On the DoF of X-Networks With Synergistic Alternating CSIT: A Step Towards Integrated Communication and Sensing

Sur le DoF des réseaux X avec CSIT alternatif synergique : une étape vers la communication et la détection intégrées

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***Abstract*— The coexistence of communication and sensing services in the next wireless communication systems, i.e., beyond 5G and 6G systems, revive the central role of interference management techniques such as interference alignment, coordinated multipoint transmission, and cell-free massive multiple-input–multiple-output (MIMO), in defeating interference and achieving the network capacity. In this article, we consider the K-user single-input–single-output (SISO) X-channel and its variants ($2 \times K$ and $K \times 2$) in fast-fading environments. This can theoretically model many practical use cases for beyond 5G and 6G networks. For instance, it can model the case of having K cars communicating with another K cars, while former cars are sensing environment using the latter ones (in a cooperative, bistatic, and active approach) over the same time and frequency resources. We assume that the transmitters have access to synergistic alternating channel state information at the transmitter (CSIT) where it alternates between three states: perfect (P), delayed (D), and no-CSIT (N), and these states are associated with fractions of time denoted by λ_P, λ_D, and λ_N, respectively. We develop novel degree-of-freedom (DoF) achievability schemes that exploit the synergy of the instantaneous CSIT and the delayed CSIT to retrospectively align interference in the subsequent channel uses. In particular, we show that the sum DoF of the K-user SISO X-channel is at least $2K/K+1$, using a two-phase transmission scheme over finite symbols channel extension and under a certain distribution of the CSIT availability of $\Lambda(\lambda_P = (1/3), \lambda_D = (1/3), \lambda_N = (1/3))$. This achievability result can be considered as a tight lower bound where it coincides with the best lower bound known for the same network but with partial output feedback instead of alternating CSIT. In addition, it shows that the role of synergistically alternating CSIT with distribution $\Lambda(1/3, 1/3, 1/3)$ is equivalent to the one of the partial output feedback. Moreover, we show the optimality of the proposed two-phase-based scheme using a simple combinatorial proof. This establishes a DoF lower bound, which is strictly better than the best lower bound known for the case of delayed CSI for all values of K. Thus, the proposed schemes offer higher DoF gain in comparison to delayed CSIT and no-CSIT.**

***Résumé*— La coexistence des services de communication et de détection dans les prochains systèmes de communication sans fil, c'est-à-dire au-delà des systèmes 5G et 6G, ravive le rôle central des techniques de gestion des interférences, telles que l'alignement des interférences, la transmission multipoint coordonnée et les entrées-multiples sorties-multiples massives sans cellule (MIMO), pour vaincre les interférences et atteindre la capacité du réseau. Dans cet article, nous considérons le canal X à K utilisateurs, à entrée unique et à sortie unique (SISO) et ses variantes ($2 \times K$ et $K \times 2$) dans des environnements à évanouissement rapide. Cela peut théoriquement modéliser de nombreux cas d'utilisation pratiques pour les réseaux au-delà de la 5G et de la 6G. Par exemple, on peut modéliser le cas où K voitures communiquent avec K autres voitures, alors que les premières voitures détectent l'environnement en utilisant les secondes (dans une approche coopérative, bi-statique et active) sur les mêmes ressources de temps et de fréquence. Nous supposons que les émetteurs ont accès à des informations synergiques sur l'état alternatif du canal au niveau de l'émetteur (CSIT) où il alterne**

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entre trois états : parfait (P), retardé (D) et sans CSIT (N), et ces états sont associés à des fractions de temps désignées par λ_P, λ_D et λ_N, respectivement. Nous développons de nouveaux schémas de réalisabilité par degré de liberté (DoF) qui exploitent la synergie du CSIT instantané et du CSIT retardé pour aligner rétrospectivement les interférences dans les utilisations ultérieures du canal. En particulier, nous montrons que la somme des DoF du canal SISO X à K utilisateurs est au moins $2K/K+1$, en utilisant un schéma de transmission à deux phases sur une extension de canal à symboles finis et sous une certaine distribution de la disponibilité du CSIT de ($\lambda_P = (1/3), \lambda_D = (1/3), \lambda_N = (1/3)$). Ce résultat de réalisabilité peut être considéré comme une limite inférieure serrée où il coïncide avec la meilleure limite inférieure connue pour le même réseau, mais avec une rétroaction de sortie partielle au lieu d'un CSIT alterné. En outre, il montre que le rôle de l'alternance synergique du CSIT avec la distribution (1/3, 1/3, 1/3) est équivalent à celui de la rétroaction de sortie partielle. De plus, nous montrons l'optimalité du schéma proposé basé sur deux phases en utilisant une preuve combinatoire simple. Cela établit une limite inférieure de DoF, qui est strictement meilleure que la meilleure limite inférieure connue pour le cas du CSI retardé pour toutes les valeurs de K. Ainsi, les schémas proposés offrent un gain de DoF plus élevé par rapport au CSIT retardé et au sans CSIT.

Index Terms— **6G communications, degrees of freedom (DoF), joint communications and sensing, multiple-input–multiple-output (MIMO) communication, precoding.**

I. INTRODUCTION

THE scarcity of the wireless spectrum, the increasing growth of high data rate demands, and the integration of sensing and communication functionalities arise the impossibility of separating the concurrent transmission in frequency and space, and imposing sharing the hardware aiming mainly at achieving the network capacity and reducing implementation and running costs of the next communication systems while creating more signal interference in wireless networks [1], [2]. Consequently, it is widely known that signal interference is the main performance-limiting factor of most wireless networks. Moreover, as the number of users in a wireless network sharing the same spectrum increases, the network becomes interference limited [2]. Therefore, establishing the performance limits of wireless networks turns out to be more challenging.

The interference alignment [3] arises the possibility of establishing the performance limits of wireless networks in terms of characterizing the sum degree of freedom (DoF) of many wireless networks. For example, in [4], it was shown that $M \times N$ X-network can achieve $MN/M+N-1$ DoF (e.g., asymptotically having MN independent messages over $M+N-1$ channel uses), the DoF upper bound of that network using simple interference alignment scheme over infinite symbols channel extension. The K-user X-network is the most comprehensive and fundamental setting for the information-theoretic study of interference alignment in multiuser wireless networks. Interestingly, this setting can be transformed into single-user MIMO, broadcast multiaccess, and Z channels using minor modifications. For instance, if one allows full cooperation between the transmitters of the K-user X-network, then the resulting setup is a K-user multiple-input–single-output (MISO) broadcast channel (BC).

A. Related Work

Considerable work in the literature on interference alignment has focused on characterizing the DoFs of X-channel and X-network. Contrary to what has been established in the context of the memory-less point-to-point channel that the channel feedback does not increase the capacity [5], the channel feedback, known as CSIT, in multiuser networks can significantly widen the capacity region and, hence, the DoF region. Throughout the literature, the CSIT plays a leading role in characterizing the DoF of wireless networks and was the canonical motif and the influential ingredient in developing the phenomenal interference alignment techniques. Under full CSIT assumption; where the transmitters have global and instantaneously perfect CSIT, the wireless networks achieve the highest DoF and enjoy the widest DoF region. In [4], it is proven that the DoF of $M \times N$-user SISO X-network with full CSIT is upper bounded by $MN/M+N-1$ also the authors proposed a partial interference alignment scheme that asymptotically approaches the upper on DoF within an $\epsilon > 0$ by considering large channel extensions. In certain cases, when the number of transmitters or receivers is equal to two, the upper bound is achievable, and perfect interference alignment is attained within finite channel extension. On the other hand, in the total lack of CSIT, the DoF region of most wireless networks collapses to the narrowest region, where its corner points are achievable simply by time or frequency-division multiplexing between users [6], [7]; however, in certain scenarios, the interference alignment is still feasible. Specifically, Jafar [8] paved the way to achieve interference alignment by exploiting only the knowledge of heterogeneous channel coherence structures associated with different users in the same network even in complete lack of knowledge of the channel at the transmitters, i.e., the X-channel without no CSIT and under the heterogeneous block fading in both time and frequency assumption; one user suffers time selectivity and other is frequency-selective, which achieves 4/3 DoF and hence coincides with the best-known DoF upper bound on it.

Extensive research efforts have been devoted to proposing middle grounds between the two extremes: full CSIT and no CSIT, such as quantized CSIT [9], [10], compound CSIT [11]–[13], and others that make use of temporal correlation, yet the most remarkable one is what is widely known as delayed CSIT. This model was first introduced by Maddah-Ali and Tse in [14] for the Gaussian MISO BC. The delayed model introduced a fundamental and rather counterintuitive observation that the completely outdated channel knowledge to the transmitters in the independent and identically distributed (i.i.d.) Rayleigh fading model, where the channels take completely independent values every time slot, creates great opportunities for interference alignment, and significantly improves the DoF of MISO BC. Maleki *et al.* [15] applied the delayed CSIT model to the distributed transmitters networks such as X-channel and interference channel. They showed that the two-user SISO X-channel and three-user SISO interference channel, under delayed CSIT assumption, can achieve $8/7$ and $9/8$ DoF, respectively. Then, Ghasemi *et al.* [16] introduced a new transmission strategy specially tailored to the distributed transmitters networks that efficiently exploited the delayed channel knowledge to provide new achievability results that outperform what has been obtained in [15]. In particular, they showed that $6/5$ and $5/4$ are achievable for the two- and three-user X-channel, respectively. In this article, we consider a two-user Gaussian X-channel where each node is equipped with a single antenna. In this channel, transmitters T_1 and T_2 have four independent messages W_{11}, W_{12}, W_{21}, and W_{22} for receivers R_1 and R_2 such that W_{ij} originates at transmitter j and is intended for receiver i. Earlier research work on the DoF of the two-user X-channel has determined that the upper bound for DoF of two-user SISO X-channel is $4/3$ and for MIMO one is $4M/3$, where M is the number of antennas per node [17]. These upper bounds are achievable with global, perfect, and instantaneous CSIT when the channel coefficients are time-varying or frequency-selective and drawn from the continuous distribution. Maleki *et al.* [15] showed that even in a fast-fading environment and for interference networks consisting of distributed transmitters and receivers, delayed CSIT channels could be beneficial and have a great impact on increasing DoF. They proved that for the two-user SISO X-channel, the $8/7$ DoF is achievable with delayed CSIT. New results have been demonstrated in [16] where the two-user SISO X-channel with delayed CSIT could achieve $6/5$ DoF and the three-user X-network could achieve $5/4$ DoF.

Recently, interference alignment has attracted a large interest. Wang and Varanasi [18] established the DoF regions of the two-user MIMO BC with a general message set that includes private and common messages. As an extension to the two-user case, Lashgari *et al.* [19] characterized the impact of the alternating CSIT on the capacity of BCs with K single-antenna receivers. They showed that the state-of-the-art achievable schemes in the literature are indeed sum-DoF optimal when restricted to linear encoding schemes. Moreover, Bazco-Nogueras *et al.* [20] studied the impact of imperfect sharing of CSIT on a network MIMO setting in which a set of M transmit antennas, possibly not co-located, jointly serve two multiantenna users endowed with N_1 and N_2 antennas. Mainly, they answered the question of how many extra cooperative antennas can help. Zhang and Wang [21] characterized the achievable DoF regions of the three-user MIMO BC with delayed CSIT. More recently, Ghasemi *et al.* [16], using real interference alignment techniques, characterized the Dof of the K-user MIMO Gaussian interference channel with M antennas at each transmitter and N antennas at each receiver.

B. Contributions

In this work, we show the possibility of theoretically modeling some integrated sensing and communication systems as K-user X-network, which facilitates the way of abstracting the features and performance and characterizing the theoretical limits of these systems. Moreover, we show the optimality of our transmission scheme by providing a combinatorial proof that any two-phase-based scheme cannot achieve more than $2K/K+1$. In addition, we show that any transmission scheme based on more than two multiphases is not beneficial in achieving more DoFs. We highlight the cooperation aspects for the X-network by showing that there is no extra DoF gain compared to the proposed alternating CSIT setting. In particular, we give an illustrative example for an improved X-network by letting one transmitter have a cognition/cooperation capability of the other transmitters; we call this node/transmitter a supernode as in [22] and show that given certain alternating CSIT pattern/distribution, this network cannot achieve more than the achievable sum DoF of the proposed schemes. Finally, we obtain the relation between the sum DoF and the CSIT distributions $\Lambda(\lambda_P, \lambda_D, \lambda_N)$ by providing closed-form expressions for the achievable sum DoF as a function of CSIT distributions.

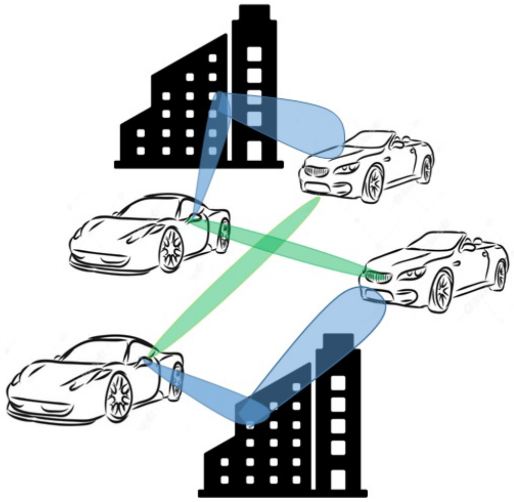


Fig. 1. Example of a 2 × 2 integrated communication system on the form of two-user X-channel/network.

C. Organization

This article is organized as follows. In Section II, the system model of the K-user X-network is presented. The DoF achievability schemes are proposed in Section III. In Section IV, we show some optimality aspects of the proposed schemes. Section V discusses the DoF of the K-user SISO X-network followed by the $2 \times K$-user SISO X-network in Section VI. Finally, we provide a comprehensive discussion and comparison with the prior art and set a conjuncture on DoF scaling in Section VII before concluding this article in Section VIII.

II. SYSTEM MODEL

Next wireless network systems are steadily showing interest in employing dual functions, i.e., communication and sensing, and nodes/terminals in their architectures [2]. In these networks, nodes/terminals are simultaneously communicating with each other and sensing the environment over the same time and frequency resources and in some cases using the same hardware. While this dual-functionality mode makes way for new use cases and applications, it also adds new challenges to be handled in order to reap the gains of the dual functionality. The chef among these challenges is how to manage the interference between communication and sensing deployed on the same resources. Our work uses a K-user X-network

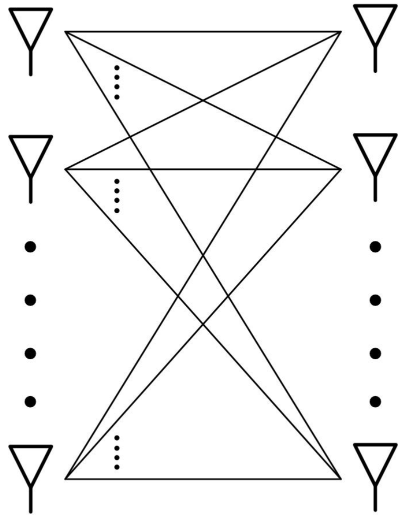


Fig. 2. K-user SISO X-network.

to represent an integrated communication and sensing system with $K \times K$ terminals. For instance, a 2×2 system is shown in Fig. 1, where two cars are communicating the status of the traffic to another two cars while sensing the environment in a bistatic mode. We note that the proposed framework is not limited only to positioning applications. For instance, a 2×2 X-network can model the case of having three cellular phones and base stations where two phones are communicating (the first is sending to the second) using side links, while the third phone is communicating with the base station using uplink. In addition, the first and third phones are sensing the environment in a bistatic mode using the second phone and the base station, respectively, as their remote sensing receivers.

In the K-user X-network as shown in Fig. 2, there are K independent transmitters $\{T_i\}_{i=1}^{K}$ communicating with and/or sensing (bistatic) K independent receivers $\{R_i\}_{i=1}^{K}$, where each node is equipped with a single antenna. This is the simplest case and it can be directly extended to the MIMO where each terminal is equipped with multiple antennas (where beamforming can be considered at each terminal). Each transmitter has an independent message (might be a sensing/communication message) for each receiver. The received signal at the ith receiver at time slot t is

$$Y_i(t) = \sum_{j=1}^{K} h_{ij}(t) X_j(t) + N_i(t) \tag{1}$$

where $X_j(t)$ is the transmitted signal from T_j at the tth time slot, which satisfies the power constraint $E\{|X_j(t)|^2\} \leq P_j$. The noise $N_i(t) \sim \mathcal{CN}(0,1)$ is the circularly symmetric complex additive white Gaussian noise with zero mean and unit variance generated at R_i at time slot t. In (1), $h_{ij}(t)$ is the channel coefficient from T_j to R_i and all channel coefficients are i.i.d. over time and drawn from a continuous distribution. We assume that the receivers know all the channel coefficients instantaneously and with infinite precision, and thus, global and perfect CSI is assumed at the receivers. In contrast, we consider three different states of the availability of CSIT: perfect (P), delayed (D), and no-CSIT (N). These states denote the availability of CSIT instantaneously and without error, with some delay $\geq$ one time slot and without error, and the unavailability of CSIT at all. It is worth mentioning that there are no differences between the CSIT required for sensing and communication functionalities since we assume that both communication and sensing functions are utilizing the same frequency and time and are being multiplexing in the space dimension and our ultimate goal is to design the spatial filter (precoder/beamformer) such that both functions coexist smoothly with no interference.

Let the state of CSIT availability of the channels to the ith receiver be denoted by S_i, where $S_i \in \{P, D, N\}$, i.e., $S_2 = P$ indicates that each transmitter j, where $j \in \{1, 2, \ldots, K\}$, has perfect and instantaneous knowledge of h_{2j}. In addition, let $S_{1,\ldots,K}$ denote the state of CSIT availability for the channels to the network, the first, second, …, Kth receiver. Therefore, $S_{1,\ldots,K} \in \{PP\cdots P, PP\cdots D, \ldots, NN\cdots N\}$. For example, $S_{123} = PDN$ refers to the case where T_j has perfect knowledge of h_{1j}, delayed knowledge of h_{2j}, and no information about h_{3j}. Moreover, we denote the CSIT availability of the channels to receiver i over n time slots of time channel extension by n-tuple $S_i^n = (S_i(1), \ldots, S_i(n))$. Similarly, the availability of CSIT for the channels to the network over n time slots channel extension, known by "CSIT pattern,"is denoted by $S_{1,\ldots,K}^n = (S_{1\cdots K}(1), \ldots, S_{1,\ldots,K}(n))$. The fraction of time associated with the state of CSIT availability for the network, denoted by λ_S where $S \in \{P, D, N\}$ is

$$\lambda_S = \frac{\sum_{t=1}^{n} \sum_{i=1}^{K} \mathbb{I}_S(S_i(t))}{nK} \tag{2}$$

where $\mathbb{I}$ denotes the indicator function and k is the number of users. Hence,

$$\sum_{S=P,D,N} \lambda_S = 1. \tag{3}$$

Furthermore, we use $\Lambda(\lambda_P, \lambda_D, \lambda_N)$ to denote the distribution of the fraction of time for the different states $\{P, D, N\}$ of CSIT availability.

Let $r_{ij}(P) = (\log_2(|W_{ij}|)/n)$ denote the rate of W_{ij} for a given transmission power P, where $|W_{ij}|$ denotes the size of the message set and n is the number of channel uses. The rate $r_{ij}(P)$ is achievable if there exists a coding scheme such that the probability of error in decoding W_{ij} goes to zero as n goes to infinity for all (i, j). The DoF region $\mathcal{D}(\Lambda)$ is defined as the set of all achievable tuples $(d_{11}, \ldots, d_{1K}, d_{21}, \ldots, d_{2K}, \ldots, d_{K1}, \ldots, d_{KK}) \in \mathbb{R}_{+}^{K^2}$, where $d_{ij} = \lim_{P\to\infty}(R_{ij}(P)/\log_2(P))$ is the DoF for message W_{ij}. The DoF of the network is defined as

$$\mathrm{DoF}(\Lambda) = \max_{(d_{11},\ldots,d_{22})\in\mathcal{D}(\Lambda)} \sum_{j=1,i=1}^{K} d_{ij}. \tag{4}$$

III. DoF Achievability Schemes

A. Achievability Scheme

In this section, we propose transmission schemes for the K-user SISO X-network. Similar to our work in [23], the transmission schemes involve two phases, namely, interference creation and interference resurrection. Utilizing the idea of interference creation and resurrection, here, we show that the K-user SISO X-channel can achieve at least $2K/K+1$ DoF.

Before we proceed to the K-user case, as an illustrative example, we show that the three-user SISO X-channel with alternating CSIT of $\Lambda(1/3, 1/3, 1/3)$ can achieve $3/2$ DoF. Let u_1, u_2, and u_3 be three independent data symbol intended to R_1 transmitted from T_1, T_2, and T_3, respectively. Also, let v_1, v_2, and v_3 be three independent data symbols intended to R_2 transmitted from T_1, T_2, and T_3, respectively. Similarly, let p_1, p_2, and p_3 be three independent data symbols intended to R_3 transmitted from T_1, T_2, and T_3, respectively. In the following paragraphs, we show that we can reliably transmit the three symbols (u_1, u_2, u_3) to receiver 1, (v_1, v_2, v_3) to receiver 2 and, finally, (p_1, p_2, p_3) to receiver 3 in six time slots.

Let us consider the alternating CSIT pattern given by $S_{123}^6 = (NDD, DND, DDN, PPN, PNP, NPP)$. Here, the delayed CSIT is distributed over three time slots. Consequently, the interference creation phase consumes three time slots, while the interference resurrection phase is executed over the other three time slots. The proposed scheme is performed in two separate phases as follows.

Phase One: For interference creation, each time slot of this phase is dedicated to each receiver where the transmitters transmit three different linear combinations of the desired messages, one term to each receiver. Since $S_{123}(1) = NDD$, the first time slot is designed such that interference is created for R_2 and R_3; hence, T_1 transmits u_1 and T_2 transmits u_2 and T_3. The received signals at R_1, R_2, and R_3 are

$$Y_1(1) = h_{11}(1)u_1 + h_{12}(1)u_2 + h_{13}(1)u_3 \equiv L_1^1(u_1, u_2, u_3) \tag{5}$$

$$Y_2(1) = h_{21}(1)u_1 + h_{22}(1)u_2 + h_{23}(1)u_3 \equiv I_2^1(u_1, u_2, u_3) \tag{6}$$

$$Y_3(1) = h_{31}(1)u_1 + h_{32}(1)u_2 + h_{33}(1)u_3 \equiv I_3^1(u_1, u_2, u_3). \tag{7}$$

Therefore, R_1 receives the linear combination $L_1^1(u_1, u_2, u_3)$ of its desired signals, while R_2 and R_3 receive only interference terms: $I_2^1(u_1, u_2, u_3)$ and $I_3^1(u_1, u_2, u_3)$. Similarly, in the next two time slots, T_1 transmits v_1, T_2 transmits v_2, and T_3 transmits v_3 in the second time slots, while T_1 transmits p_1, T_2 transmits p_2, and T_3 transmits p_3 in the third time slot. Then, the received signals at R_1, R_2, and R_3 are

$$Y_1(2) = h_{11}(2)v_1 + h_{12}(2)v_2 + h_{13}(2)v_3 \equiv I_1^1(v_1, v_2, v_3) \tag{8}$$

$$Y_2(2) = h_{21}(2)v_1 + h_{22}(2)v_2 + h_{23}(2)v_3 \equiv I_2^1(v_1, v_2, v_3) \tag{9}$$

$$Y_3(2) = h_{31}(2)v_1 + h_{32}(2)v_2 + h_{33}(3)v_3 \equiv I_3^1(v_1, v_2, v_3) \tag{10}$$

where R_2 receives the first linear combination $L_2^1(v_1, v_2, v_3)$ of its desired signals, while R_1 and R_3 receive the first interference terms: $I_2^1(v_1, v_2, v_3)$ and $I_2^1(v_1, v_2, v_3)$. In the third time slot

$$Y_1(3) = h_{11}(3)p_1 + h_{12}(3)p_2 + h_{13}(3)p_3 \equiv I_1^1(p_1, p_2, p_3) \tag{11}$$

$$Y_2(3) = h_{21}(3)p_1 + h_{22}(3)p_2 + h_{23}(3)p_3 \equiv I_2^1(p_1, p_2, p_3) \tag{12}$$

$$Y_3(3) = h_{31}(3)p_1 + h_{32}(3)p_2 + h_{33}(3)p_3 \equiv L_3^1(p_1, p_2, p_3) \tag{13}$$

where R_3 receives the first linear combination $L_3^1(p_1, p_2, p_3)$ of its desired signals, while R_1 and R_2 receive the first interference terms: $I_1^1(p_1, p_2, p_3)$ and $I_2^1(p_1, p_2, p_3)$. By the end of time slot 3, each receiver receives one linear combination term from its intended message, and as a by-product, the other two receivers receive two interference terms. Now, we have six interference terms available to the three receivers. In the interference resurrection phase, we will utilize these interference terms to provide the receivers with sufficient information to successfully decode their messages; specifically, each receiver needs another two linear combinations. Trivially, it requires six time slots to deliver six independent linear combinations. However, as we will show in the following that it will take only three time slots by using the interference resurrection, exploiting interference as common messages.

Phase Two: In the fourth time slot, interference resurrection phase begins, and the transmitters utilize the channel knowledge in PPN to reconstruct $I_2^1(u_1, u_2, u_3)$ at R_2 while reconstructing $I_1^1(v_1, v_2, v_3)$ at R_1. As a result, R_1 and R_2 receive their second linear combination terms $L_1^2(u_1, u_2, u_3)$ and $L_2^2(v_1, v_2, v_3)$, while R_3 receives pure interference. In particular, the transmitted signals are

$$X_1(4) = h_{21}^{-1}(4)h_{21}(1)u_1 + h_{11}^{-1}(4)h_{11}(2)v_1 \tag{14}$$

$$X_2(4) = h_{22}^{-1}(4)h_{22}(1)u_2 + h_{12}^{-1}(4)h_{12}(2)v_2 \tag{15}$$

$$X_3(4) = h_{23}^{-1}(4)h_{23}(1)u_3 + h_{13}^{-1}(4)h_{13}(2)v_3. \tag{16}$$

Note that the transmitted signals, in interference resurrection phase, are beamformed signals—not random linear combinations like in interference creation phase-dependent on both the current channel knowledge and the outdated channel knowledge formerly received at interference creation phase. For an instance, to construct $X_1(4)$, T_1 utilizes the instantaneous knowledge of $h_{11}(4)$ and the delayed knowledge of $h_{11}(2)$.

Therefore, the received signals at R_1, R_2, and R_2 are

$$Y_1(4) \equiv I_1^1(v_1, v_2, v_3) + L_1^2(u_1, u_2, u_3) \tag{17}$$

$$Y_2(4) \equiv I_2^1(u_1, u_2, u_3) + L_2^2(v_1, v_2, v_3) \tag{18}$$

$$Y_3(4) \equiv I_3^2(u_1, u_2, u_3) + I_3^2(v_1, v_2, v_3). \tag{19}$$

In the fifth time slot, interference resurrection phase for users 1 and 3 begins, the transmitters utilize the channel knowledge in PNP to reconstruct $I_3^1(u_1, u_2, u_3)$ at R_3 while reconstructing $I_1^1(p_1, p_2, p_3)$ at R_1. As a result, R_1 receives its third linear combination term $L_1^3(u_1, u_2, u_3)$ and R_3 receives its second interference term $L_3^2(p_1, p_2, p_3)$, while R_2 receives pure interference. In particular, the transmitted signals are

$$X_1(5) = h_{31}^{-1}(5)h_{31}(1)u_1 + h_{11}^{-1}(5)h_{11}(3)p_1 \tag{20}$$

$$X_2(5) = h_{32}^{-1}(5)h_{32}(1)u_2 + h_{12}^{-1}(5)h_{12}(3)p_2 \tag{21}$$

$$X_3(5) = h_{33}^{-1}(5)h_{33}(1)u_3 + h_{13}^{-1}(5)h_{13}(3)p_3. \tag{22}$$

As a result, the received signals at R_1, R_2, and R_2 are

$$Y_1(5) \equiv I_1^1(p_1, p_2, p_3) + L_1^3(u_1, u_2, u_3) \tag{23}$$

$$Y_2(5) \equiv I_2^2(u_1, u_2, u_3) + I_2^2(p_1, p_2, p_3) \tag{24}$$

$$Y_3(5) \equiv I_3^1(u_1, u_2, u_3) + L_3^2(p_1, p_2, p_3). \tag{25}$$

In the sixth time slot, interference resurrection phase for users 2 and 3 begins, and the transmitters utilize the channel knowledge in NPP to reconstruct $I_2^1(p_1, p_2, p_3)$ at R_2 while reconstructing $I_3^1(v_1, v_2, v_3)$ at R_3. As a result, R_2 and R_3 receive their second linear combination terms $L_2^2(v_1, v_2, v_3)$ and $L_3^2(u_1, u_2, u_3)$, while R_1 receives pure interference. In particular, the transmitted signals are

$$X_1(6) = h_{31}^{-1}(6)h_{31}(2)v_1 + h_{21}^{-1}(6)h_{21}(3)p_1 \tag{26}$$

$$X_2(6) = h_{32}^{-1}(6)h_{32}(2)v_2 + h_{22}^{-1}(6)h_{22}(3)p_2 \tag{27}$$

$$X_3(6) = h_{33}^{-1}(6)h_{33}(2)v_3 + h_{23}^{-1}(6)h_{23}(3)p_3. \tag{28}$$

As a result, the received signals at R_1, R_2, and R_2 are given, respectively, by

$$Y_1(6) \equiv L_1^2(u_1, u_2, u_3) + I_1^1(v_1, v_2, v_3) \tag{29}$$

$$Y_2(6) \equiv I_2^1(u_1, u_2, u_3) + L_2^2(v_1, v_2, v_3) \tag{30}$$

$$Y_3(6) \equiv I_3^2(u_1, u_2, u_3) + I_3^2(v_1, v_2, v_3). \tag{31}$$

Theorem 1: The DoF of the K-user SISO X-channel with synergistic alternating CSIT under any distribution $\in \Lambda(\lambda_P \geq (1/3), \lambda_D \geq (2/3) - \lambda_P)$ is lower bounded as follows:

$$\mathrm{DoF}^X_{K\times K}\left(\lambda_P \geq \frac{1}{3}, \lambda_D \geq \frac{2}{3} - \lambda_P\right) \geq \frac{K^2}{K + \binom{K}{2}} = \frac{2K}{K+1}. \tag{32}$$

Proof: The transmission scheme starts with the transmission of information symbols in phase one, the interference creation phase, in a certain way that guarantees to create reconstructable interference terms while providing receivers with linear combinations of their intended data symbols. This phase consumes K time slots to deliver K different linear combinations of the data symbols to K different receivers while creating $K \times (K-1)$ reconstructable interference terms. In contrast, phase two, the interference creation phase, this phase consumes $\binom{K}{2}$ time slots to deliver $K \times (K-1)$ new linear combinations of the data symbols to the indented receivers in order to successfully decode K^2 data symbols.

Phase One—"Interference Creation": This phase is associated with the delayed CSIT and might have one to K subphases where each subphase consumes one time slot. The number of subphases depends on whether the delayed CSIT of the channels to the two receivers occurs simultaneously or not. In the first case where the delayed CSIT occurs in the same time slot, i.e., $S_{1\cdots K} = D\cdots D$, phase one has only one subphase in which all data symbols are greedily transmitted, and thus, interference creation happens. Consequently, each receiver has one equation consisting of K terms, and the first term is a linear combination from the desired symbols, while others are the interference term. On the other hand, when the delayed CSIT does not occur simultaneously, i.e., $S_{1\cdots K} \in \{D\cdots N, N\cdots D, D\cdots P, P\cdots D\}$, phase one includes a number of subphases greater than one. Each subphase is dedicated to transmitting the data symbols of one receiver. Consequently, each receiver has K different equations over K time slots: one of them is a linear combination of the desired symbols without interference and the others are interference terms only.

Phase Two—"Interference Resurrection": This phase is associated with perfect CSIT. Similar to phase one, this phase might have one or two subphases depending on whether the perfect the CSIT occurs simultaneously or not. In this phase, the transmitters reconstruct the old interference by exploiting the delayed CSIT received in phase one. When the two transmitters have perfect CSIT simultaneously, phase two has only one subphase in which the two transmitters reconstruct the old interference received in phase one. Then, the transmitters transmit two independent messages exploiting the combined perfect CSIT. On the other hand, when the perfect CSIT is distributed over two time slots, phase two consists of two subphases where each subphase is dedicated to resurrecting the interference for one receiver. Unlike combined perfect CSIT, transmitters consume two time slots to totally reconstruct the old interference and provide a new linear combination of the desired symbols to the receivers. □

Remark 1: We note that this lower bound is tight for $K = 2$, for which the two-user X-channel with alternating CSIT pattern of $\Lambda(1/3, 1/3, 1/3)$ achieves the upper bound of $4/3$ on the DoF of the two-user X-channel with perfect instantaneous CSIT, i.e., $\Lambda(1, 0, 0)$. Although the lower bound stated in Theorem 1 does not asymptotically scale with K, it is strictly better than the best known lower bound for the X-network with only delayed CSIT, i.e., $\Lambda(0, 1, 0)$, where $\mathrm{DoF} \geq (4/3) - (3/2(2k-1))$ for all values of K [16].

IV. OPTIMALITY OF TWO-PHASE-BASED SCHEME

In this section, we show that any two-phase-based scheme cannot achieve more than $2K/K+1$ DoF. In addition, we discuss the cooperation between transmitters in the X-networks with alternating CSIT setting. Motivated by the combinatorial proof of [14], we derive our analysis and results. Generally, phase j takes symbols of order j and generates symbols of order $j+1$. The jth phase takes $(K-j+1)\binom{K}{j}$ common symbols (messages) of order j and yields $\binom{K}{j+1}$ of order $j+1$. This phase consumes $\binom{K}{j}$ time slots, with each time slot dedicated to a subset of receivers S, where $|S| = j$. Then, the DoF of order-j is as follows:

$$\mathrm{DoF}_j(K) = \frac{(K-j+1)\binom{K}{j}}{\binom{K}{j} + \frac{j\binom{K}{j+1}}{\mathrm{DoF}_{j+1}(K)}}. \tag{33}$$

1) For the two-phase-based scheme, we can write $D_\Sigma(K)$ as follows:

$$\mathrm{DoF}_1(K) = D_\Sigma(K) = \frac{K^2}{K + \binom{K}{2}} = \frac{2K}{K+1}. \tag{34}$$

2) For the case of three-phase-based scheme

$$\mathrm{DoF}_2(K) = \frac{(K-1)\binom{K}{2}}{\binom{K}{2} + \frac{2\binom{K}{3}}{\mathrm{DoF}_3(K)}} = \frac{3K-3}{2K-1}(\geq 1), \forall K > 1 \tag{35}$$

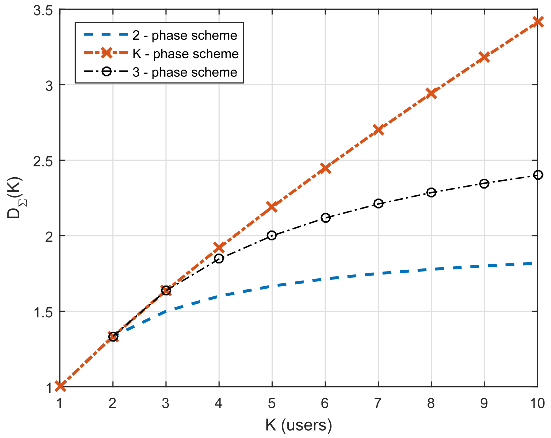


Fig. 3. Comparison between the possible DoF gain of multiphase ICR schemes.

for $\text{DoF}_3(K) = 1$, $D_\Sigma(K)$ can be written as follows:

$$\text{DoF}_1(K) = D_\Sigma(K) = \frac{6K}{2\ K+5}. \tag{36}$$

Fig. 3 shows the DoF gain attained as the number of phases in the transmission scheme becomes larger.

A. Proof of $\text{DoF}_2(K) < 1$

In this section, we show that $\text{DoF}_2(K) = 1$ for the K-user X-network. It is worth noting that in order to employ a K phases scheme, $\text{DoF}_{K-1}(K) > 1$ is required for the scheme to be beneficial in terms of DoF. Focusing on the three-user case, in phase one, the overheard equations at the three receivers are given as follows.

At At R_1: $I_1^1(v_1, v_2, v_3)$ and $I_1^1(p_1, p_2, p_3)$.
At R_2: $I_2^1(u_1, u_2, u_3)$ and $I_2^1(p_1, p_2, p_3)$.
At R_3: $I_3^1(u_1, u_2, u_3)$ and $I_3^1(v_1, v_2, v_3)$.

Then, we define the order-2 common messages as follows.

1) *To* R_1 *and* R_2: $u_{12} = I_2^1(u_1, u_2, u_3) + I_1^1(v_1, v_2, v_3)$.
2) *To* R_1 *and* R_3 : $u_{13} = I_3^1(u_1, u_2, u_3) + I_1^1(p_1, p_2, p_3)$.
3) *To* R_2 *and* R_3 : $u_{23} = I_3^1(v_1, v_2, v_3) + I_2^1(p_1, p_2, p_3)$.

It is worth noting that phase one generates three order-2 messages; however, we can provide six order-2 messages through repeating phase one with new input variables. Then, the other three order-2 messages are v_{12}, v_{13}, and v_{23}. By the end of phase two, we have the following observation: the overheard equations at receivers are given as follows.

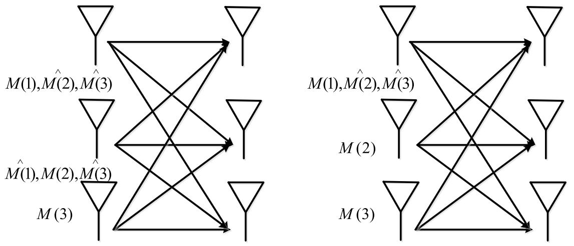
1) *At* R_1 : $I_1^1(u_{23}, v_{23})$.
2) *At* R_2 : $I_2^1(u_{13}, v_{13})$.
3) *At* R_3 : $I_3^1(u_{12}, v_{12})$.

Then, we define the order-3 messages intended to the receivers as follows.

1) *To* R_1, R_2, *and* R_3 : $u_{123} = \alpha_1 I_1^1(u_{23}, v_{23}) + \alpha_2 I_2^1(u_{13}, v_{13}) + \alpha_3 I_3^1(u_{12}, v_{12})$.
2) *To* R_1, R_2, *and* R_3 : $v_{123} = \beta_1 I_1^1(u_{23}, v_{23}) + \beta_2 I_2^1(u_{13}, v_{13}) + \beta_3 I_3^1(u_{12}, v_{12})$.

Here, the constants $\{\alpha_i\}_{i=1}^3$ and $\{\beta_i\}_{i=1}^3$ are known at the receivers. Then, as discussed in [14], each receiver will have enough equations to solve these messages (six order-2 messages) in five time slots. However, to achieve $\text{DoF}_2(K) > 1$, this transmission scheme will be attained if the following conditions hold.

1) X-network is improved as in Fig. 4(a).
2) CSIT pattern S_{123}^6 = (NDD, DND, DDN, PPD, PDP, DPP).



(a) (b)

Fig. 4. X-networks with different enhancements. (a) Improved SISO X-network. (b) Three-user supernode partially cooperative X-network.

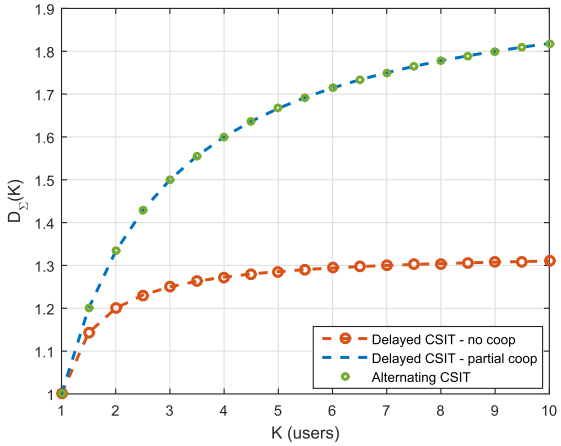


Fig. 5. DoF comparison of schemes with different CSIs.

B. Partial Cooperation in Alternating CSIT

In this section, we show that even if there is an enhancement in the X-network, one cannot achieve the joint processing gain as in the broadcast network, i.e., $\text{DoF}_j(K) < 1, \forall j > 1$.

Theorem 2: The K-user X-network with partial cooperation under synergistic alternating CSIT with distribution $\in \Lambda(\lambda_P \geq (1/3), \lambda_D \geq (2/3) - \lambda_P)$ can achieve almost surely

$$D_\Sigma(K) = \frac{2K}{K+1} \tag{37}$$

Proof (Supernode Partial Cooperation): In the K-user X-networks, we define $M(i), \forall i = 1, 2, \ldots, K$ as the message set of transmitter i, and $\tilde{M}(i) \subseteq M(i)$ denotes the subset of $M(i)$. Moreover, there is one supernode among the K transmitters that can access the message sets of other $K-1$ transmitters. Without loss of generality, we assume that transmitter 1 is the supernode. Message sets $\tilde{M}(2), \tilde{M}(3), \ldots, \tilde{M}(K)$ can be accessed by transmitter 1 due to message sharing. We call the networks with above properties as supernode partially cooperative X-network. Fig. 4(b) shows an example for three users where the first transmitter has a cognition capability of knowing the other two transmitters.

Transmission Scheme: We use the same ICR transmission scheme; however, in phase II, the supernode will transmit the following in three time slots. Define the message set of transmitter i as $M(i) = \{u_i, v_i, p_i\}$

$$X_1(4) = I_2^1(u_1, u_2, u_3) + I_1^1(v_1, v_2, v_3) \tag{38}$$

$$X_1(5) = I_3^1(u_1, u_2, u_3) + I_1^1(p_1, p_2, p_3) \tag{39}$$

$$X_1(6) = I_3^1(v_1, v_2, v_3) + I_2^1(p_1, p_2, p_3). \tag{40}$$

□

It is worth noting that it was shown in [24] that cooperation has no benefits in terms of DoF under perfect CSIT. However, in [22], it was shown that the partial cooperation has DoF gain under delayed CSIT. Fig. 5 shows the achieved DoF by employing cooperation for perfect CSIT, delayed CSIT, and alternating CSIT.

Moreover, we note that in order to derive an upper bound on the DoF of X-network with alternating CSIT, new mathematical machinery should be developed in order to incorporate the three states and their time fraction into the canonical converse proof methods and typical information theoretical inequalities. In the absence of these machineries, one has to upper bound the obtained DoF using some upgraded setups (e.g., assuming cooperation/supernodes or MISO with only delayed CSIT)

C. DoF for K-User X-Network With Alternating CSIT

Generally, for the K-user network, the relation between D_Σ, λ_P, and λ_D is as follows:

$$D_\Sigma(K) = 1 + \frac{1}{4}(\gamma_P + \gamma_D) = 1 + \frac{1}{2} \times \frac{K(K-1)}{K + \binom{K}{2}} \tag{41}$$

$$= 1 + \frac{K-1}{K+1} = \frac{2K}{K+1} \tag{42}$$

where $\gamma_j = (\sum_{t=1}^{n} I(S_i(t) = j)/n), \forall i = 1, \ldots, K, j \in \{P, D\}$, and $\gamma_P = \gamma_D = (K(K-1)/K + \binom{K}{2})$. For $K \geq 3$ and CSIT fractions equally distributed among users, we can write $\{d_i\}_{i=1}^{K}$ as

$$d_i = \frac{1}{K} + \frac{1}{4K}(\gamma_P + \gamma_D) = \frac{2}{K+1}. \tag{43}$$

V. $K \times 2$-USER SISO X-NETWORK

Toward characterizing the DoF for the $K \times 2$-user SISO X-network with alternating CSIT, first, we provide the achievability schemes for 3×2 and 4×2 SISO X-channel as illustrative examples. Then, we generalize our achievability scheme to the K-user case.

A. Consider a 3×2-User X-Network

With synergistic alternation under certain distribution, $\Lambda(1/3, 1/3, 1/3)$, in order to achieve 4/3 DoF, similar to the aforementioned schemes, the transmission strategy is executed in two distinctive phases: interference creation and interference resurrection phases, nevertheless, with minor modifications. Typically, the information symbols are avidity fed to some receivers in the interference creation phase in a form of random linear combinations while creating interference to the other receivers. During the interference resurrection phase, the old interference terms, formerly created, are sent as new linear combinations to some receivers, ensuing interference alignment to other receivers. By the end, all the receivers have the desired number of equations in terms of their intended information symbols. In particular, to achieve 4/3 DoF, we send six independent symbols to each receiver over nine time slots and the alternating CSIT pattern is given by $S_{12}^{9} = (DD, DD, DD, PN, PN, PN, NP, NP, NP)$. Here, the delayed CSIT is collocated over three time slots. Consequently, the interference creation phase consumes three time slots to generate six linear combinations and six independent interference terms, three per each receiver, while the interference resurrection phase is executed over the other six time slots to align the six interference terms formerly created and, as a by-product, generate new three linear combinations to each receiver. The proposed scheme is performed in two separate phases as follows.

Phase One: For interference creation, unlike previous interference creation phases in previous schemes, here, each time slot of this phase is dedicated to both the receivers where we send four different information symbols two for each receiver in each time slot. Since $S_{12}^{3} = (DD, DD, DD)$, each time slot is designed such that interference is created for R_1 and R_2, and hence, T_1 transmits u_1^1 and v_1^1, T_2 transmits v_2^1, and T_3 transmits u_3^1. The received signals at R_1 and R_2 are

$$\begin{aligned} Y_1(1) &= h_{11}(1)u_1^1 + h_{13}(1)u_3^1 + h_{11}(1)v_1^1 + h_{12}(1)v_2^1 \\ &\equiv L_1^1\big(u_1^1, u_3^1\big) + I_1^1\big(v_1^1, v_2^1\big) \end{aligned} \tag{44}$$

$$\begin{aligned} Y_2(1) &= h_{21}(1)u_1^1 + h_{23}(1)u_3^1 + h_{21}(1)v_1^1 + h_{22}(1)v_2^1 \\ &\equiv I_2^1\big(u_1^1, u_3^1\big) + L_2^1\big(v_1^1, v_2^1\big). \end{aligned} \tag{45}$$

Therefore, R_1 receives the first linear combination $L_1^1(u_1^1, u_3^1)$ of its desired signals in addition to interference term $I_1^1(v_1^1, v_2^1)$, while R_2 receives the first linear combination $L_2^1(v_1^1, v_2^1)$ of its desired signals along with interference term $I_2^1(u_1^1, u_3^1)$. Similarly, in the next two time slots, T_1 transmits u_1^2, T_2 transmits u_2^1 and v_2^2, and T_3 transmits v_3^1 in the second time slots, while T_1 transmits v_1^2, T_2 transmits u_2^2, and T_3 transmits u_3^2 and v_3^2 in the third time slot. As a result, the received signals at R_1 and R_2 are

$$\begin{aligned} Y_1(2) &= h_{11}(2)u_1^2 + h_{13}(2)u_2^1 + h_{11}(2)v_2^2 + h_{12}(2)v_3^1 \\ &\equiv L_2^1\big(u_1^1, u_3^1\big) + I_1^2\big(v_1^1, v_2^1\big) \end{aligned} \tag{46}$$

$$\begin{aligned} Y_2(2) &= h_{21}(2)u_1^2 + h_{23}(2)u_2^1 + h_{21}(2)v_2^2 + h_{22}(2)v_3^1 \\ &\equiv I_2^2\big(u_2^1, u_1^2\big) + L_2^2\big(v_3^1, v_2^2\big) \end{aligned} \tag{47}$$

$$\begin{aligned} Y_1(3) &= h_{12}(3)u_2^2 + h_{13}(3)u_3^2 + h_{11}(3)v_1^2 + h_{13}(3)v_3^2 \\ &\equiv L_1^3\big(u_2^2, u_3^2\big) + I_1^3\big(v_1^2, v_3^2\big) \end{aligned} \tag{48}$$

$$\begin{aligned} Y_2(3) &= h_{22}(3)u_2^2 + h_{23}(3)u_3^2 + h_{21}(3)v_1^2 + h_{23}(3)v_3^2 \\ &\equiv I_2^3\big(u_2^2, u_3^2\big) + L_2^3\big(v_1^2, v_3^2\big) \end{aligned} \tag{49}$$

where R_3 receives the first linear combination $L_3^1(p_1, p_2, p_3)$ of its desired signals, while R_1 and R_2 receive the first interference terms $I_1^1(p_1, p_2, p_3)$ and $I_2^1(p_1, p_2, p_3)$.

By the end of time slot 3, each receiver receives three linear combination terms from its intended information symbols along with three interference terms. Now, we have six interference terms available to the two receivers. Then, the interference resurrection phase takes these interference terms to generate six common messages between the two receivers. Resurrecting interference terms is beneficial to the two receivers; one receiver utilizes it by eliminating the interference terms from its received signals in phase one, while the other receiver receives it as a new linear combination from its information symbols. After the interference resurrection phase, the receivers have access to sufficient information to successfully decode their messages. Specifically, each receiver needs six independent linear combinations from its information symbols.

1) Phase Two: In the fourth time slot, interference resurrection phase begins, and the transmitters utilize the channel knowledge in PN to reconstruct $I_1^1(v_1^1, v_2^1)$ at R_1. As a result, R_2 receives the fourth linear combination term $L_2^4(v_1^1, v_2^1)$, while R_1 extracts its first linear combination term $L_1^1(u_1^1, u_3^1)$ by subtracting $Y_1(4)$ from $Y_1(1)$. In particular, the transmitted signals are

$$X_1(4) = h_{11}^{-1}(4)h_{11}(1)v_1^1 \tag{50}$$

$$X_2(4) = h_{12}^{-1}(4)h_{12}(1)v_2^1. \tag{51}$$

Note that the transmitted signals, in interference resurrection phase, are beamformed signals—not random linear combinations like in interference creation phase, dependent on both the current channel knowledge and the outdated channel knowledge formerly received at the interference creation phase. Therefore, the received signals at R_1, R_2, and R_2 are

$$Y_1(4) \equiv I_1^1\big(v_1^1, v_2^1\big) \tag{52}$$

$$\begin{aligned} Y_2(4) &= h_{21}(4)h_{11}^{-1}(4)h_{11}(1)v_1^1 + h_{22}(4)h_{12}^{-1}(4)h_{12}(1)v_2^1 \\ &\equiv L_4^2\big(v_1^1, v_2^1\big). \end{aligned} \tag{53}$$

In the fifth time slot, interference resurrection phase for user 2, the transmitters utilize the channel knowledge in PN to reconstruct the interference term $I_2^1(u_1^1, u_3^1)$ at R_2. As a result, R_2 receives the same interference term received at time slot one and, thereby, R_2 extracts its first linear combination term $L_2^1(v_1^1, v_2^1)$ by subtracting $Y_5(3)$ from $Y_2(1)$, while, as a by-product, R_1 receives its fourth linear combination term $L_1^4(u_1^1, u_3^1)$. In particular, the transmitted signals are

$$X_1(5) = h_{21}^{-1}(5)h_{21}(1)u_1^1 \tag{54}$$

$$X_3(5) = h_{23}^{-1}(5)h_{23}(1)u_3^1. \tag{55}$$

As a result, the received signals at R_1 and R_2 are

$$\begin{aligned} Y_1(5) &= h_{11}(5)h_{21}^{-1}(5)h_{21}(1)u_1^1 + h_{13}(5)h_{23}^{-1}(5)h_{23}(1)u_3^1 \\ &\equiv L_1^4\big(u_1^1, u_3^1\big) \end{aligned} \tag{56}$$

$$Y_2(5) \equiv I_2^1\big(u_1^1, u_3^1\big). \tag{57}$$

In the sixth time slot, the transmitters utilize the channel knowledge in PN to reconstruct $I_1^2(v_2^2, v_3^1)$ at R_1. As a result, R_2 receives the fifth linear combination term $L_2^5(v_2^2, v_3^1)$, while R_1 extracts its second linear combination term $L_1^2(u_1^2, u_2^1)$ by subtracting $Y_1(6)$ from $Y_1(2)$. In particular, the transmitted signals are

$$X_2(6) = h_{12}^{-1}(6)h_{12}(2)v_2^2 \tag{58}$$

$$X_3(6) = h_{13}^{-1}(6)h_{13}(2)v_3^1. \tag{59}$$

Therefore, the received signals at R_1 and R_2 are

$$Y_1(6) \equiv I_1^2\big(v_2^2, v_3^1\big) \tag{60}$$

$$\begin{aligned} Y_2(6) &= h_{22}(6)h_{12}^{-1}(6)h_{12}(2)v_2^2 + h_{23}(6)h_{13}^{-1}(6)h_{13}(2)v_3^1 \\ &\equiv L_5^2\big(v_2^2, v_3^1\big). \end{aligned} \tag{61}$$

In the seventh time slot, interference resurrection phase for user 2, the transmitters utilize the channel knowledge in NP to reconstruct the interference term $I_2^2(u_1^2, u_2^1)$ at R_2. As a result, R_2 receives the same interference term received at time slot two and thereby R_2 able to extract its second linear combination term $L_2^2(v_2^2, v_3^1)$ by subtracting $Y_2(7)$ from $Y_2(2)$, while, as a by-product, R_1 receives its fifth linear combination term $L_1^5(u_1^2, u_2^1)$. In particular, the transmitted signals are

$$X_1(7) = h_{21}^{-1}(7)h_{21}(2)u_1^2 \tag{62}$$

$$X_2(7) = h_{22}^{-1}(7)h_{22}(2)u_2^1. \tag{63}$$

As a result, the received signals at R_1 and R_2 are given, respectively, by

$$\begin{aligned} Y_1(7) &= h_{11}(7)h_{21}^{-1}(5)h_{21}(1)u_1^2 + h_{12}(7)h_{22}^{-1}(7)h_{22}(1)u_2^1 \\ &\equiv L_1^4\big(u_1^2, u_2^1\big) \end{aligned} \tag{64}$$

$$Y_2(7) \equiv I_2^2\big(u_1^2, u_2^1\big). \tag{65}$$

In the eighth time slot, the transmitters utilize the channel knowledge in PN to reconstruct $I_1^2(v_1^2, v_3^2)$ at R_1. As a result, R_2 receives the sixth linear combination term $L_2^6(v_1^2, v_3^2)$, while R_1 extracts its third linear combination term $L_1^3(u_2^2, u_3^2)$ by subtracting $Y_1(8)$ from $Y_1(3)$. In particular, the transmitted signals are

$$X_1(8) = h_{11}^{-1}(8)h_{11}(3)v_1^2 \tag{66}$$

$$X_3(8) = h_{13}^{-1}(8)h_{13}(3)v_3^2. \tag{67}$$

Therefore, the received signals at R_1 and R_2 are

$$Y_1(8) \equiv I_3^2\big(v_1^2, v_3^2\big) \tag{68}$$

$$\begin{aligned} Y_2(8) &= h_{21}(8)h_{11}^{-1}(8)h_{11}(3)v_1^2 + h_{23}(8)h_{13}^{-1}(8)h_{13}(3)v_3^2 \\ &\equiv L_6^2\big(v_1^2, v_3^2\big). \end{aligned} \tag{69}$$

In the ninth time slot, interference resurrection phase for user 2, the transmitters utilize the channel knowledge in NP to reconstruct the interference term $I_2^2(u_2^2, u_3^2)$ at R_2. As a result, R_2 receives the same interference term received at time slot 2 and thereby R_2 able to extract its third linear combination term $L_2^3(u_2^2, u_3^2)$ by subtracting $Y_2(9)$ from $Y_2(3)$, while, as a by-product, R_1 receives its sixth linear combination term $L_1^6(u_2^2, u_3^2)$. In particular, the transmitted signals are

$$X_2(9) = h_{22}^{-1}(9)h_{22}(3)u_2^2 \tag{70}$$

$$X_3(9) = h_{23}^{-1}(9)h_{23}(3)u_3^2. \tag{71}$$

As a result, the received signals at R_1 and R_2 are

$$\begin{aligned} Y_1(9) &= h_{12}(9)h_{22}^{-1}(9)h_{22}(3)u_2^2 + h_{13}(9)h_{23}^{-1}(9)h_{23}(3)u_3^2 \\ &\equiv L_1^4\big(u_2^2, u_3^2\big) \qquad (72) \\ Y_2(9) &\equiv I_3^2\big(u_2^2, u_3^2\big). \qquad (73) \end{aligned}$$

By the end of the ninth slot, each receiver has access to sufficient information to successfully decode its symbols. Specifically, each receiver has six independent equations (linear combinations) in six variables and six interference terms aligned in three dimensions.

2) Achievability Scheme to Even Number of Transmitters: Consider a 4 × 2-user X-network with synergistic alternation under certain distribution, $\Lambda((1/3), (1/3), (1/3))$. In order to achieve (4/3) DoF, similar to the aforementioned schemes, the transmission strategy is executed in two distinctive phases, nevertheless, without duplicating the number of transmitted symbols. In particular, to achieve (4/3) DoF, we send six independent symbols to each receiver over nine time slots and the alternating CSIT pattern is given by $S_{12}^9 = (DD, DD, DD, PN, PN, PN, NP, NP, NP)$. Here, the delayed CSIT is collocated over two time slots. Consequently, the interference creation phase consumes two time slots to generate four linear combinations and four independent interference terms, three per each receiver, while the interference resurrection phase is executed over the other four time slots to align the four interference terms formerly created and, as a by-product, generate new two linear combinations to each receiver. The proposed scheme is performed in two separate phases as follows.

Phase One: For interference creation, unlike previous interference creation phases in previous schemes, here, each time slot of this phase is dedicated to both the receivers (send random linear combinations from the desired signals and create interference for both receivers simultaneously) where we send four different information symbols two for each receiver in each time slot. Since $S_{12}^2 = (DD, DD)$, each time slot is designed such that interference is created for R_1 and R_2; hence, T_1 transmits u_1, T_2 transmits u_2 and v_2, and T_3 transmits v_3. The received signals at R_1 and R_2 are

$$\begin{aligned} Y_1(1) &= h_{11}(1)u_1 + h_{12}(1)u_2 + h_{12}(1)v_2 + h_{13}(1)v_3 \\ &\equiv L_1^1(u_1, u_2) + I_1^1(v_2, v_3) \qquad (74) \\ Y_2(1) &= h_{21}(1)u_1 + h_{22}(1)u_2 + h_{22}(1)v_2 + h_{23}(1)v_3 \\ &\equiv I_2^1(u_1, u_2) + L_2^1(v_2, v_3). \qquad (75) \end{aligned}$$

Therefore, R_1 receives the first linear combination $L_1^1(u_1, u_2)$ of its desired signals in addition to interference term $I_1^1(v_2, v_3)$, while R_2 receives the first linear combination $L_2^1(v_2, v_3)$ of its desired signals along with interference term $I_2^1(u_2, u_2)$. Similarly, in the next time slots, T_3 transmits u_3, T_4 transmits u_4 and v_4, and T_1 transmits v_1. As a result, the received signals at R_1 and R_2 are

$$\begin{aligned} Y_1(2) &= h_{13}(2)u_3 + h_{14}(2)u_4 + h_{11}(2)v_1 + h_{14}(2)v_4 \\ &\equiv L_2^1(u_3, u_4) + I_1^2(v_1, v_4) \qquad (76) \\ Y_2(2) &= h_{23}(2)u_3 + h_{24}(2)u_4 + h_{21}(2)v_1 + h_{24}(2)v_4 \\ &\equiv I_2^2(u_3, u_4) + L_2^2(v_1, v_4). \qquad (77) \end{aligned}$$

By the end of time slot 2, end of interference creation phase, each receiver receives two linear combination terms from its intended information symbols along with two interference terms. Now, we have four interference terms available to the two receivers. Then, the interference resurrection phase takes these interference terms to generate four common messages between the two receivers. Resurrecting interference terms is beneficial to the two receivers; one receiver utilizes it by eliminating the interference terms from its received signals in phase one, while the other receiver receives it as a new linear combination from its information symbols. After the interference resurrection phase, the receivers have access to sufficient information to successfully decode their messages. Specifically, each receiver needs four independent linear combinations from its information symbols.

Phase two: In the third time slot, the interference resurrection phase begins and extends for four time slots, and here, we execute the interference resurrection phase in two separate stages as follows.

Stage 1 (Interference Resurrection for R_1): The transmitters utilize the channel knowledge in PN, received in time slots 3 and 4, to reconstruct $I_1^1(v_2, v_3)$ and $I_1^1(v_1, v_4)$ at R_1. As a result, R_2 receives the third and fourth linear combination terms $L_2^3(v_2, v_3)$ and $L_2^4(v_1, v_4)$, respectively, while R_1 extracts its first and second linear combination terms $L_1^1(u_1, u_2)$ and $L_1^2(u_3, u_4)$ by subtracting $Y_1(3)$ from $Y_1(1)$ and $Y_1(4)$ from $Y_1(2)$, respectively. In particular, the transmitted signals are

$$X_2(3) = h_{12}^{-1}(3)h_{12}(1)v_2 \qquad (78)$$

$$X_3(3) = h_{13}^{-1}(3)h_{13}(1)v_3 \qquad (79)$$

$$X_1(4) = h_{11}^{-1}(4)h_{11}(1)v_1 \qquad (80)$$

$$X_4(4) = h_{14}^{-1}(4)h_{14}(1)v_4. \qquad (81)$$

As a result, the received signals at R_1 and R_2, over the third and fourth time slots, are

$$\begin{aligned} Y_1(3) &\equiv I_1^2(v_2, v_3) \qquad (82) \\ Y_2(3) &= h_{21}(3)h_{12}^{-1}(3)h_{12}(3)v_2 + h_{23}(3)h_{13}^{-1}(3)h_{13}(2)v_3 \\ &\equiv L_2^3(v_2, v_3) \qquad (83) \\ Y_1(4) &\equiv I_1^2(v_1, v_4) \qquad (84) \\ Y_2(4) &= h_{21}(4)h_{11}^{-1}(4)h_{11}(2)v_1 + h_{24}(4)h_{14}^{-1}(4)h_{14}(2)v_4 \\ &\equiv L_2^4(v_1, v_4). \qquad (85) \end{aligned}$$

Stage 2 (Interference Resurrection for R_2): In the fifth and sixth time slots, the transmitters utilize the channel knowledge in PN to reconstruct the interference terms $I_2^1(u_1, u_2)$ and $I_2^2(u_3, u_4)$ at R_2. As a result, R_2 receives the same interference terms received at interference creation phase and, thereby, R_2 extracts its first and second linear combination term $L_2^1(v_2, v_3)$ and $L_2^2(v_1, v_4)$ by subtracting $Y_2(5)$ from $Y_2(1)$ and $Y_2(6)$ from $Y_2(2)$, while, as a by-product, R_1 receives its third and fourth linear combination terms $L_1^3(u_1, u_2)$ and

$L_1^4(u_3, u_4)$(new information). In particular, the transmitted signals are

$$X_1(5) = h_{21}^{-1}(5)h_{21}(1)u_1 \tag{86}$$
$$X_2(5) = h_{22}^{-1}(5)h_{22}(1)u_2 \tag{87}$$
$$X_3(6) = h_{23}^{-1}(6)h_{23}(1)u_3 \tag{88}$$
$$X_4(6) = h_{23}^{-1}(6)h_{23}(1)u_4. \tag{89}$$

As a result, the received signals at R_1 and R_2, over the fifth and sixth time slots, are

$$\begin{aligned} Y_1(5) &= h_{11}(5)h_{21}^{-1}(5)h_{21}(1)u_1 + h_{12}(5)h_{22}^{-1}(5)h_{22}(1)u_2 \\ &\equiv L_1^3(u_1, u_2) \end{aligned} \tag{90}$$
$$Y_2(5) \equiv I_2^1(u_1, u_2) \tag{91}$$
$$\begin{aligned} Y_1(6) &= h_{13}(6)h_{23}^{-1}(6)h_{23}(2)u_3 + h_{14}(6)h_{24}^{-1}(5)h_{24}(2)u_4 \\ &\equiv L_1^4(u_3, u_4) \end{aligned} \tag{92}$$
$$Y_2(6) \equiv I_2^2(u_3, u_4). \tag{93}$$

By the end of the sixth time slot, each receiver has access to sufficient information to successfully decode its information symbols. Specifically, each receiver has four independent equations (linear combinations) in four variables and four interference terms seized in only two dimensions.

3) Generalization to $K \times 2$-User SISO X-Network: In this section, we describe the extension to the interference creation–resurrection transmission strategy for the $K \times 2$-user SISO X-Channel with synergistic alternating CSIT under $\Lambda(1/3, 1/3, 1/3)$. The transmission scheme is a two-phase scheme, such as the previous one, but with many stages in each phase. New random linear combinations are sent to the receivers: in phase one, interference creation phase, in a certain way that guarantees fed the receivers with a certain number of equations of the information symbols as well as creating common messages between the receivers; in our case, it is the interference itself, and in phase two, interference resurrection, responsible for delivering the common messages to the receivers and thereby providing each receiver with the required number of equations to successfully decode its intended information symbols. In the $K \times 2$-user SISO X-channel, each transmitter in the network has an independent message to be communicated to each receiver, and therefore, the network has multiple of $2K$ independent messages communicating between its nodes. This directly implies that each receiver interest in decoding K independent messages over the successful communication time (a certain number of time slots of channel use) consequently each receiver requires K independent equations to resolve its own messages.

Phase One: In the interference creation phase, the K transmitters send their messages in a certain way to provide each receiver with $K/2$ random linear combinations of K information symbols corrupted by $K/2$ interference terms in $K/2$ time slots. Specifically, we divide the $2K$ information symbols available at transmitters into $K/2$ batches; each batch has four different symbols. In each batch, there are two different groups of two symbols, and each group has symbols that are intended to a certain receiver but generated at different transmitters. This strategy in diving the information symbols guarantees that the interference terms created in phase one are beneficial when resurrecting in phase two. Here, beneficial means that these interference terms can work as a common message for the two receivers. By the end of phase one, each receiver has access to $K/2$ independent linear combinations of its own symbols corrupted with $K/2$ constituent (constructible and beneficial) interference terms.

Phase Two: In the interference resurrection phase, the transmitters utilize the delayed CSIT sent in phase one and the instantaneous CSIT to generate and broadcast common messages to the receiver by reconstructing the constituent interference terms formerly received in phase one. In particular, the transmitters generate and send K common messages, $K/2$ messages for each receiver over two stages. Creating one common message (constituent interference term) directly implies providing one receiver with old interference term to extract new linear combination from an interference-corrupted linear combination formerly received in phase one while providing the other receiver with the new linear combination. Almost sure, all the transmitters have sufficient channel knowledge to create the common message (old constituent interference term) at certain receiver; the delayed CSIT received in phase one provides the transmitters with the old channel coefficient, while the instantaneous CSIT enable them to nullify the effect of current channel coefficient and, thereby, the old interference term can be resurrected. Sending such a common message only consumes one time slot; consequently, the interference resurrection phase consumes K time slots. By the end of phase two, each receiver receives $K/2$ new linear combinations of its own symbols in a certain stage while receiving $K/2$ constituent interference terms in the other stage, used to extract $K/2$ linear combinations. After delivering all these common messages, every receiver has access to K linear combinations of its intended information symbols. It is straightforward to show that these K linear combinations are linearly independent almost surely, and thus, each receiver can resolve all its K information symbols. Hence, the DoF of the $K \times 2$-user SISO X-channel with synergistic alternating CSIT is lower bounded as follows:

$$\mathrm{DoF}_{K\times 2}^{X}(\lambda_P \geq 1/3, \lambda_D \geq 2/3 - \lambda_P) \geq \frac{2K}{K/ + K} = \frac{4}{3}.$$

This lower bound is tight for $K = 2$, for which the two-user X-channel achieves the upper bound on the DoF of $4/3$.

VI. 2 × K-USER SISO X-NETWORK

In order to characterize the DoF for the $2 \times K$-user SISO X-network with synergistic alternating CSIT, we provide the achievability schemes for 2×3 and 2×4 SISO X-network as illustrative examples. Then, we generalize our achievability scheme to the $2 \times K$-user case. We note that the $2 \times K$ case can model the scenario where one network node, e.g., base station, is communicating with K-users, while another network node, e.g., positioning station/anchor, is sensing the same K-users using the same time and frequency resources. This scenario is in the downlink direction. However, the

$K \times 2$ case can model the same scenario but in the uplink direction.

A. *Consider a* 2×3*-User X-Network*

With synergistic alternation under certain distribution, $\Lambda(2/9, 2/9, 5/9)$, in order to achieve $4/3$ DoF, similar to the aforementioned schemes, the transmission strategy is executed in two distinctive phases, interference creation and interference resurrection phases, nevertheless, with minor modifications. In particular, to achieve $4/3$ DoF, especially in 2×3-case, we send eight independent symbols, four symbols from each transmitter over six time slots, and the alternating CSIT pattern is given by $S_{12}^6 = (\mathrm{NDN}, \mathrm{DNN}, \mathrm{NND}, \mathrm{NDN}, \mathrm{PPN}, \mathrm{NPP})$. Here, the delayed CSIT is distributive over four time slots. Consequently, the interference creation phase consumes four time slots to generate four linear combinations and four independent interference terms: one for R_1, two for R_2, and one for R_3, while the interference resurrection phase is executed over the last two time slots to align the four interference terms formerly created and, as a by-product, generate new linear combinations to each receiver. The proposed scheme is performed in two separate phases as follows.

Phase One: For interference creation, unlike interference creation phases in $K \times 2$-user case, here, each time slot of this phase is dedicated to only one receiver where we send two different information symbols intended to certain receiver in each time slot. Since $S_{12}^4 = (\mathrm{NDN}, \mathrm{DNN}, \mathrm{NND}, \mathrm{NDN})$, each time slot is designed such that interference is created for R_1 or R_2 or R_3 depending on the transmitted signal intended to which receiver, hence, in each time slot of the interference creation phase, T_1 and T_2 transmit information symbols intended to certain receiver. As a result, one receiver receives a linear combination from its desired symbols (without interference), while the others receive only interference term. For example, in time slot 1, T_1 transmits u_1^1 and T_2 transmits u_1^1; consequently, the received signals are

$$Y_1(1) = h_{11}(1)u_1^1 + h_{12}(1)u_2^1 \equiv L_1^1\big(u_1^1, u_2^1\big) \tag{94}$$

$$Y_2(1) = h_{21}(1)u_1^1 + h_{22}(1)u_2^1 \equiv I_2^1\big(u_1^1, u_2^1\big) \tag{95}$$

$$Y_3(1) = h_{31}(1)u_1^1 + h_{32}(1)u_2^1 \equiv I_3^1\big(u_1^1, u_2^1\big). \tag{96}$$

Therefore, R_1 receives the first linear combination $L_1^1(u_1^1, u_2^1)$ of its desired signals, while R_2 and R_3 receive only interference terms $I_2^1(u_1^1, u_2^1)$ and $I_3^1(u_1^1, u_2^1)$, respectively. Similarly, in the next three time slots, the transmitters send two independent linear combinations to R_2 and one linear combination to R_3. In particular, T_1 transmits v_1^1 and T_2 transmits v_2^1 in the second time slots, while, in the third time slot, T_1 transmits v_1^2 and T_2 transmits v_2^2; after that, in the fourth time slot, the transmitters are dedicated to R_3, i.e., T_1 transmits p_1^1 and T_2 transmits p_2^1. As a result, the received signals at R_1, R_2, and R_3 are given by

$$Y_1(2) = h_{11}(2)v_1^1 + h_{12}(2)v_2^1 \equiv I_1^1\big(v_1^1, v_2^1\big) \tag{97}$$

$$Y_2(2) = h_{21}(2)v_1^1 + h_{22}(2)v_2^1 \equiv L_2^1\big(v_1^1, v_2^1\big) \tag{98}$$

$$Y_3(2) = h_{31}(2)v_1^1 + h_{32}(2)v_2^1 \equiv I_3^1\big(v_1^1, v_2^1\big) \tag{99}$$

$$Y_1(3) = h_{11}(3)v_1^2 + h_{12}(3)v_2^2 \equiv L_1^2\big(v_1^2, v_2^2\big) \tag{100}$$

$$Y_2(3) = h_{21}(3)v_1^2 + h_{22}(3)v_2^2 \equiv L_2^2\big(v_1^2, v_2^2\big) \tag{101}$$

$$Y_3(3) = h_{31}(3)v_1^2 + h_{32}(3)v_2^2 \equiv I_3^2\big(v_1^2, v_2^2\big) \tag{102}$$

$$Y_1(4) = h_{11}(4)p_1^1 + h_{12}(4)p_2^1 \equiv I_1^1\big(p_1^1, p_2^1\big) \tag{103}$$

$$Y_2(4) = h_{21}(4)p_1^1 + h_{22}(4)p_2^1 \equiv I_2^1\big(p_1^1, p_2^1\big) \tag{104}$$

$$Y_3(4) = h_{31}(4)p_1^1 + h_{32}(4)p_2^1 \equiv L_3^1\big(p_1^1, p_2^1\big). \tag{105}$$

At the end of the fourth time-slot, the first and the third receivers receive two linear combination terms from their intended information symbols and four interference terms; two of four are useful interference terms and the others are useless ones. On the other hand, receiver 2 receives two linear combination terms from its intended information symbols; in addition to two interference terms, all of them are useful terms. Now, we have two interference terms available to the second receiver in addition to two interference terms available to receivers 1 and 3. Then, the interference resurrection phase takes the four interference terms to generate two common messages between the receivers. We note that resurrecting interference terms in each time slot is beneficial to two receivers only, while the other one receives only interference. Contrary to the role of common messages in the $K \times 2$-user scheme, one receiver utilizes it by eliminating the interference terms from its received signals in phase one, while the other receiver receives it as a new linear combination from its information symbols; here, common messages are the sum of two terms (old interference terms) where each receiver pair simultaneously receives it as new linear combination from their information symbols after eliminating the interference terms formerly received in the interference creation phase. After the interference resurrection phase, all receivers have access to sufficient information to decode their messages.

Phase Two: In the fifth time slot, interference resurrection phase begins, the transmitters utilize the channel knowledge of PPN, to simultaneously reconstruct $I_1^1(v_1^1, v_2^1)$ at R_1 and $I_2^1(u_1^1, u_2^1)$ at R_2. As a result, R_2 receives the third linear combination term $L_2^3(v_1^1, v_2^1)$ along with old interference term $I_2^1(u_1^1, u_2^1)$, while R_1 receives its second linear combination term $L_1^2(u_1^1, u_2^1)$ in addition to old interference term $I_1^1(v_1^1, v_2^1)$. In particular, the transmitted signals are

$$X_1(5) = h_{21}^{-1}(5)h_{21}(1)u_1^1 + h_{11}^{-1}(5)h_{11}(2)v_1^1 \tag{106}$$

$$X_2(5) = h_{22}^{-1}(5)h_{22}(1)u_2^1 + h_{12}^{-1}(5)h_{12}(2)v_2^2. \tag{107}$$

Therefore, the received signals at R_1, R_2, and R_2 are

$$Y_1(5) = L_1^2\big(u_1^1, u_2^1\big) + I_1^1\big(v_1^1, v_2^1\big) \tag{108}$$

$$Y_2(5) = L_2^3\big(v_1^1, v_2^1\big) + I_2^1\big(u_1^1, u_2^1\big) \tag{109}$$

$$Y_3(5) = I_3^3\big(v_1^1, v_2^1\big) + I_3^2\big(u_1^1, u_2^1\big). \tag{110}$$

In the sixth time slot, we continue with interference resurrection phase for R_2 and R_3; similarly, the transmitters utilize the channel knowledge in NPP to make interference resurrection simultaneously possible for R_2 and R_3, in particular, reconstructing the interference term $I_2^1(p_1^1, p_2^1)$ at R_2 and $I_3^1(v_1^2, v_2^2)$ at R_3. As a result, R_2 receives the fourth linear combination term $L_2^4(v_1^2, v_2^2)$ along with old interference term

$I_2^1(p_1^1, p_2^1)$, while R_3 receives its second linear combination term $L_3^2(p_1^2, p_2^1)$ in addition to old interference term $I_3^2(v_1^2, v_2^2)$. In particular, the transmitted signals are

$$X_1(6) = h_{21}^{-1}(6)h_{21}(7)p_1^1 + h_{31}^{-1}(6)h_{31}(3)v_1^2 \tag{111}$$

$$X_2(6) = h_{22}^{-1}(6)h_{22}(7)p_2^1 + h_{32}^{-1}(6)h_{32}(3)v_2^2. \tag{112}$$

Therefore, the received signals at R_1, R_2, and R_3 are

$$Y_1(6) = I_1^2\left(p_1^1, p_2^1\right) + I_1^3\left(v_1^3, v_2^3\right) \tag{113}$$

$$Y_2(6) = L_2^4\left(v_1^2, v_2^2\right) + I_2^1\left(p_1^1, p_2^1\right) \tag{114}$$

$$Y_3(6) = I_3^2\left(v_1^2, v_2^2\right) + L_3^2\left(p_1^1, p_2^1\right). \tag{115}$$

By the end of the sixth slot, each receiver has access to sufficient information to successfully decode its symbols. Specifically, R_1 and R_3 have two independent equations (linear combinations) in two variables, while R_2 has four independent equations in four variables and four interference terms aligned into two dimensions.

B. Achievability Scheme to Even Number of Receivers

Consider a 2 × 4-user X-network with synergistic alternation under certain distribution; $\Lambda(1/6, 1/6, 4/6)$. In order to achieve 4/3 DoF, similar to the previous scheme, the transmission strategy is executed in two distinctive phases, interference creation and interference resurrection phases, nevertheless, insert new variables q_1 and q_2 intended to R_4 instead of duplicating the number of transmitted symbols of the second receiver in the 2 × 3-user scheme. In particular, to achieve 4/3 DoF, we send two independent symbols to each receiver over six time slots and the alternating CSIT pattern is given by $S_{12}^6 =$ (NDNN, DNNN, NNDN, NNND, PPNN, NNPP). Here also, the delayed CSIT is distributed over four time slots. Consequently, the interference creation phase consumes four time slots to generate four linear combinations and 12 independent interference terms, while the interference resurrection phase is executed over the last two time slots to align the four interference terms formerly created and, as a by-product, generate new linear combinations to each receiver. The proposed scheme is exactly the same as the previous scheme in the 2 × 3-user case. We note that the required CSIT decreases with K, the number of receivers.

C. Generalization to $2 \times K$*-User SISO X-Network*

In this section, we describe the extension to the interference creation–resurrection transmission strategy for the $2 \times K$-user SISO X-channel with synergistic alternating CIST under $\Lambda(2/3K, 2/3K, (3K-4)/3K)$. The transmission scheme is a two-phase scheme, such as the previous one, but with many stages in each phase. New random linear combinations are sent to the receivers: in phase one, interference creation phase, in a certain way that guarantees fed the receivers with a certain number of equations of the information symbols as well as creating common messages between the receivers; in our case, it is the interference itself; and phase two, interference resurrection, responsible for delivering the common messages to the receivers and thereby providing each receiver with the required number of equations to successfully decode its intended information symbols. In the $2 \times K$-user SISO X-network, each transmitter in the network has an independent message to be communicated to each receiver; therefore, the network has multiple of $2K$ independent messages communicating between its nodes. This directly implies that each receiver interests in decoding K independent messages over the successful communication time (a certain number of time slots of channel uses); consequently, each receiver requires K independent equations to resolve its own messages.

Phase One: In the interference creation phase, the K transmitters send their messages in a certain way to provide each receiver with $K/2$ random linear combinations of K information symbols corrupted by $K/2$ interference terms in $K/2$ time slots. Specifically, we divide the $2K$ information symbols available at transmitters into $K/2$ batches; each batch has four different symbols. In each batch, there are two different groups of two symbols, and each group has symbols that are intended to a certain receiver but generated at different transmitters. This strategy in diving the information symbols guarantees that the interference terms created in phase one are beneficial when resurrecting in phase two. Here, beneficial means that these interference terms can work as a common message for the two receivers. By the end of phase one, each receiver has access to only one linear combination of its own symbols as well as $(K-1)$ reconstructable interference terms.

Phase Two: In the interference resurrection phase, the transmitters utilize the delayed CSIT sent in phase one and the instantaneous CSIT to generate and broadcast common messages to the receivers by reconstructing the constituent interference terms formerly received in phase one. In particular, the transmitters generate and send K common messages by adding $2K$ constituent interference terms generated in phase one. Creating one common message (constituent interference term) directly implies providing two receivers with old interference terms along with new linear combination. Almost sure, all the transmitters have sufficient channel knowledge to create the common message(old constituent interference term) at certain receiver; the delayed CSIT received in phase one provides the transmitters with the old channel coefficient, while the instantaneous CSIT enables them to nullify the effect of current channel coefficient and, thereby, the old interference term can be resurrected. Sending such a common message only consumes one time slot; consequently, the interference resurrection phase consumes $K/2$ time slots. By the end of phase K, receivers receive K new linear combinations. After delivering all these common messages, every receiver has access to two linear combinations of its intended information symbols. Then, these two linear combinations are linearly independent almost surely, and thus, each receiver can resolve all its two information symbols. Hence, the DoF of the $2 \times K$-user SISO X-channel with synergistic alternating CSIT is lower bounded by

$$\mathrm{DoF}_{2\times K}^{X}(\lambda_P \geq 1/3, \lambda_D \geq 2/3 - \lambda_P) \geq \frac{2K}{K/2 + K} = \frac{4}{3}.$$

Similar to the $K \times 2$ case, the lower bound is tight for $K = 2$ and the upper bound DoF of 4/3 is achievable.

VII. PRIOR ART COMPARISON AND DISCUSSION

A. Numerical Comparison

In the lack of tight upper bounds for the K-user, $K \times 2$-user, and $2 \times K$-user SISO X-network under the alternating CSIT assumption, we cannot claim any optimality to our achievability schemes. However, in the following, we present a comprehensive comparison between our achievability schemes and the main schemes in the literature on X-networks. First, we begin with comparing our achievability schemes and results, under the alternating CSIT assumption, to their peers but under perfect CSIT. Indeed, we recall the results of [4], which present the tightest upper bounds of $K^2/2K-1$ and $2K/K+1$ for the K-user and $2 \times K$-user or $K \times 2$-user SISO X-network with perfect CSIT, respectively. In addition to these upper bounds, the authors developed an asymptotic interference alignment scheme for the $M \times N$-user SISO X-network (K-user when $M = N$), which partially aligns the interference to asymptotically achieve the previous upper bound within a constant gap, $\epsilon > 0$, over infinite channel extension. Specifically, they constructed an achievable scheme to achieve $(M-1)Nn^{\Gamma} + N(n+1)^{\Gamma}$ DoF over an $(M-1)Nn^{\Gamma} + N(n+1)^{\Gamma}$ symbol extension of the channel, where $\Gamma = (M-1)(N-1)$ so that the achievable DoFs are arbitrary close to $MN/M+N-1$ when $n \to \infty$. For the sake of comparison, the number of time slots utilized in the channel extension to achieve a certain ultimate DoF can be a good metric to measure how practical the communication scheme is. For instance, to achieve $2K/K+1$ DoF for the K-user SISO X-network using the achievable scheme of [4], it requires sending $K(K-1)n^{(K-1)^2} + K(n+1)^{(K-1)^2}$ over a $(K-1)n^{(K-1)^2} + K(n+1)^{(K-1)^2}$ symbol channel extension and perfect CSIT, while our proposed scheme simply sends K^2 independent messages over $(K + \binom{K}{2})$. Another example, for three-user SISO X-channel, to achieve $3/2$ DoF, it requires almost sending 9365 message over 6243 time slots, while our scheme requires sending nine messages over six time slots. Moreover, since our achievability scheme essentially creates K^2 point-to-point links over a $(K + \binom{K}{2})$ symbol extension of the channel, it provides a $\mathcal{O}(1)$ capacity characterization of the K-user X-network, while the asymptomatic interference alignment scheme only yields a capacity characterization within $\mathcal{O}(\log(\mathrm{SNR}))$.

Second, Ghasemi *et al.* [16] showed the possibility of distributed retrospective interference alignment for the K-user SISO X-network with delayed CSIT. They proposed a two-phase transmission scheme to tackle the main bottleneck of the distributed network, loss of joint signal processing of the signals at transmitters (each transmitter has access only to its own symbols). In particular, it proved that the K-user SISO X-network can achieve $(4/3) - (2/3(3K-1))$ DoF under the delayed CSIT assumption. However, it was proven in [25] that this result is tight for $K = 2$, and there is no evidence so far to confirm that for$K > 2$. Trivially, our achievability result for the K-user case is strictly higher than this one, i.e., for our case, the achievable DoF $\to 2$, while for the Ghasemi scheme, it tends to $4/3$ when $K \to \infty$.

B. Conjecture of DoF Scaling

Our achievable DoFs for the K-user, $K \times 2$-user, and $2 \times K$-user SISO X-network under the alternating CSIT assumption are tight for $K = 2$, for which we achieve the upper bound of DoF of $4/3$ for the SISO X-channel with perfect CSIT. Moreover, in light of [23, Remark 2], the impact of the synergy between instantaneous CSIT and delayed CSIT is to enhance the achievable DoFs of the X-channel by defeating its distributed nature at the transmitters' side. In other words, this synergy enables virtual joint signal processing at transmitters, thereby upgrading the two-user X-channel with synergistic alternating CSIT to a two-user MISO BC with delayed CSIT. Therefore, these insights pose that the synergistic alternating CSIT, in terms of characterizing the DoF, could enhance the K-user X-network to a K-user MISO BC with delayed CSIT as it is beneficial in the two-user case. Consequently, we conjecture that, under the synergistic alternating CSIT with $\Lambda((1/3), (1/3), (1/3))$, the K-user SISO X-channel can achieve $K/1 + (1/2)$, the upper bound on the DoFs of the K-user MISO BC with delayed CSIT. However, it does not seem to be possible through our achievability schemes (two-phase schemes), and we believe that the multiphase schemes have many things to offer to the K-user SISO X-networks with synergistic alternating CSIT. Indeed, multiphase achievability schemes are a perfect match to the multi-interferer nature of K-user X-networks.

VIII. CONCLUSION

We investigated the synergistic benefits of alternating CSIT for different settings of the K-user X-networks. Specifically, we proposed two-phase interference alignment schemes and obtained new DoF achievability results for $K \times 2$-user, $2 \times K$-user, and K-user SISO X-networks under synergistic alternating CSIT assumption. We showed that the proposed two-phase DoF achievability schemes are optimal such that there are no two-phase schemes that can achieve higher DoF. Moreover, we conjectured that the K-user X-network with alternating CSIT can achieve $K/1 + (1/2)$ DoF, i.e., the upper bound on the DoF of the K-user MISO BC (an upgrade to the X channel where all the transmitters are collocated and work in a cooperative manner). Interestingly, these DoF achievability schemes show that sensing systems can be integrated into the current communication systems and they can utilize the same time and frequency resources while employing simple interference alignments techniques to manage the interference between communication and sensing functionalities. Finally, an important future direction of this work is to develop new tighter upper bounds on the DoF of K-user X-networks that have been studied here.

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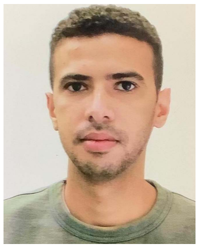

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