MHD EFFECTS ON NANOFLUID WITH ENERGY AND HYDROTHERMAL BEHAVIOR BETWEEN TWO COLLATERAL PLATES
Application of New Semi Analytical Technique

by

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In this study, heat and mass transfer characteristic of unsteady nanofluid flow between parallel plates is investigated. The important effect of Brownian motion and thermophoresis has been included in the model of nanofluid. The governing equations are solved via differential transformation method. The validity of this method was verified by comparison previous work which is done for viscous fluid. The analytical investigation is carried out for different governing parameters namely: the squeeze number, Hartmann number, Schmidt number, Brownian motion parameter, thermophoretic parameter, and Eckert number. The results indicate that skin friction coefficient has direct relationship with Hartmann number and squeeze number. Also it can be found that Nusselt number increases with increase of Hartmann number, Eckert number, and Schmidt number but it is decreases with augment of squeeze number.

Key words: *magnetohydrodynamic, nanofluid, Brownian motion, Eckert number, thermophoresis, differential transformation method, Schmidt number*

Introduction

Control of heat transfer in many energy systems is crucial due to the increase in energy prices. In recent years, nanofluid technology is proposed and studied by some researchers experimentally or numerically to control heat transfer in a process. The nanofluid can be applied to engineering problems, such as: heat exchangers, cooling of electronic equipment, and chemical processes. Almost all of the researchers assumed that nanofluid treated as the common pure fluid and conventional equations of mass, momentum, and energy are used and the only effect of nanofluid is its thermal conductivity and viscosity which are obtained from the theoretical models or experimental data. Abu-Nada *et al*. [1] investigated natural convection heat transfer enhancement in horizontal concentric annuli field by nanofluid. They found that for low Rayleigh numbers, nanoparticles with higher thermal conductivity cause more enhancement in heat transfer. Jou and Tzeng [2] numerically studied the natural convection heat transfer enhancements of nanofluids within a 2-D enclosure. They analyzed heat transfer performance using Khana-Fer's model for various parameters, like volume fraction, Grashof number, and aspect ratio of the enclosure. Results showed that increasing the buoyancy parameter and volume fraction

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of nanofluids cause an increase in the average heat transfer coefficient. Rashidi *et al.* [3] considered the analysis of the second law of thermodynamics applied to an electrically conducting incompressible nanofluid fluid flowing over a porous rotating disk. They concluded that using magnetic rotating disk drives has important applications in heat transfer enhancement in renewable energy systems and industrial thermal management. Sheikholeslami and Rashidi [4] studied the effect space dependent magnetic field on free convection of Fe_3O_4-water nanofluid. They showed that Nusselt number decreases with increase of Lorentz forces. Sheikholeslami *et al.* [5] applied LBM to simulate 3-D nanofluid flow and heat transfer in presence of magnetic field. They indicated that adding magnetic field leads to decrease in rate of heat transfer. Recently, several authors used nanofluid and other passive methods in order to enhance rate of heat transfer [6-36].

All the previous studies assumed that there are not any slip velocities between nanoparticles and fluid molecules and assumed that the nanoparticle concentration is uniform. It is believed that in natural convection of nanofluids, the nanoparticles could not accompany fluid molecules due to some slip mechanisms such as Brownian motion and thermophoresis, so the volume fraction of nanofluids may not be uniform anymore and there would be a variable concentration of nanoparticles in a mixture. Nield and Kuznetsov [37] studied the natural convection in a horizontal layer of a porous medium saturated by a nanofluid. The analysis reveals that for a typical nanofluid (with large Lewis number) the prime effect of the nanofluids is via a buoyancy effect coupled with the conservation of nanoparticles, the contribution of nanoparticles to the thermal energy equation being a second-order effect. Khan and Pop [38] published a paper on boundary-layer flow of a nanofluid past a stretching sheet as a first paper in that field. Their model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. They indicated that the reduced Nusselt number is a decreasing function of each dimensionless number. Sheikholeslami and Abelman [39] used two phase simulation of nanofluid flow and heat transfer in an annulus in the presence of an axial magnetic field. Recently several authors used two phase model in their studies [40-43].

The study of heat and mass transfer unsteady squeezing viscous flow between two parallel plates in motion normal to their own surfaces independent of each other and arbitrary with respect to time has been regarded as one of the most important research topics due to its wide spectrum of scientific and engineering applications such as hydro-dynamical machines, polymer processing, lubrication system, chemical processing equipment, formation and dispersion of fog, damage of crops due to freezing, food processing, and cooling towers. The first work on the squeezing flow under lubrication approximation was reported by Stefan [44]. Mahmood *et al.* [45] investigated the heat transfer characteristics in the squeezed flow over a porous surface.

One of the semi-exact methods which do not need small parameters is the differential transformation method (DTM). This method constructs an analytical solution in the form of a polynomial. It is different from the traditional higher order Taylor series method. The Taylor series method is computationally expensive for large orders. The DTM is an alternative procedure for obtaining an analytic Taylor series solution of differential equations. The main advantage of this method is that it can be applied directly to non-linear differential equations without requiring linearization, discretization and therefore, it is not affected by errors associated to discretization. The concept of DTM was first introduced by Zhou [46], who solved linear and non-linear problems in electrical circuits. Jang *et al.* [47] applied the 2-D DTM to the solution of partial differential equations. Analytical and numerical methods were successfully applied to various application problems [48, 49].

The main purpose of this study is to investigate the problem of unsteady nanofluid flow between parallel plates using DTM. The influence of the squeeze number, Hartmann number, Schmidt number, Brownian motion parameter, thermophoretic parameter, and Eckert number on temperature and concentration profiles is investigated.

Governing equations

Heat and mass transfer analysis in the unsteady 2-D squeezing flow of nanofluid between the infinite parallel plates is considered, fig. 1. The two plates are placed at $\ell(1-\gamma t)^{1/2} = h(t)$. When $\gamma > 0$ the two plates are squeezed until they touch $t = 1/\gamma$ and for $\gamma < 0$ the two plates are separated. The viscous dissipation effect, the generation of heat due to friction caused by shear in the flow, is retained. Also, it is also assumed that the uniform magnetic field ($\vec{B} = B\vec{e}_y$) is applied, where $\vec{e}_y$ is unit vectors in the Cartesian co-ordinate system. The electric current J and the electromagnetic force F are defined by $J = \sigma(\vec{V}\times\vec{B})$ and $F = \sigma(\vec{V}\times\vec{B})\times\vec{B}$, respectively.

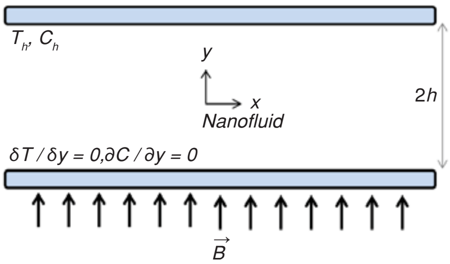


Figure 1. Geometry of problem

The governing equations for mass, momentum, energy, and mass transfer in unsteady 2-D flow of nanofluid are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\partial \tag{1}$$

$$\rho_f\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial v} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \sigma B^2 u \tag{2}$$

$$\rho_f\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu}{(\rho c_p)_f}\left[4\left(\frac{\partial u}{\partial x}\right)^2\right] + \frac{(\rho c_p)_p}{(\rho c_p)_f}\left[\begin{matrix} D_B\left\{\frac{\partial C}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right\} \\ + \frac{D_T}{T_c}\left\{\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right\}\end{matrix}\right] \tag{4}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \left(\frac{D_T}{T_c}\right)\left\{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right\} \tag{5}$$

Here u and v are the velocities in the x- and y-directions respectively, T – the temperature, C – the concentration, p – the pressure, ρ_f – the base fluid density, μ – the dynamic viscosity, k – the thermal conductivity, c_p – the specific heat of nanofluid, and D_B – the diffusion coefficient of the diffusing species. The relevant boundary conditions are:

$$\begin{aligned} &C = 0, \quad v = v_w = \frac{\mathrm{d}h}{\mathrm{d}t} \quad T = T_\mathrm{H}, \quad C = C_\mathrm{H} \quad at\ y = h(t) \\ &v = \frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = \frac{\partial C}{\partial y} = 0 \qquad at\ y = 0 \end{aligned} \tag{6}$$

We introduce these parameters:

$$\left.\begin{aligned} \eta=\frac{y}{[\ell\sqrt{1-\gamma t}\,]}, \qquad u=\frac{\gamma x}{[2(1-\gamma t)]}f'(\eta),\\ v=-\frac{\gamma\ell}{[2\sqrt{1-\gamma t}\,]}f(\eta), \quad \theta=\frac{T}{T_{\mathrm{H}}},\\ \phi=\frac{C}{C_{\mathrm{h}}} \end{aligned}\right\} \tag{7}$$

Substituting the previous variables into eqs. (2) and (3) and then eliminating the pressure gradient from the resulting equations give:

$$f^{iv}-S\left(\eta f'''+3f''+f'f''-ff'''\right)-\mathrm{Ha}^2 f''=0 \tag{8}$$

Using eq. (7), eqs. (4) and (5) reduce to the following differential equations:

$$\theta''+\Pr S(f\theta'-\eta\theta')+\Pr\mathrm{Ec}(f''^2)+N_b\,\phi'\theta'+N_t\,\theta'^2=0 \tag{9}$$

$$\phi''+S\,\mathrm{Sc}(f\phi'-\eta\phi')+\frac{N_t}{N_b}\theta''=0 \tag{10}$$

With these boundary conditions:

$$\left.\begin{aligned} f(0)=0, \qquad f''(0)=0, \quad \theta'(0)=0,\\ \phi'(0)=0, \qquad f(1)=1, \qquad f'(1)=0,\\ \theta(1)=\phi(1)=1, \end{aligned}\right\} \tag{11}$$

where S is the squeeze number, Pr – the Prandtl number, Ec – the Eckert number, Sc – the Schmidt number, Ha – Hartman number of nanofluid, N_b – the Brownian motion parameter, and N_t – the thermophoretic parameter, which are defined:

$$\left.\begin{aligned} S=\frac{\beta l^2}{2\mu}\rho_{\mathrm{f}}, \quad \Pr=\frac{\mu}{\rho_{\mathrm{f}}\alpha}, \quad \mathrm{Ec}=\frac{1}{c_p}\left(\frac{\beta x}{2(1-\beta t)}\right)^2\\ \mathrm{Sc}=\frac{\mu}{\rho_{\mathrm{f}}D}, \quad \mathrm{Ha}=\ell B\sqrt{\frac{\sigma}{\mu}(1-\beta t)},\\ N_b=(\rho c)_p D_B(C_{\mathrm{h}})/[(\rho c)_{\mathrm{f}}\alpha]\ ,\\ N_t=(\rho c)_p D_T(T_{\mathrm{H}})/[(\rho c)_{\mathrm{f}}\alpha T_{\mathrm{c}}]. \end{aligned}\right\} \tag{12}$$

Nusselt number is defined:

$$\mathrm{Nu}=\frac{-\ell k\left(\dfrac{\partial T}{\partial y}\right)_{y=h(t)}}{kT_{\mathrm{H}}} \tag{13}$$

In terms of eq. (7), we obtain:

$$\mathrm{Nu}^{*} = \sqrt{1-\alpha t}\,\mathrm{Nu} = -\theta'(1) \tag{14}$$

Differential transform method

Basic of DTM

Basic definitions and operations of differential transformation are introduced as follows. Differential transformation of the function $f(\eta)$ is defined:

$$F(k) = \frac{1}{k!}\left[\frac{\mathrm{d}^k f(\eta)}{\mathrm{d}\eta^k}\right]_{\eta=\eta_0} \tag{15}$$

where $f(\eta)$ is the original function and $F(k)$ is the transformed function which is called the T-function (it is also called the spectrum of the $f(\eta)$ at $\eta = \eta_o$, in the k domain). The differential inverse transformation of $F(k)$ is defined:

$$f(\eta) = \sum_{k=0}^{\infty} F(k)(\eta-\eta_0)^k \tag{16}$$

by combining eqs. (15) and (16) $f(\eta)$ can be obtained:

$$f(\eta) = \sum_{k=0}^{\infty}\left[\frac{\mathrm{d}^k f(\eta)}{\mathrm{d}\eta^k}\right]_{\eta=\eta_0}\frac{(\eta-\eta_0)^k}{k!} \tag{17}$$

Equation (17) implies that the concept of the differential transformation is derived from Taylor's series expansion, but the method does not evaluate the derivatives symbolically. However, relative derivatives are calculated by an iterative procedure that is described by the transformed equations of the original functions. From the definitions of eqs. (15) and (16), it is easily proven that the transformed functions comply with the basic mathematical operations shown below. In real applications, the function $f(\eta)$ in eq. (17) is expressed by a finite series and can be written:

$$f(\eta) = \sum_{k=0}^{N} F(k)(\eta-\eta_0)^k \tag{18}$$

Equation (4) implies that:

$$f(\eta) = \sum_{k=N+1}^{\infty}\left[F(k)(\eta-\eta_0)^k\right]$$

is negligibly small, where N is series size.

Theorems to be used in the transformation procedure, which can be evaluated from eqs. (15) and (16), are given in tab. 1.

Table 1. Some of the basic operations of DTM

Original function	Transformed function
$f(\eta) = \alpha g(\eta) \pm \beta h(\eta)$	$F[k] = \alpha G[k] \pm \beta H[k]$
$f(\eta) = \dfrac{\mathrm{d}^n g(\eta)}{\mathrm{d}\eta^n}$	$F[k] = \dfrac{(k+n)!}{k!}G[k+n]$
$f(\eta) = g(\eta)h(\eta)$	$F[k] = \sum_{m-0}^{k} F[m]H[k-m]$
$f(\tau) = \sin(\varpi\eta+\alpha)$	$F[k] = \dfrac{\varpi^k}{k!}\sin\left(\dfrac{\pi k}{2}+\alpha\right)$
$f(\tau) = \cos(\varpi\eta+\alpha)$	$F[k] = \dfrac{\varpi^k}{k!}\cos\left(\dfrac{\pi k}{2}+\alpha\right)$
$f(\eta) = \mathrm{e}^{\lambda\eta}$	$F[k] = \dfrac{\lambda^k}{k!}$
$F(\eta) = (1+\eta)^m$	$F[k] = \dfrac{m(m-1)...(m-k+1)}{k!}$
$f(\eta) = \eta^m$	$F[k] = \delta(k-m) = \begin{cases} 1, k = m \\ 0, k \neq m \end{cases}$

Solution with DTM

Now DTM into governing equations has been applied. Taking the differential transforms of eqs. (8)-(10) with respect to χ and considering $h = 1$ gives:

$$\left.\begin{array}{c}(k+1)(k+2)(k+3)(k+4)F[k+4]+S\sum_{m=0}^{k}\left\{\Delta[k-m-1](m+1)(m+2)(m+3)F[m+3]\right\}-\\ -3S(k+1)(k+2)F[k+2]-S\sum_{m=0}^{k}\left\{[k-m+1]F[k-m+1](m+1)(m+2)F[m+2]\right\}+\\ +S\sum_{m=0}^{k}\left\{F[k-m](m+1)(m+2)(m+3)F[m+3]\right\}-\mathrm{Ha}^2(k+1)(k+2)F[k+2]=0\\ \Delta[m]=\begin{cases}1 & m=1\\ 0 & m\neq 1\end{cases}\end{array}\right\}\tag{19}$$

$$F[0]=0,\ F[1]=a_1,\ F[2]=0,\ F[3]=a_2\tag{20}$$

$$\left.\begin{array}{c}(k+1)(k+2)\Theta[k+2]+\Pr S\sum_{m=0}^{k}\left\{F[k-m](m+1)\Theta[m+1]\right\}-\\ -\Pr S\sum_{m=0}^{k}\left\{\Delta[k-m](m+1)\Theta[m+1]\right\}+\\ +\Pr \mathrm{Ec}\sum_{m=0}^{k}\left\{(k-m+1)(k-m+2)F[k-m+2](m+1)(m+2)F[m+2]\right\}+\\ +N_b\sum_{m=0}^{k}\left\{\Phi[k-m](m+1)\Theta[m+1]\right\}+N_t\sum_{m=0}^{k}\left\{\Theta[k-m](m+1)\Theta[m+1]\right\}=0\\ \Delta[m]=\begin{cases}1 & m=1\\ 0 & m\neq 1\end{cases}\end{array}\right\}\tag{21}$$

$$\Theta[0]=a_3,\ \Theta[1]=0\tag{22}$$

$$\left.\begin{array}{c}(k+1)(k+2)\Phi[k+2]+\mathrm{Sc}\,S\sum_{m=0}^{k}\left\{F[k-m](m+1)\Phi[m+1]\right\}+\\ +\mathrm{Sc}\,S\sum_{m=0}^{k}\left\{\Delta[k-m](m+1)\Phi[m+1]\right\}+\dfrac{N_t}{N_b}(k+1)(k+2)\Theta[k]=0\\ \Delta[m]=\begin{cases}1 & m=1\\ 0 & m\neq 1\end{cases}\end{array}\right\}\tag{23}$$

$$\Phi[0]=a_4,\ \Phi[1]=0\tag{24}$$

where $F[k]$, $\Theta[k]$, and $\Phi[k]$, are the differential transforms of $f(\eta)$, $\theta(\eta)$, $\varphi(\eta)$, and a_1, a_2, a_3, a_4, are constants which can be obtained through boundary condition. This problem can be solved:

$$\left.\begin{array}{l}F[0]=0, F[1]=a_1, F[2]=0, F[3]=a_2, F[4]=0\\ F[5]=\dfrac{3}{20}Sa_2+\dfrac{1}{20}Sa_1a_2+\dfrac{1}{20}a_1a_2+\dfrac{1}{20}\mathrm{Ha}^2\,a_2,\ldots\end{array}\right\}\tag{25}$$

$$\left.\begin{array}{c}\Theta[0]=a_3,\Theta[1]=0,\Theta[2]=0,\Theta[3]=0,\\ \Theta[4]=-3\,\mathrm{Pr}\,\mathrm{Ec}\,a_2^2-2\,\mathrm{Pr}\,\mathrm{Ec}\,a_1a_2,\Theta[5]=0,\\ \Theta[6]=\dfrac{2}{5}\mathrm{Pr}^2\,S\,a_1\,\mathrm{Ec}\,a_2^2-\dfrac{6}{5}\mathrm{Pr}\,\mathrm{Ec}\,a_2^2S-\dfrac{2}{5}\mathrm{Pr}\,\mathrm{Ec}\,a_2^2S\,a_1-\dfrac{2}{5}a_2^2\,a_1\,\mathrm{Pr}\,\mathrm{Ec}-\\ -\dfrac{2}{5}\mathrm{Pr}\,\mathrm{Ec}\,a_2^2\mathrm{Ha}^2+\dfrac{4}{5}N_b\,\mathrm{Ec}\,a_2^2\Phi[2]...\end{array}\right\} \tag{26}$$

$$\left.\begin{array}{c}\Phi[0]=a_4,\Phi[1]=0,\Phi[2]=0,\Phi[3]=0,\Phi[4]=\dfrac{3N_t\,\mathrm{Pr}\,\mathrm{Ec}a_2^3}{N_b},\Phi[5]=0,\\ \Phi[6]=\dfrac{2N_t}{5N_b}\mathrm{Pr}\,\mathrm{Ec}a_2^2\left(-\mathrm{Sc}Sa_1-\mathrm{Pr}\,Sa_1+3S+Sa_1+a_1+\mathrm{Ha}^2\right)...\end{array}\right\} \tag{27}$$

The process is continuous. By substituting eqs. (25)-(27) into the main equation based on DTM, it can be obtained that the closed form of the solutions is:

$$F(\eta)=a_1\eta+a_2\eta^3+\left(\frac{3}{20}Sa_2+\frac{1}{20}Sa_1a_2+\frac{1}{20}a_1a_2+\frac{1}{20}\mathrm{Ha}^2a_2\right)\eta^5+... \tag{28}$$

$$\theta(\eta)=a_3+\left(-3\mathrm{PrEc}a_2^2-2\mathrm{PrEc}a_1a_2\right)\eta^3.$$

$$\left(\frac{2}{5}\mathrm{Pr}^2Sa_1\mathrm{Ec}a_2^2-\frac{6}{5}\mathrm{PrEc}a_2^2S-\frac{2}{5}\mathrm{PrEc}a_2^2Sa_1-\frac{2}{5}a_2^2a_1\mathrm{PrEc}-\frac{2}{5}\mathrm{PrEc}a_2^2\mathrm{Ha}^2\right)\eta^6+... \tag{29}$$

$$\phi(\eta)=a_3+\left(\frac{3N_t\mathrm{PrEc}a_2^3}{N_b}\right)\eta^4+\left[\frac{2N_t}{5N_b}\mathrm{PrEc}a_2^2\left(-\mathrm{Sc}Sa_1-\mathrm{Pr}Sa_1+3S+Sa_1+a_1+\mathrm{Ha}^2\right)\right]\eta^6+... \tag{30}$$

by substituting the boundary condition from eq. (11) into eqs. (28)-(30) in point $\eta = 1$ it can be obtained the values of a_1, a_2, a_3, a_4 .

For example when $S = 0.5$, $\mathrm{Pr} = 10$, $\mathrm{Ec} = 0.1$, $\mathrm{Sc} = 0.5$, $N_t = N_b = 0.1$, and $\mathrm{Ha} = 1$ constant values are obtained:

$$\begin{array}{l}a_1=1.403081712, a_2=-0.3181618412,\\ a_3=1.317426833, a_4=\ 0.6953650785\end{array} \tag{31}$$

By substituting obtained a_1, a_2, a_3, a_4 into eqs (28)-(30), it can be obtained the expression of $f(\eta)$, $\Theta(\eta)$, and $\Phi(\eta)$.

Results and discussion

In this study, nanofluid flow and heat transfer in the unsteady flow between parallel plates is investigated considering thermophoretic and Brownian motion effects. The effects of the squeeze number, Hartmann number, Schmidt number, Brownian motion parameter, thermophoretic parameter, and Eckert number on heat and mass characteristics are examined. The present DTM code is validated by comparing the obtained results with other works reported in [48]. As shown in tab. 2 they are in a very good agreement.

Table 2. Comparison of $-\theta'(1)$ between the present results and analytical results obtained by Mustafa *et al.* [48] for viscous fluid $S = 0.5$ and $\delta = 0.1$

Pr	Ec	Mustafa *et al.* [48]	Present work
0.5	1	1.522368	1.52236749518
2	1	5.98053	5.98053039715
5	1	14.43941	14.4394132325
1	1.2	3.631588	3.63158826816
1	2	6.052647	6.05264710721
1	5	15.13162	15.1316178324

Effect of the squeeze number on the velocity profiles is shown in fig. 2. It is important to note that the squeeze number, S, describes the movement of the plates ($S > 0$ corresponds to the plates moving apart, while $S < 0$ corresponds to the plates moving together (the so-called squeezing flow). In this study positive values of S are considered. As S increases, horizontal velocity decreases. The S has different effect on vertical velocity profile near each plate. The f' increases with increases S of when $\eta > 0.5$ but opposite trend is observed when $\eta < 0.5$. Also, fig. 2 shows that $f''(1)$ increase with increase of S which means that the S has direct relationship with the absolute values of skin friction coefficient.

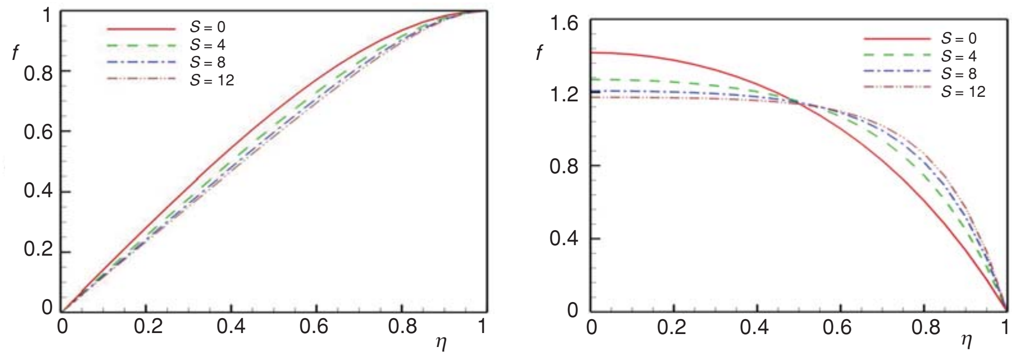


Figure 2. Effect of S on the velocity profiles when Ha = 2

Figure 3 shows the effect of the Hartmann number on the velocity profiles. It is worthwhile mentioning that the effect of magnetic field is to decrease the value of the velocity magnitude throughout the enclosure because the presence of magnetic field introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction. This type of resisting force slows down the fluid velocity. Also it can be concluded that skin friction coefficient increases with increase of Hartmann number.

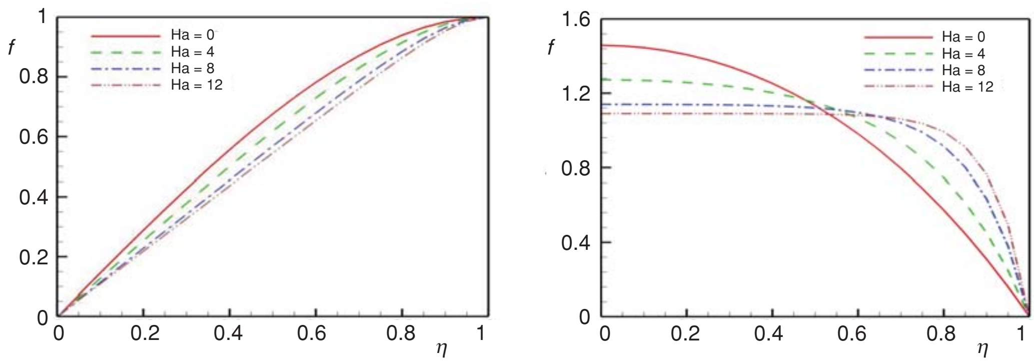
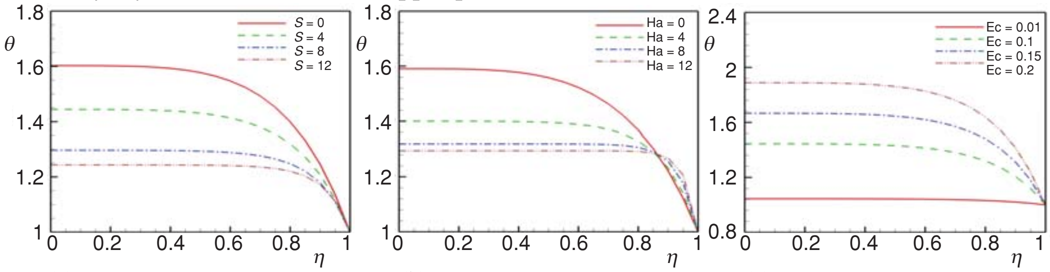


Figure 3. Effect of the Hartmann number on the velocity profiles when S = 0.5

Figure 4 shows the effect of the squeeze number, S, Hartmann number, and Eckert number on the temperature profile. An increase in the S can be related with the decrease in the kinematic viscosity, an increase in the distance between the plates and an increase in the speed at which the plates move. Thermal boundary-layer thickness increases as the S increases.

Temperature profiles have meeting point near $\eta = 0.82$ for different values of Hartmann number. Increasing Hartmann number leads to increase in temperature profile gradient near the hot plate. The presence of viscous dissipation effects significantly increases the temperature.

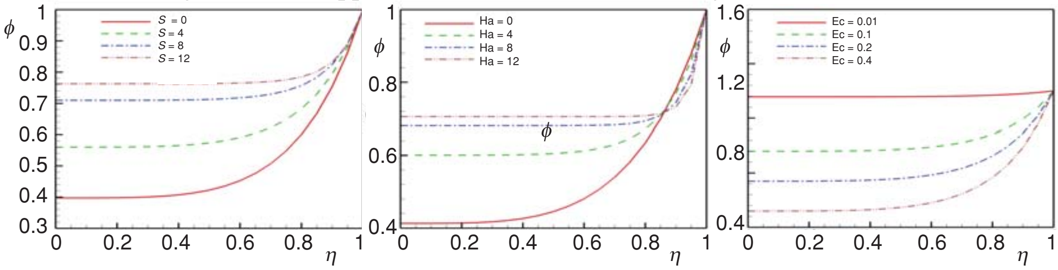
Nusselt number increase with increase of Eckert number because of reduction of thermal boundary layer thickness near the upper plate.



(a) Ec = 0.1, Ha = 2 (b) S = 0.5, Ec = 0.1 (c) S = 0.5, Ha = 2

Figure 4. Effects of *S*, Hartmann number, and Eckert number on the temperature profile when Sc = 0.5, $N_t = N_b$ = 0.5, and Pr = 10

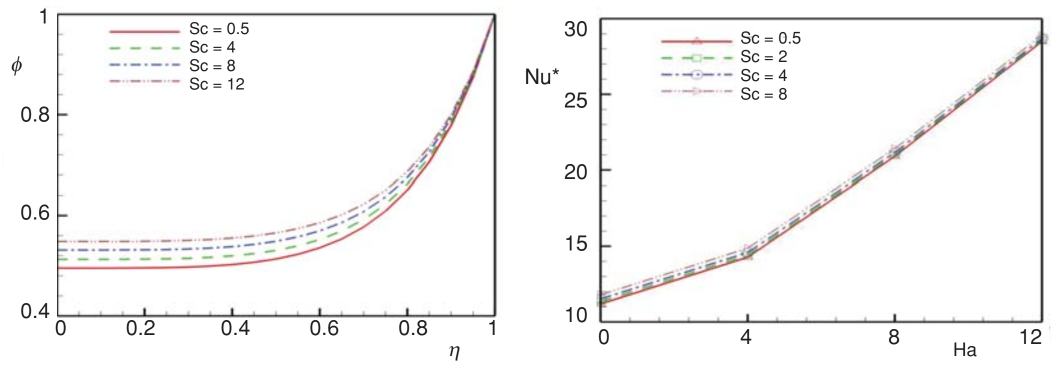
Effects of the squeeze number, S, Hartmann number, and Eckert number on the concentration profile are shown in fig. 5. Effects of these parameters on concentration profile are reverse to temperature profile. It means that concentration profile increase with augment of S but it decreases with increase of Eckert number. As Hartmann number increases, concentration increases when $\eta < 0.82$ but opposite behavior observed when $\eta > 0.82$.



(a) Ec = 0.1, Ha = 2 (b) S = 0.5, Ec = 0.1 (c) S = 0.5, Ha = 2

Figure 5. Effects of *S*, Hartmann number, and Eckert number on the concentration profile when Sc = 0.5, $N_t = N_b$ = 0.5, and Pr = 10

Effect of Schmidt number on the concentration profile and Nusselt number is shown in fig. 6. Increasing Schmidt number causes the concentration profile to increase. As Schmidt number enhances thermal boundary layer thickness near the hot plate decreases slightly and in turn Nusselt number increases with increase of Schmidt number.



Ha = 2, S = 0.5, Ec = 0.1 S = 0.5, Ec = 0.1

Figure 6. Effect of Schmidt number on the concentration profile and Nusselt number when $N_t = N_b$ = 0.5, and Pr = 10

Figure 7 shows the effect of N_t and N_b on the concentration profile. Concentration profile enhances as N_b increases but it reduces when N_t increases. The N_t and N_b have little effect on temperature profile.

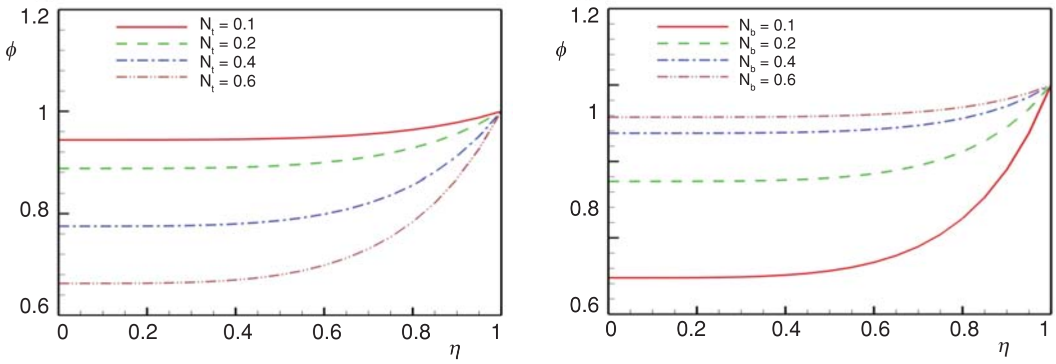


Figure 7. Effect of N_t and N_b on the concentration profile when S = 0.5, Sc = 0.5 m Ha = 2, and Pr = 10

The corresponding polynomial representation of such model for Nusselt number is:

$$\begin{aligned}\mathrm{Nu}^* &= a_{13} + a_{23}Y_1 + a_{33}Y_2 + a_{43}Y_1^2 + a_{53}Y_2^2 + a_{63}Y_1Y_2 \\ Y_1 &= a_{11} + a_{21}S + a_{31}\mathrm{Ha} + a_{41}S^2 + a_{51}\mathrm{Ha}^2 + a_{61}S\,\mathrm{Ha} \\ Y_2 &= a_{12} + a_{22}\mathrm{Ec} + a_{32}\mathrm{Ha} + a_{42}\mathrm{Ec}^2 + a_{52}\mathrm{Ha}^2 + a_{62}\mathrm{Ec}\,\mathrm{Ha}\end{aligned} \tag{32}$$

Also a_{ij} can be found in tab. 3 for example a_{21} equals to (4.975737). Variation of Nu* for various values of squeeze number, Hartmann number and Eckert number is shown in fig. 8. Nusselt number is an increasing function of Hartmann and Eckert numbers. As S increases Nusselt number decreases slightly.

Table 3. Constant coefficient for using eq. (31)

a_{ij}	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
j = 1	0.512104	24.40298	–0.39176	0.957187	0.032832	3.673337
j = 2	4.975737	–0.16995	0.258367	0.027757	0.032832	0.000889
j = 3	–0.56318	0.897206	0.24523	–0.00032	–0.01829	0.010858

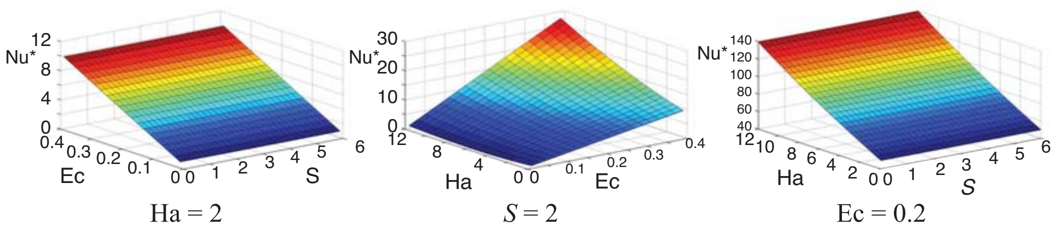


Figure 8. Variation of Nu* for various values of squeeze number, Hartmann number, and Eckert number when Sc = 2, N_t = N_b = 0.5, and Pr = 10 (*for color image see journal web site*)

Conclusion

Unsteady nanofluid flow between parallel plates is investigated. In order to simulate nanofluid, two phase model is considered. Differential Transformation Method is used to solve the governing equations. The effects of the squeeze number, Hartmann number, Schmidt number, Brownian motion parameter, thermophoretic parameter and Eckert number on temperature and concentration profiles are examined. The results show that skin friction coefficient increases with augment of Hartmann number and squeeze number. Also it can be concluded that Nusselt number is an increasing function of Hartmann number, Eckert number, and Schmidt number but it is decreasing function of squeeze number.

Nomenclature

$\vec{B}$ – magnetic field, [kgs^{-2}A^{-1}]
C – nanofluid concentration, [–]
c_p – specific heat at constant pressure, [Jkg^{-1}K^{-1}]
D_B – Brownian diffusion coefficient, [–]
D_T – thermophoretic diffusion coefficient [–]
Ec – Eckert number, [–]
F – transformation of f, [ms^{-1}]
f – skin friction coefficient
Ha – Hartmann number, [–]
k – thermal conductivity, [kgms^{-3}K^{-1}]
Le – Lewis number, [$=\alpha/D_B$]
N_b – Brownian motion parameter, [–]
N_t – thermophoretic parameter, [–]
Nu – Nusselt number, [–]
Pr – Prandtl number, [–]
p – pressure, [Pa]
S – squeeze number, [–]
Sc – Schmidt number, [Le Pr]
T – fluid temperature, [K]
$\vec{V}$ – velocity field, [ms^{-1}]
u, v – velocity components in the x- and y-direction, [ms^{-1}]

Greek symbols

α – thermal diffusivity, [l]
Θ – transformation of θ, [K]
θ – dimensionless temperature, [–]
μ – dynamic viscosity, [Pa·s]
ρ – density, [kgm^{-3}]
σ – electrical conductivity of nanofluid, [kgms^{-3}K^{-1}]
Φ – transformation of ϕ, [–]
ϕ – dimensionless concentration, [–]

Subscripts

C – cold, [c]
H – hot, [h]
h – high, [h]
f – base fluid, [b]

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