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A Comparative Study on the Efficiency of Reliability Methods for the Probabilistic Analysis of Local Scour at a Bridge Pier in Clay-Sand-Mixed Sediments

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Abstract: In this work, the performance of reliability methods for the probabilistic analysis of local scour at a bridge pier is investigated. The reliability of bridge pier scour is one of the important issues for the risk assessment and safety evaluation of bridges. Typically, the depth prediction of bridge pier scour is estimated using deterministic equations, which do not consider the uncertainties related to scour parameters. To consider these uncertainties, a reliability analysis of bridge pier scour is required. In the recent years, a number of efficient reliability methods have been proposed for the reliability-based assessment of engineering problems based on simulation, such as Monte Carlo simulation (MCS), subset simulation (SS), importance sampling (IS), directional simulation (DS), and line sampling (LS). However, no general guideline recommending the most appropriate reliability method for the safety assessment of bridge pier scour has yet been proposed. For this purpose, we carried out a comparative study of the five efficient reliability methods so as to originate general guidelines for the probabilistic assessment of bridge pier scour. In addition, a sensitivity analysis was also carried out to find the effect of individual random variables on the reliability of bridge pier scour.

Keywords: bridge pier; scour; reliability analysis; failure probability; simulation; subset simulation (SS)

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1. Introduction

Pier foundations are crucial for the reliable bed support of bridge structures [1,2]. However, flowing water can lead to erosive action that subsequently transports materials from around the abutments and piers of these structures [3,4]. This phenomenon is called scour, which can cause the failure of bridges. Besides, more severe conditions such as flood events make bridges more vulnerable to failure by scour hydraulic deficiencies [5]. Therefore, maintaining the safety levels of bridge piers under natural scour conditions is an important task to be carried out with precise estimations and predictions [6,7]. To prevent such unwanted events, usually the scour depth must not reach the pier foundation depth [8]. Thus, an accurate prediction of the maximum allowable scour depth for specific bridge piers can highly improve the level of bridge safety and ensure reliable design of pier foundations for bridges.

Over the past decades, several researchers have investigated the natural phenomenon called scour and its effects on the pier foundation of bridges. Here, some of the most important findings in this field are presented. For a given value of a pier width and the size of sediment after the scour peak, the maximum depth of the scour slightly decreases with velocity, as found by Chabert and Engeldinger [9] and confirmed using Laursen's [10] data. The latter study reports that there are no dependencies between the local scour depth and the contraction ratio before the overlap of scour holes to neighboring piers. The higher velocities increase the scour depth to more than the threshold condition, as reported by Jain and Fischer [11]. A lower depth of scour can be observed using shorter and higher forms of bed, as concluded by Chee [12]. Based on Melville's [13] studies, a strong dependency is found on bed formation, scour, and velocity depth, whereas the maximum scour depth occurs under transition flat-bed conditions for non-ripple-forming sands and at a threshold pick for ripple-forming sands. The scour depth equation is formulated using sediment parameters and flow intensity, as reported by Raudkivi [14].

All the above findings led to various experimental programs and numerical studies that resulted in several equations for modeling the maximum scour depth for a bridge pier. Among the various well-known formulations for modeling the maximum scour pier depth, we cite Kothyari et al. [15], Sheppard and Miller [16], Zhao et al. [17], Dixen et al. [18], Ettmer et al. [19], Hong et al. [20], and Radice and Lauva [21]. The developed models appear to have limited specific ranges of utilization based on different parameters such as flow intensity (V/Vc) [16]. Moreover, the deterministic form of these models cannot involve the related uncertainties that surround the scour depth prediction and the bridge pier [4]. Thus, a more efficient framework based on probabilistic analysis should be proposed to overcome these drawbacks.

As the modeling of the maximum depth of local scour is based on experimental and fitting techniques, the reliability of bridge piers under scour conditions, taking into account the surrounded uncertainties, presents a highly nonlinear form, in which accurate estimations of the failure probability need the utilization of robust and stable approaches [22]. In structural reliability analysis, conventional approaches utilized for probabilistic analysis are no longer efficient for a limit state function with highly nonlinear, complex, or high-dimensional problems [23,24]. Usually, structural reliability approaches can be categorized into two families: analytical-based approaches and simulation-based approaches. The first include the first-/second-order reliability method (FORM/SORM), in which the main challenge is to find the most probable point (MPP) for failure on the surface of the limit state function [25,26]. However, FORM has been proven by several studies to have lower accuracy and imprecise failure probability results whenever dealing with highly nonlinear problems with non-normal random variables, as the presented case of a bridge pier under scour conditions. Among simulation approaches, Monte Carlo simulation (MCS) is widely used to solve several reliability analysis problems in civil engineering. Despite the simplicity and straightforwardness of the method for all distribution types, the approximation in reliability using the MCS is considered computationally expensive, especially for a limit state function (LSF) with low probability or finite element method calls. To improve the MCS performance, several methods have been developed, such as subset simulation (SS), importance sampling (IS), directional simulation (DS), and line sampling (LS). These methods exhibit less computational costs compared to the original MCS in several fields. Although there are several structural reliability approaches used for probabilistic analysis, no studies have been conducted to investigate the abilities of simulation techniques for solving the complex problem of bridge priers under scour conditions. Consequently, providing a consistent investigation of the most suitable simulation techniques with stable and accurate reliability results for applicable complex reliability analysis of bridge priers under scour conditions can highly improve the safety of such structures and predict undesired failure events with more accuracy and robustness.

Considering the above discussion and arguments, the present paper attempts mainly to investigate the performance of several structural reliability methods for accurate and precise estimation of the failure probability for bridge piers under scour conditions. To achieve the goals, different simulation approaches are investigated, including Monte Carlo simulation (MCS), subset simulation (SS), importance sampling (IS), directional simulation (DS), and line sampling (LS). Moreover, the selected case study is local scour of a bridge pier in clay-sand-mixed sediments, whereas the scour depth estimation was modeled using the Muzzammil et al. [27] formula. Besides, the proposed limit state function includes several random variables to take into consideration the relevant uncertainties. In addition, the influence of various input variables on structural safety is investigated through a sensitivity analysis. The paper structure is as follows: Section 2 describes the current problem and formulates the limit state function of the local scour for bridge piers. Besides, Section 2 introduces various simulation techniques; the case study is reported in detail in Section 3. In addition, the application, results, and investigations are discussed as well in Section 3, and finally, findings and conclusions are presented in Section 4.

2. Materials and Methods

2.1. Scour Depth Prediction Models

In recent years, artificial-intelligence-based methods have been widely employed in many civil engineering fields for modeling different kinds of complex phenomena and problems [28–30]. The artificial intelligence (AI) approaches are considered a viable alternative to replace the excited models, in which these models are developed using classical methods [31,32]. Thus, using laboratory and experimental tests, AI methods are utilized for the prediction of scour depth. Sharafati et al. [33], Sreedhara et al. [34], Khan et al. [35], Odeyemi et al. [36], Danish et al. [37], and Muzzammil et al. [27] have carried out various studies on the scour depth around bridge piers under different bed conditions and developed several deterministic prediction models. In this study, the most recent correlation is employed for modeling the limit state function, as described in the following sections.

2.1.1. Deterministic Scour Depth Prediction Model

Debnath and Chaudhuri [2] investigated the effect of water content, clay content, and sand size on the local scour of cylindrical bridge piers. They used a substrate composed of sticky clay and sand with d_{50} = 180 mm in all their experiments. The tests were conducted in a flume that is 0.9 m wide, 0.9 m deep, and 18.5 m long, with a constant slope of 0.001. They also proposed a regression equation as a function of clay content, water content, pier Froude number, and bed shear strength to predict the maximum depth of scour (Equation (1)).

$$\frac{d_s}{D} = 8.2C^{-0.28}W_C^{0.15}F_r^{0.79}\hat{\tau}_s^{-0.38} \tag{1}$$

where d_s denotes the maximum scour depth (m), D is the pier diameter (m), C is the clay content, W_c is the water content, F_r represents the pier Froude number, while $\hat{\tau}_s$ represents a dimensionless factor of the bed shear strength. Muzzammil et al. [27], based on Debnath and Chaudhuri's [2] laboratory data, presented a more powerful model using the genetic expression programming (GEP) method to predict the depth of scour, which is given as follows:

$$\frac{d_s}{D} = 0.656 + 2F_r - 3C + W_s + \frac{1}{\hat{\tau}_s} \tag{2}$$

This model is used for further investigations of the failure probability of bridge piers under scour conditions in this work.

2.1.2. Probabilistic Scour Depth Prediction Model

The probabilistic assessment of each system is based on a limit state function (LSF). Therefore, if in a system, where $\boldsymbol{X} = [x_1, x_2, \ldots, x_n]^T$ is a vector of random variables affecting its resistance (R) and load (L), the LSF can be expressed using the following equation:

$$g(X) = R - L \tag{3}$$

If the load of the system exceeds its resistance, the system will fail, which is expressed mathematically as formulated in Equation (4):

$$P_f = P[g(\boldsymbol{X}) \le 0] = \int_{g(\boldsymbol{X}) \le 0} f(\boldsymbol{X})\, d\boldsymbol{X} \tag{4}$$

where $f(\boldsymbol{X})$ is the probability density function (PDF) of vector $\boldsymbol{X}$ of the random variables and $g(\boldsymbol{X}) \le 0$ represents the failure set. In this study, for the bridge scour problem, the LSF can be written in the form of Equation (5):

$$g(\boldsymbol{X}) = d_r - d_s \tag{5}$$

where d_r and d_s are the depth of the foundation and the maximum scour depth, respectively. The expression for d_s with a model correction factor (λ) in the LSF can be extracted as follows:

$$g(\boldsymbol{X}) = d_r - \lambda D\left(0.656 + 2F_r - 3C + W_S + \frac{1}{\hat{\tau}_s}\right) \tag{6}$$

where λ is the model correction factor expressed as the ratio of the observed-to-predicted scour depth [38].

2.2. *Review of the Simulation Methods for the Reliability Analysis*

This section briefly reviews the five examined simulation-based methods for solving the presented structural reliability problem in Section 2. These approaches include Monte Carlo simulation (MCS), importance sampling (IS), subset simulation (SS), line sampling (LS), and directional simulation (DS).

2.2.1. Monte Carlo Simulation (MCS)

Generally, the basis of simulation methods for accurate estimation of the probability of failure is the production of random samples in accordance with the distribution of the random variables for the problem at hand, where the response of the system is calculated for each set of random variables generated [39,40]. Monte Carlo simulation (MCS) is considered the most widely used reliability approach for solving problems in several fields. The method was proposed by Metropolis and Ulam [41] based on the coverage of all possible space by the produced samples [42]. In this method, random samples are produced based on the statistical distribution functions of various random variables, and then the LSF is assessed based on each set of samples, and the probability of system failure is calculated by dividing the number of states $g(\boldsymbol{X}) \le 0$ by the total number of sample sets (Equations (7) and (8)) [43].

$$P_f = P[g(X_1, X_2, \ldots,\ X_n) \le 0] = \frac{1}{N}\sum_{i=1}^{N} I(X_1, X_2, \ldots,\ X_n) \tag{7}$$

where N is the total number of simulations and $I(X_1, X_2, \ldots,\ X_n)$ is a function defined by:

$$I(X_1, X_2, \ldots,\ X_n) = \begin{cases} 1 & if\ g(X_1, X_2, \ldots,\ X_n) \le 0 \\ 0 & if\ \ g(X_1, X_2, \ldots,\ X_n) \le 0 \end{cases} \tag{8}$$

2.2.2. Importance Sampling (IS)

Importance sampling (IS) is a method based on the reduction of variance that has a significant effect on reducing the computational cost while using the MCS, in which the SS utilizes an alternative function $h(v)$ instead of the original distribution function of the random variables. In this method, selecting a suitable alternative function is of great importance. The probability of failure in the IS is calculated by the following equation [44]:

$$P_f = \int_{g(\boldsymbol{V})\le 0} \frac{I(V_1, V_2, \ldots, V_n) f(v)}{h(v)} h(v) dv = \frac{1}{N} \sum_{i=1}^{N} I[g(V_1, V_2, \ldots, V_n)] \frac{f_X(V_i)}{h_V(V_i)} \quad (9)$$

where $\boldsymbol{V}$ is the vector of a variable with the probability density function $h(v)$.

2.2.3. Subset Simulation (SS)

Subset simulation (SS) is one of the most efficient methods for calculating the probability of failure for engineering problems that have a small probability of failure [45]. In this method, the probability of failure is expressed by multiplying large amounts of conditional probabilities. Thus, the failure probability of the problem becomes a sequence of conditional events. In the simulation process, conditional samples are generated using the Markov chain and the Metropolis algorithm to cover the failure area [46]. According to Figure 1, if $b_1 > b_2 > \ldots > b_m = 0$ and the intermediate events are defined as $F_k = \{\boldsymbol{X} : g(\boldsymbol{X}) < b_k\}; (k = 1, 2, \ldots, m)$ [47,48]:

$$F_1 \supset F_2 \supset \ldots \supset F_m \quad (10)$$

$$F_1 \bigcap_{i=1}^{k} F_i \quad (11)$$

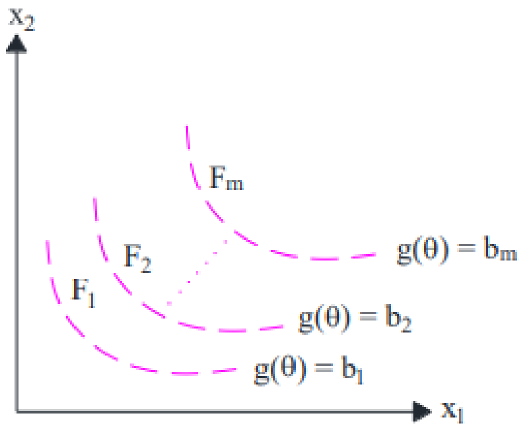

Figure 1. Subset simulation method [49].

Consequently, the probability of event F_k can be computed by the product of the conditional probabilities as follows:

$$P_f = P(F) = P(F_m|F_{m-1}) = \ldots = P(F_1) \prod_{i=1}^{m-1} P(F_{i+1}|F_i) \quad (12)$$

$$P_f = P(F) = P(F_m|F_{m-1}) = \ldots = P(F_1) \prod_{i=1}^{m-1} P(F_{i+1}|F_i) \quad (13)$$

2.2.4. Line Sampling (LS)

The basis of the line sampling method is transferring the variables to the normal standard space, finding the direction of the most probable point (MPP) and producing samples in the importance direction [50]. This method combines the reliability and simulation algorithms. If $\boldsymbol{a}$ is an important vector (Figure 2), the failure area is expressed as follows:

$$F = \left\{ \boldsymbol{X} \in \mathbb{R}^n : x_{\boldsymbol{a}} \in F_{\boldsymbol{a}}\left(X_1^{\perp}, \ldots, X_{n-1}^{\perp} \right) \right\} \tag{14}$$

where $x_{\boldsymbol{a}}$ is a realization of the random variable $\boldsymbol{X}_{\boldsymbol{a}}$, which is defined along $\boldsymbol{a}$; $x^{\perp} \in \mathbb{R}^{n-1}$ is a realization of a vector of random variables orthogonal to $\boldsymbol{a}$, denoted as $\boldsymbol{X}^{\perp}$; and $F_{\boldsymbol{a}}$ is a function representing the failure domain along $\boldsymbol{a}$, defined on $\mathbb{R}^{n-1}$. Then, P_f can be expressed as follows:

$$P_f = \int_{\mathbb{R}^n} I_F(\boldsymbol{X}) \phi_Z(\boldsymbol{X}) d\boldsymbol{X} = E_{x^{\perp}}\left[\Phi\left(F_{\boldsymbol{a}}\left(\boldsymbol{X}^{\perp} \right) \right) \right] \tag{15}$$

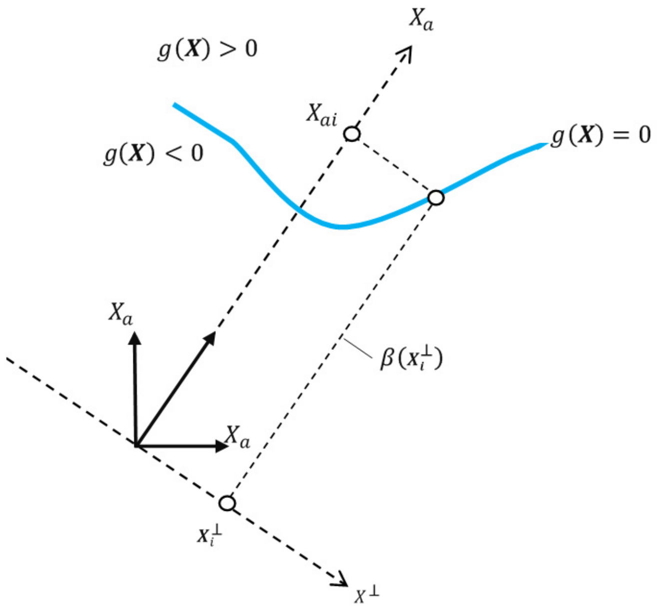

Figure 2. Line sampling (LS) method [51].

In the case that $F_{\boldsymbol{a}}\left(\boldsymbol{X}^{\perp}\right)$ lies within the half open interval $[\beta\left(\boldsymbol{X}^{\perp}\right), \infty)$, the one-dimensional conditional failure probability can be evaluated as $\Phi(F_{\boldsymbol{a}}\left(\boldsymbol{X}^{\perp}\right) = \Phi\left(-\beta\left(\boldsymbol{X}^{\perp}\right)\right)$, where $\beta\left(\boldsymbol{X}^{\perp}\right)$ is a reliability index, as indicated in Figure 2. An unbiased estimate of P_f is calculated on a set of samples $\left\{ \boldsymbol{X}_1^{\perp} \sim \phi_{\boldsymbol{X}^{\perp}}\left(\boldsymbol{X}^{\perp}\right) : i = 1, \ldots, N \right\}$ as:

$$\hat{P}_f = \frac{1}{N} \sum_{i=1}^{N} \Phi(F_{\boldsymbol{a}}\left(\boldsymbol{X}_i^{\perp}\right) = \frac{1}{N} \sum_{i=1}^{N} \Phi(-\beta\left(\boldsymbol{X}_i^{\perp}\right) = \frac{1}{N} \sum_{i=1}^{N} P_{F_i} \tag{16}$$

where $P_{F_i} = \Phi\left(-\beta\left(\boldsymbol{X}_i^{\perp}\right)\right)$.

2.2.5. Directional Sampling (DS)

In the DS method, the variables are transferred to a Cartesian coordinate system. The idea of the DS approach is to use directions instead of samples. If $\boldsymbol{U}$ is an n-dimensional vector with Gaussian distribution as $\boldsymbol{U} = \boldsymbol{RA}$ ($\boldsymbol{R} > 0$), where $\boldsymbol{A}$ is a random unit vector uniformly distributed on the unit sphere, R^2 is a chi-square-distributed random variable with n degrees of freedom. Conditioning on $\boldsymbol{A} = \boldsymbol{a}$, the probability of failure can be written as [50]:

$$P_f = \int_{a \in \Omega_n} P\{g(\boldsymbol{RA}) \leq 0 | \boldsymbol{A} = \boldsymbol{a}\}\, f_{\boldsymbol{A}}(\boldsymbol{a}) d\boldsymbol{a} = \int_{a \in \Omega_n} P\{g(\boldsymbol{Ra}) \leq 0\}\, f_{\boldsymbol{A}}(\boldsymbol{a}) d\boldsymbol{a} \tag{17}$$

where $f_A(\boldsymbol{a})$ is the probability function density of $\boldsymbol{A}$ on the unit sphere. The failure probability (P_f) is now calculated by performing N simulations of the unit vector $\boldsymbol{A}$, and then, with the sample values $p_i = P\{g(\boldsymbol{Ra}) \leq 0\}$, P_f is obtained by the unbiased estimator:

$$P_f \approx E[P_F] = \frac{1}{N}\sum_{i=1}^{N} p_i \tag{18}$$

3. Implementation, Results, and Discussions

3.1. Case Study

To evaluate the previous described reliability methods mentioned above, the database of Debnath and Chaudhuri [2] was utilized in this study. The statistical characteristics of the random variables are reported in Table 1. Given that the reliability of a system is expressed in the form of the reliability index (β) in various studies, here the reliability index based on the studied reliability methods is examined for different safety factor values. The relationship between the reliability index (β) and the probability of failure (P_f) for a given vector of random variables, $\boldsymbol{X}$, is expressed as follows:

$$P_f = \Phi(-\beta) \tag{19}$$

Table 1. Distribution properties of the utilized input random variables [3,4].

Variables	Symbol	Unit	$X_{\mathbf{mean}}$	COV	Distribution
Model correction factor	λ	-	1.00	0.06	Normal
Pier diameter	D	m	0.09	0.05	Lognormal
Pier Froude number	F_r	-	0.29	0.003	Normal
Clay fraction	C	-	0.19	0.010	Lognormal
Water content	W_c	-	0.22	0.002	Normal
Bed shear strength	τ_s	$\left(\frac{\mathrm{N}}{\mathrm{m}^2}\right)$	5271.42	0.06	Lognormal

CoV = Coefficient of variation.

The safety factor is also a traditional parameter for expressing the safety of the system against design uncertainties. Thus, the safety factor of scour is defined as the ratio of the maximum scour depth to the depth of the foundation.

$$SF = \frac{d_f}{d_s} \tag{20}$$

3.2. Reliability Methods Performance

In assessing the reliability analysis of a complex problem, the first step after defining the limit state function and determining the distribution functions of the input random variables is to select the appropriate setting parameters or the number of samples required for the simulation process for each reliability method. The selected values of simulation in each method (i.e., MCS, SS, IS, LS, and DS) in this study are presented in Table 2. Besides, the diagram of convergence for the MCS is shown in Figure 3. According to the reported values in Table 2 and Figure 3, the MCS needs more than one million simulations to achieve accurate results of the failure probability of the presented problem of a bridge pier under local scour conditions. In contrast, a suggested number of 1000, 6000, 100, and 150 simulations were attributed to the IS, SS, LS, and DS, respectively, in order to reduce the computational cost burden compared to the one provided by the MCS. It is worth mentioning that these selected values in Table 2 are obtained by the same process as that illustrated in Figure 3 for all reliability methods, where the number of simulations is increased until the probability of failure value stabilizes. Thus, this procedure indicates the highly delicate nature and sensitivity of the reliability analysis process to the selection of a simulation number for accurate failure probability results.

Table 2. The required simulation number for reliability analysis.

Methods	Number of Simulations
Monte Carlo simulation (MCS)	1,000,000
Importance sampling (IS)	1000
Subset simulation (SS)	6000
Line sampling (LS)	100
Directional simulation (DS)	150

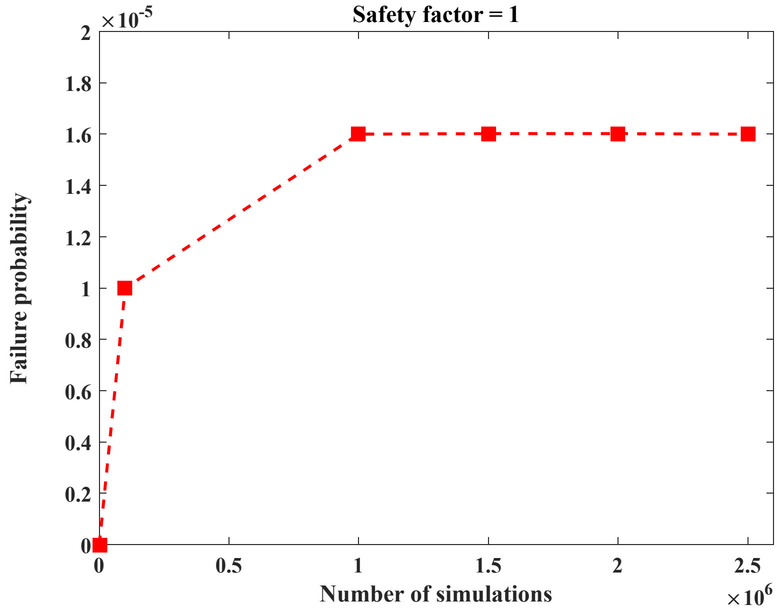

Figure 3. The coverage curve of the MCS.

After the optimum selection of a suitable number of simulation samples for each reliability method, the reliability analysis of a local scour at a bridge pier formulated using the LSF in Equation (6) was performed using MCS, where the safety factor takes values in the range between 1 and 1.5 with a separated distance of 0.1. The obtained reliability analysis results are presented in Figure 4. It is obvious that the results clearly show a decrease in P_f values with an increase in safety factor values. Moreover, it can be indicated that the value of the reliability index (β) increased from 4.15 with a safety factor value equal to 1 to 4.52 with a safety factor equal to 1.5.

In the second step, based on the results obtained by using the MCS, the other reliability methods (i.e., SS, IS, DS, and LS) were evaluated for different safety factor values, and their performance results compared to the ones by the MCS are reported in Table 3 and shown in Figures 5–8. According to the estimated results by the proposed simulation methods, it is clear that both the DS (Figure 5) and the LS (Figure 6) failed to estimate the probability of failure values of the studied problem due to the complexity and nonlinearity of the LSF (Equation (6)). Note that in these two methods, it is very important to find the optimum values of the direction vector $\boldsymbol{a}$, and generally, as the complexity of the problem increases, it becomes very difficult to obtain an optimum value of the direction vector $\boldsymbol{a}$ for these methods, which is considered one of their major weaknesses. Moreover, the IS (Figure 7) has proven in some cases to be unable to make a good estimation of the probability of failure, as in the case of a safety factor value equal to 1.1, in which the obtained probability of failure by the IS is equal to 1.84×10^{-5}, while using the MCS it is 1.20×10^{-5}. However, the IS shows poor estimations compared to almost all the other cases compared to the MCS. Among the proposed simulation methods for the reliability analysis of a bridge pier under local scour conditions, the SS (Figure 8) is the only one that has been able to produce a good approximation of the failure probability for almost all the safety factor values. As an example of the reported results in Table 3 and Figure 8, the P_f using the SS for $SF = 1.4$ is $P_f = 2.90 \times 10^{-6}$, while using the MCS, $P_f = 5.00 \times 10^{-6}$, which is relatively very close.

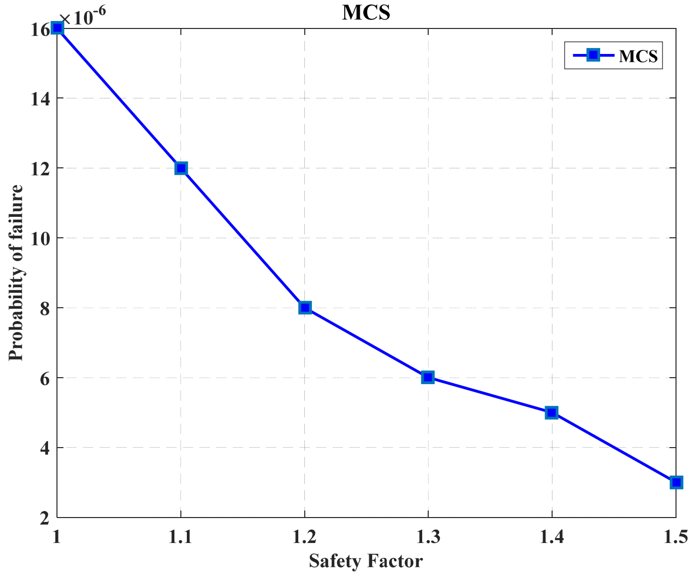

Figure 4. Variation of the probability of failure versus the safety factor using the MCS.

Table 3. Reliability index and failure probability values for different safety factor values using all proposed simulation methods.

Safety Factor	1		1.1		1.2		1.3		1.4		1.5	
	P_f	β	P_f	β	P_f	β	P_f	β	P_f	β	P_f	β
MCS	1.60×10^{-5}	4.1587	1.20×10^{-5}	4.224	8.00×10^{-6}	4.3145	6.00×10^{-6}	4.3776	5.00×10^{-6}	4.4172	3.00×10^{-6}	4.5264
DS	0.076985	1.43	9.02×10^{-2}	1.3394	1.02×10^{-1}	1.2711	6.42×10^{-2}	1.5203	7.13×10^{-2}	1.466	0.043372	1.7128
LS	1.05×10^{-5}	4.2546	1.05×10^{-5}	4.2546	1.05×10^{-5}	4.2546	1.05×10^{-5}	4.2546	1.05×10^{-5}	4.2546	1.05×10^{-5}	4.2546
IS	3.25×10^{-5}	3.994	1.84×10^{-5}	4.1268	9.51×10^{-5}	3.7318	6.52×10^{-5}	3.8257	1.46×10^{-5}	4.1802	1.69×10^{-5}	4.1464
SS	1.85×10^{-5}	4.126	6.36×10^{-6}	4.1643	7.67×10^{-6}	4.2749	7.56×10^{-6}	4.3269	2.90×10^{-6}	4.3954	2.85×10^{-6}	4.4727

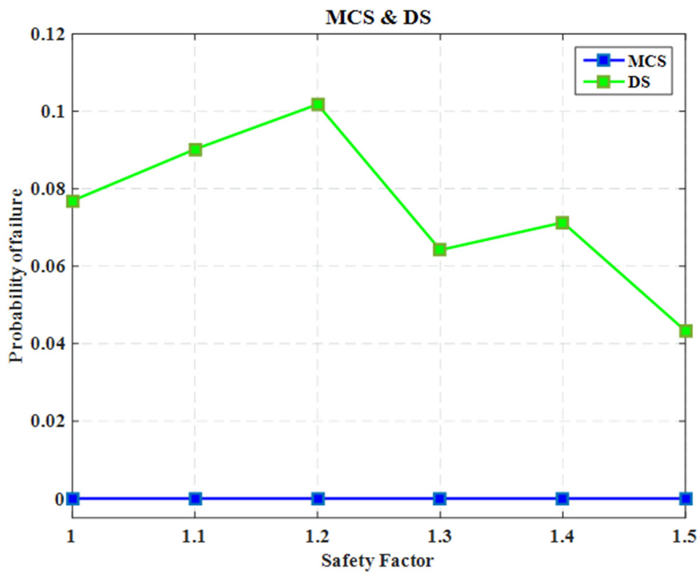

Figure 5. Variation of the probability of failure versus the safety factor using the DS.

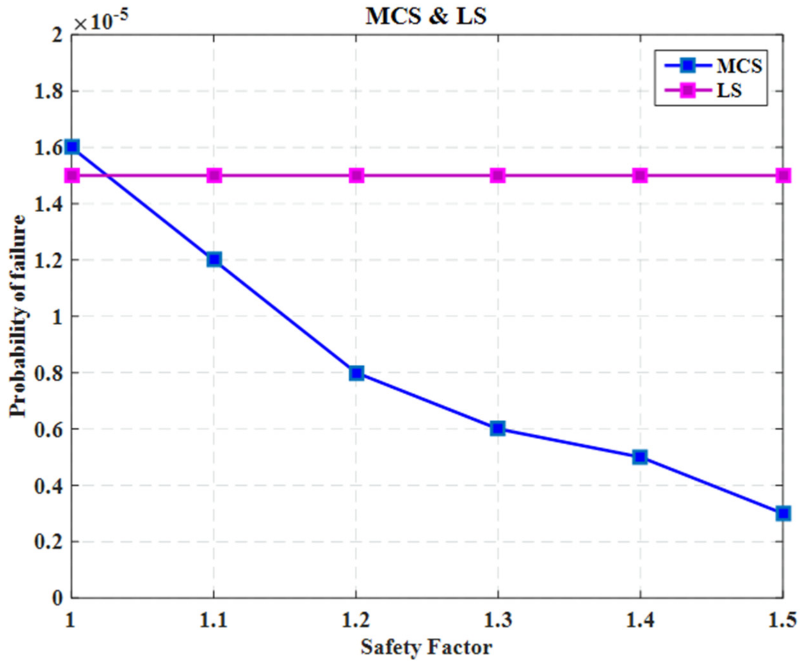

Figure 6. Variation of the probability of failure versus the safety factor using the LS.

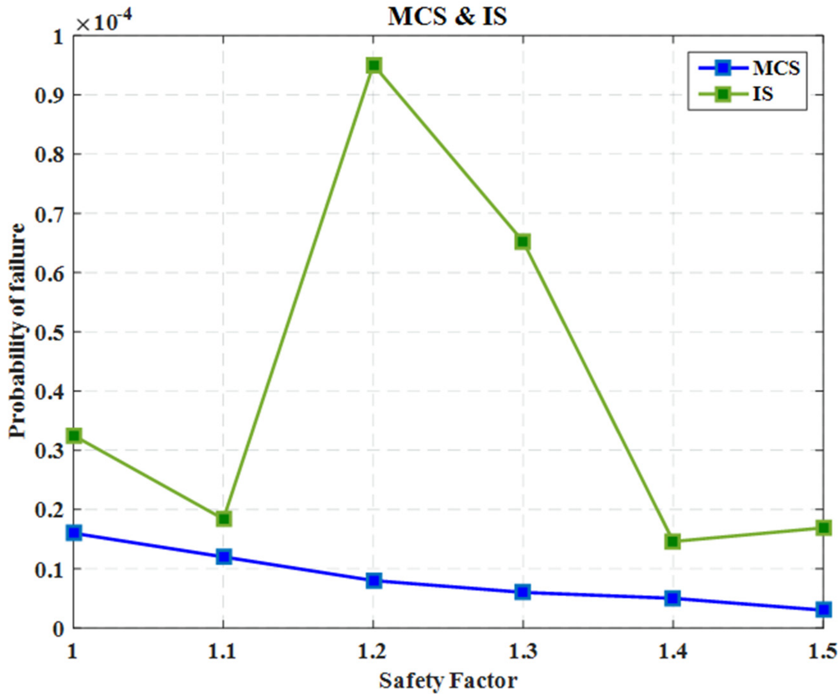

Figure 7. Variation of the probability of failure versus the safety factor using the IS.

In addition, the calculated relative error (%) based on the reliability index between the proposed simulation and MCS methods was also investigated and is presented in Table 4. It can be seen from the tabulated results that the DS yielded the highest relative errors, which is mostly higher than 60% of all safety factor values, while the LS failed to estimate the reliability index (β) that yielded the same value for all safety factor cases (i.e., $\beta = 4.2546$). The IS revealed an average relative error between 2.3% and 13.51%, which is considered moderate performance. Finally, it can be seen that the SS method was the most efficient simulation method compared to the others, where the highest yielded relative error was equal to 1.41%. Thus, it can be concluded that the SS manages to solve the complexities for the problem of the local scour of a bridge pier with high accuracy as the MCS, with only 6000 simulations instead of one million using the MCS.

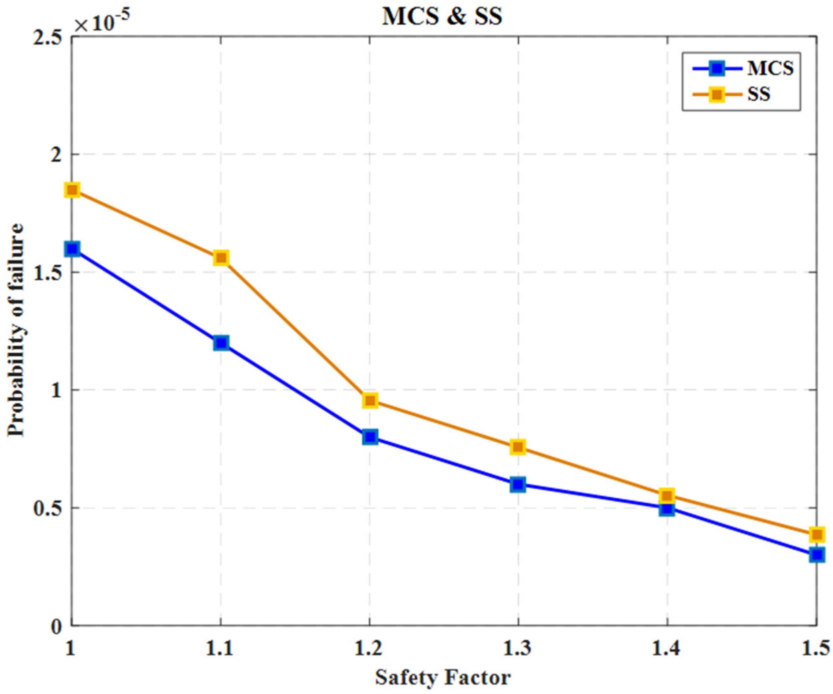

Figure 8. Variation of the probability of failure versus the safety factor using the SS.

Table 4. Comparative percentage relative error of the proposed simulation methods and the MCS for different safety factor values.

Method	DS		LS		IS		SS	
Safety Factor	β	Error (%)	β	Error (%)	β	Error (%)	β	Error (%)
1	1.43	65.61	4.2546	−2.31	3.994	3.96	4.126	0.79
1.1	1.3394	68.29	4.2546	−0.72	4.1268	2.30	4.1643	1.41
1.2	1.2711	70.54	4.2546	1.39	3.7318	13.51	4.2749	0.92
1.3	1.5203	65.27	4.2546	2.81	3.8257	12.61	4.3269	1.16
1.4	1.466	66.81	4.2546	3.68	4.1802	5.37	4.3954	0.49
1.5	1.7128	62.16	4.2546	6.00	4.1464	8.40	4.4727	1.19

3.3. Uncertainty Effect of Scouring Parameters

After the evaluation of the proposed simulation methods for solving the problem of the local scour of a bridge pier, the influence of the different parameters included in the LSF (Equation (6)) formulation on the reliability index was carried out. The influence of the scouring parameters on the reliability index was investigated for variations in a range from 0.5 to 1.5 (i.e., 50–150%). These parameters include the model's correction factor (λ), pier diameter (D), Froude number (F_r), clay fraction (C), water content (W_c), and bed shear strength (τ_s). The results of the sensitivity analysis against uncertainties are plotted in Figure 9. From the results, it should be clear that the reliability index value decreases with an increase in the uncertainties from 0.5 to 1.5 for the Froude number, bed shear strength, and water content. Based on the reported analysis results, it can be seen that the Froude number has the greatest impact on the reliability index compared to the other input variables. In this way, with increasing uncertainty of F_r from 0.5 to 1.5, the reliability index decreased from 4.26 to 4.14. In other words, the scour depth increases with an increase in the Froude number. Thereafter, the bed shear strength, clay fraction, and water content affect the bridge pier, causing failure. The bed shear strength and the pier diameter have a similar behavior to the Froude number with regard to their uncertainty in scouring. In contrast, with uncertainty increasing in the clay fraction, the reliability index slightly increases, so at first, with an uncertainty of 0.5, the reliability is 4.2, and in the end, with an uncertainty of 1.5, it is equal to 4.3.

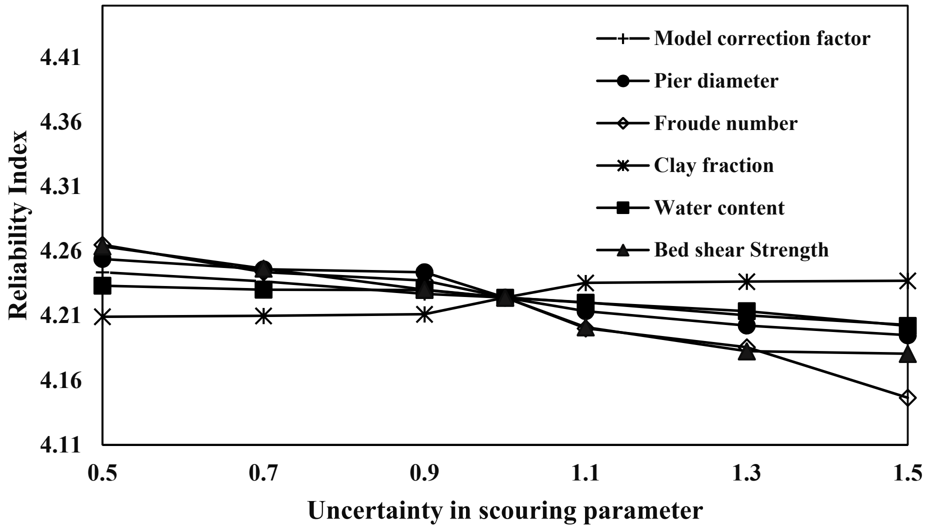

Figure 9. Influence of bridge pier scour parameters on reliability index values.

4. Conclusions

Maintaining the safety levels of bridge piers under local scour conditions is an important task, especially in clay-sand-mixed sediments. Thus, the present work mainly focused on the appropriate selection of structural reliability approaches for accurate and precise estimation of the failure probability of these structures. A probabilistic analysis of local scour at a bridge pier in clay-sand-mixed sediments using five simulation methods was conducted. These approaches included Monte Carlo simulation as a reference, subset simulation (SS), directional sampling (DS), importance sampling (IS), and line sampling (LS). Moreover, a large database of real experimental data that comprised 250 real samples was employed. Several findings and remarks are as follows:

- The reliability index increases with an increase in the safety factor, whereas an increase in the safety factor causes a decrease in failure probability.
- The results showed that subset simulation (SS) has excellent performance for the accurate and precise estimation of failure probability compared to the IS, LS, and DS. The SS yielded the lowest relative error, relative to MCS, in which the highest error was 1.41% for a safety factor between 1 and 1.5.
- The DS showed the lowest performance for solving this engineering problem, with the highest relative error of 70.54% (SF = 1.2) compared to other simulation methods. In addition, the LS failed to deal with this current problem and was unable to approximate the reliability index for different safety factor values.
- The sensitivity analysis revealed that the model correction factor and water content are resisting factors related to reliability. The Froude number was found to be a dominant parameter related to the bridge pier failure.

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Abbreviations

d_s:	Maximum scour depth (m)
D:	Pier diameter (m)
C:	Clay content
$\boldsymbol{W_C}$:	Water content
F_r:	Pier Froude number
$\boldsymbol{\tau_s}$:	Bed shear strength
$\boldsymbol{\hat{\tau}_s}$:	Dimensionless factor of the bed shear strength ($\frac{\tau_s}{\rho V^2}$)
ρ:	Mass density of water
V:	Depth averaged velocity
LSF:	Limit state function
$\boldsymbol{P_f}$:	Failure probability
$\boldsymbol{d_r}$:	Depth of foundation
MCS:	Monte Carlo simulation
DS:	Directional simulation
LS:	Line sampling
SS:	Subset simulation
IS:	Importance sampling
SF:	Safety factor
$\boldsymbol{f(X)}$:	Probability density function

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