16b6+2b7+n38-

p38=2500000

b0+0b1+4b2+141.26b4+2b5+16b6+2b7+n39-

p39=2500000

b0+0b1+4b2+134.78b4+2b5+16b6+2b7+n40-

p40=2500000

end

free b0

free b1

free b2

free b4

free b5

free b6

free b7

Appendix D. Lindo File Result for Apartments

LP OPTIMUM FOUND AT STEP 59

OBJECTIVE FUNCTION VALUE

1) 5338364.

VARIABLE	VALUE	REDUCED COST
N1	0.000000	2.000000
N2	0.000000	2.000000
N3	0.000000	1.871056
N4	0.000000	2.000000
N5	124422.257812	0.000000
N6	101719.132812	0.000000
N7	0.000000	2.000000
N8	110231.015625	0.000000
N9	0.000000	2.000000
N10	0.000000	1.118930
N11	0.000000	2.000000
N12	15124.439453	0.000000
N13	0.000000	1.242868
N14	22547.933594	0.000000
N15	0.000000	1.128944
N16	122547.937500	0.000000
N17	0.000000	2.000000
N18	0.000000	2.000000
N19	138722.890625	0.000000
N20	2316.355469	0.000000
N21	10000.000000	0.000000
N22	0.000000	0.881070
N23	47123.265625	0.000000
N24	0.000000	0.000000
N25	0.000000	2.000000
N26	0.000000	2.000000
N27	0.000000	2.000000
N28	0.000000	2.000000
N29	362821.281250	0.000000
N30	0.000000	1.757132
N31	0.000000	2.000000
N32	52807.597656	0.000000

N33	52807.597656	0.000000
N34	500000.000000	0.000000
N35	0.000000	2.000000
N36	0.000000	2.000000
N37	295485.593750	0.000000
N38	0.000000	2.000000
N39	0.000000	0.000000
N40	48419.300781	0.000000
P1	280129.218750	0.000000
P2	293627.937500	0.000000
P3	0.000000	0.128944
P4	280129.218750	0.000000
P5	0.000000	2.000000
P6	0.000000	2.000000
P7	109770.812500	0.000000
P8	0.000000	2.000000
P9	50811.890625	0.000000
P10	0.000000	0.881070
P11	138729.187500	0.000000
P12	0.000000	2.000000
P13	0.000000	0.757132
P14	0.000000	2.000000
P15	0.000000	0.871056
P16	0.000000	2.000000
P17	3805.000244	0.000000
P18	172715.921875	0.000000
P19	0.000000	2.000000
P20	0.000000	2.000000
P21	0.000000	2.000000
P22	0.000000	1.118930
P23	0.000000	2.000000
P24	0.000000	2.000000
P25	157803.812500	0.000000
P26	280129.218750	0.000000
P27	7872.324707	0.000000
P28	293627.937500	0.000000
P29	0.000000	2.000000
P30	0.000000	0.242868
P31	124598.796875	0.000000
P32	0.000000	2.000000
P33	0.000000	2.000000
P34	0.000000	2.000000

P35	659558.437500	0.000000
P36	453672.906250	0.000000
P37	0.000000	2.000000
P38	24284.371094	0.000000
P39	0.000000	2.000000
P40	0.000000	2.000000
B0	191280.984375	0.000000
B1	96873.320312	0.000000
B2	61277.113281	0.000000
B4	7472.114258	0.000000
B5	775156.687500	0.000000
B6	4485.793457	0.000000
B7	-306993.156250	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	1.000000
4)	0.000000	0.871056
5)	0.000000	1.000000
6)	0.000000	-1.000000
7)	0.000000	-1.000000
8)	0.000000	1.000000
9)	0.000000	-1.000000
10)	0.000000	1.000000
11)	0.000000	0.118930
12)	0.000000	1.000000
13)	0.000000	-1.000000
14)	0.000000	0.242868
15)	0.000000	-1.000000
16)	0.000000	0.128944
17)	0.000000	-1.000000
18)	0.000000	1.000000
19)	0.000000	1.000000
20)	0.000000	-1.000000
21)	0.000000	-1.000000
22)	0.000000	-1.000000
23)	0.000000	-0.118930
24)	0.000000	-1.000000
25)	0.000000	-1.000000
26)	0.000000	1.000000
27)	0.000000	1.000000
28)	0.000000	1.000000
29)	0.000000	1.000000

30)	0.000000	-1.000000
31)	0.000000	0.757132
32)	0.000000	1.000000
33)	0.000000	-1.000000
34)	0.000000	-1.000000
35)	0.000000	-1.000000
36)	0.000000	1.000000
37)	0.000000	1.000000
38)	0.000000	-1.000000
39)	0.000000	1.000000
40)	0.000000	-1.000000
41)	0.000000	-1.000000

NO. ITERATIONS= 59

Appendix E. Lindo Code for Townhouses

Lindo Code – Townhouses

B1= Location ‘0’ for Viva Bahria and ‘1’ for Porto Arabia

B2= Stands for View ‘4’ is the best view and ‘1’ is the least favorable view.

B3= Type in this case there is only one type that is Townhouses represented as ‘0’.

B4= No. of bedrooms.

B5= Area in meter square.

B6= Balcony space in meter square.

B7= No of parking’s.

min n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12+p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12

st

b0+1b1+4b2+146b4+2b5+10b6+2b7+n1-p1=2800000

b0+0b1+4b2+114.49b4+1b5+17b6+1b7+n2-p2=2550000

b0+1b1+4b2+474b4+3b5+97b6+2b7+n3-p3=9000000

b0+1b1+2b2+146b4+2b5+10b6+2b7+n4-p4=3250000

b0+1b1+4b2+146b4+2b5+10b6+2b7+n5-p5=2400000

b0+0b1+4b2+114.49b4+1b5+17b6+1b7+n6-p6=2575000

b0+1b1+1b2+146b4+2b5+10b6+2b7+n7-p7=2350000 b0+1b1+4b2+116.5b4+1b5+9.5b6+1b7+n8-p8=2100000

b0+1b1+4b2+146b4+2b5+0b6+2b7+n9-p9=2800000

b0+1b1+4b2+146b4+2b5+10b6+2b7+n10-p10=3150000

b0+0b1+4b2+117.47b4+1b5+8b6+1b7+n11-p11=2400000

b0+1b1+4b2+116.5b4+1b5+9.5b6+2b7+n12-p12=2400000

end

free b0

free b1

free b2

free b4

free b5

free b6

free b7

Appendix F. Lindo File Result for Town Houses

LP OPTIMUM FOUND AT STEP 13

OBJECTIVE FUNCTION VALUE

1) 1662731.

VARIABLE	VALUE	REDUCED COST
N1	0.000000	2.000000
N2	0.000000	1.337521
N3	0.000000	0.993386
N4	681134.375000	0.000000
N5	0.000000	2.000000
N6	25000.000000	0.000000
N7	0.000000	1.666667
N8	0.000000	1.000000
N9	0.000000	0.346561
N10	143403.171875	0.000000
N11	0.000000	1.662479
N12	0.000000	0.993386

P1	206596.828125	0.000000
P2	0.000000	0.662479
P3	0.000000	1.006614
P4	0.000000	2.000000
P5	606596.812500	0.000000
P6	0.000000	2.000000
P7	0.000000	0.333333
P8	0.000000	1.000000
P9	0.000000	1.653439
P10	0.000000	2.000000
P11	0.000000	0.337521
P12	0.000000	1.006614
B0	-597876.125000	0.000000
B1	-319291.875000	0.000000
B2	218865.609375	0.000000
B4	12059.443359	0.000000
B5	240513.406250	0.000000
B6	20659.681641	0.000000
B7	300000.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000

3)	0.000000	0.337521
4)	0.000000	-0.006614
5)	0.000000	-1.000000
6)	0.000000	1.000000
7)	0.000000	-1.000000
8)	0.000000	0.666667
9)	0.000000	0.000000
10)	0.000000	-0.653439
11)	0.000000	-1.000000
12)	0.000000	0.662479
13)	0.000000	-0.006614

NO. ITERATIONS= 1