

A Goal Programming Model for Capital Rationing with a Linear Cash Fluctuations Measure

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1- Introduction

Capital rationing arise in situations where the total available resources (capital, labor, materials, etc.) is less than the resource requirements for all investment opportunities being considered by management [1]. Therefore firms need to devise procedures for rationing in order to select the optimal group⁽¹⁾ of investments under the restriction of scarce resources. Several capital rationing techniques for the ranking of alternatives is presented in the literature [1-3].

Most of these approaches will provide good means of ranking the alternatives. Put this way, management then selects from the list until either the list or the available resources is exhausted. Optimality is not guaranteed through this procedure. Consider the following example.

Example 1

Consider Table (1) below:

Proposal	1	2	3
Capital	20,000	12,000	9,000
NPV	4,000	2,500	2,200

Table (1)

Example 1 data

Note that investment proposals are ranked according to the values of the *Net Present Value* (NPV). If the capital available is

⁽¹⁾ Optimality is considered with respect to the objective function being evaluated. Different objectives in most cases will yield different optimal groups.

25,000 then only the first alternative will be selected. In this case a total NPV of 4,000 is realized. If on the other hand, investment opportunities 2 and 3 are selected (total capital expenditure would be 21,000) then a total NPV of 4,700 is realized.

Other techniques as *linear programming* may be used. In this case the objective function is to maximize the total NPV realized.

In many cases the need arises for achieving more than one objective. *Goal programming* provided a way of realizing these objectives *simultaneously*. The concept of goal programming was initially developed in the early sixties by Charnes and Cooper [4]. In 1965, Ijiri [5] introduced additional definitions and refinements of the technique. Several applications and extensions were later given by Ignizio [6] and Lee [7]. The concept was applied to capital rationing to allow for multiple objective rationing [3, 8-15].

In this paper a goal programming model is developed that selects a group of investment opportunities which maximized the annual cash flow fluctuations. Both objective functions are linear mixed integer functions. Several measures of deviation are introduced, compared, and analyzed via computer runs using RISK [16]. Test problem were randomly generated by BUDGEN [17]. Implementation of the model, test examples, and concluding remarks are presented. Special constraints and cases of the capital rationing problem are presented in Appendix A.

2- Notation

The following notation is used throughout the paper:

$I \equiv$ the set of all investments i ; $0 \leq i \leq m$ (where $i=0$ represent investments).

$$x_i \left\{ \begin{array}{l} 1, \text{ if investment } i \text{ is selected} \\ 0, \text{ otherwise} \end{array} \right\}^{(2)}$$

$f_{ij} \equiv$ cash flow of investment i in year j ; $1 \leq j \leq n$

$F_j =$ annual combined cash flows of current and selected investments in year j .

$$F_j = \sum_{i \in I} f_{ij} x_i \quad ; 1, j, n \quad (1)$$

$\sigma \equiv$ standard deviation of the combined annual cash flows of current and selected investments.

$P_i \equiv$ net present value of n_i cash flows generated by investment opportunity i .

$CL \equiv$ total capital available in local currency.

$CF \equiv$ total capital available in foreign currency.

$C \equiv$ total capital available, irrespective of currency.

$cl_i \equiv$ capital required for proposal i in local currency.

$cf_i \equiv$ capital required for proposal i in foreign currency.

$c_i \equiv$ capital required for investment opportunity i , irrespective of currency.

3- Assumptions

The proposed model relies on the following set of assumptions:

a. Indivisible investment opportunities: proposed investments can

⁽²⁾ The variable x_0 represents the current investments and will always equal to 1. This means, it is a constant not a variable. Yet, to simplify the notation and the calculation of the cash fluctuation function, it will be considered a variable in the notation and will be treated as a constant in the development of the model as will be shown later.

not be broken up into parts; either it is undertaken as a whole or it is not undertaken at all.

- b. Single period budgeting; only projects requiring capital expenditures at the present are considered.
- c. Equal lives: all proposed investment opportunities and all current investments are assumed to have equal lives.

4. Model Development

we will first develop the simplified single objective models, the maximization and minimization models. Both models will have the same set of constraints, and differ only in the objective function. The first model, the Capital Rationing Maximization model (CRMAX), maximized the NPV of the firm. The second model, the Capital Rationing Minimization model (CRMIN) minimizes the fluctuations of annual combined cash flows.

The goal programming model (CRGP) is then presented and tested using LINDO [30].

4.1 The Capital Rationing Maximization Problem

In this model, the maximization of the total NPV of cash flows of current and selected investments is considered. This presentation would not be complete without a note on the selection of a Minimum Acceptable Rate of Return (MARR), that will be used in the calculation of the NPV. The literature presented many alternatives for determining MARR [1-3, 18-27].

It is beyond the scope of this work to discuss the controversy related to the selection of MARR. A reasonable discount factor that

will be used is the cost of capital inquiry. Hence, it is assumed that the discount factors for all investment opportunities are equal.

The mathematical model of the Capital Rationing Maximization problem (CRMAX) is given by:

$$\begin{aligned}
 \text{(CRMAX)} \quad & \text{Maximize} \quad \sum_{i \in I} P_i x_i \\
 & \text{subject to:} \quad \sum_{i \in I} c_i x_i \leq C \\
 & \quad \quad \quad x_0 = 1 \\
 & \quad \quad \quad x_i = 0, 1 \quad ; \quad \forall i \in I, i \neq 0
 \end{aligned}$$

4.2 The Capital Rationing Minimization Problem

Risk analysis allows the notion of the degree of deviation of possible outcomes of a financial element from their calculated mean. Risk considered in this paper, measures the fluctuations in annual cash flows of a certain group of investment opportunities. Thus, the optimum group of investments, under the minimization model, is the one that yields cash flows with the least possible fluctuations from year to year, subject to the constraint of limited resources.

To evaluate the rate of fluctuation in a given set of cash flows, two measures are applied: the *Standard Deviation*, and the *Average Absolute deviation*. Let \bar{F} be the mean of n annual cash flows of a selected group of investment opportunities, then:

$$\bar{F} = \frac{\sum_{j=1}^n F_j}{n} \quad (2)$$

Hence, the standard deviation (σ) may be calculated as follows:

$$\sigma = \sqrt{\frac{\sum_{j=1}^n (F_j - \bar{F})^2}{n}} \quad (3)$$

and the average absolute deviation (A) may be calculated as follows:

$$A = \frac{\sum_{j=1}^n |F_j - \bar{F}|}{n} \quad (4)$$

Extending equations (2), (3) and (4) to represent their elementary components, x_i and f_{ij} , results in the following:

$$\bar{F} = \frac{\sum_{j=1}^n \sum_{i \in I} f_{ij} x_i}{n} \quad (5)$$

$$\sigma = \sqrt{\frac{\sum_{j=1}^n \left(\sum_{i \in I} f_{ij} x_i - \frac{\sum_{j=1}^n \sum_{i \in I} f_{ij} x_i}{n} \right)^2}{n}} \quad (6)$$

$$A = \frac{\sum_{j=1}^n \left| \sum_{i \in I} f_{ij} x_i - \frac{\sum_{j=1}^n \sum_{i \in I} f_{ij} x_i}{n} \right|}{n} \quad (7)$$

Measuring fluctuations in cash flows, using (6) results in a nonlinear objective function:

$$\text{Minimize } \sigma = \sqrt{\frac{\sum_{j=1}^n \left(\sum_{i \in I} f_{ij} x_i - \frac{\sum_{j=1}^n \sum_{i \in I} f_{ij} x_i}{n} \right)^2}{n}} \quad (8)$$

The average absolute deviation measure, as given by (7), provides a mean of measuring the desired fluctuations via linear relations, therefore, a 0-1 integer programming model may be constructed. This should be easier to solve than the nonlinear model.

Therefore, the objective function will take the form:

$$\text{Minimize } A = \frac{\sum_{j=1}^n \left| \sum_{i \in I} f_{ij} x_i - \frac{\sum_{j=1}^n \sum_{i \in I} f_{ij} x_i}{n} \right|}{n} \quad (9)$$

The minimum value of A subject to the constraints of the problem is equal to the minimum value of $n \cdot A$ subject to the same set of constraints, therefore, (9) may be reduced to the following:

$$\text{Minimize } A = \sum_{j=1}^n \left| \sum_{i \in I} f_{ij} x_i - \frac{\sum_{j=1}^n \sum_{i \in I} f_{ij} x_i}{n} \right| \quad (10)$$

4.2.1 Comparison between the two Measures of Risk

It will be shown that the average absolute deviation model yields results that are very close to those provided by the standard deviation model. Because either measure simply determines how different alternatives compare in terms of cash fluctuations, close results are considered satisfactory for risk measurement.

It should be noted that the standard deviation does provide a more accurate measure of dispersion (the degree to which a set of values vary about their mean) than the average absolute deviation on the same sets of data [28]. Therefore, it is valid to say that selections made by the two measures, may be different. We need to answer two questions: how much do they differ? And, how often?

The program BUDGEN [17] was used to randomly generate hundreds of test cases. The program RISK [16] was then used to test

and evaluate these cases. Each case consists of a predefined number of alternatives and their annual cash flows. The following measures were computed:

(A) The *Consecutive Absolute Difference*:

$$\sum_{j=1}^{n-1} |F_j - F_{j+1}| \quad (11)$$

(B) The *Relative Absolute Difference*:

$$\sum_{j=1}^{n-1} \left| \frac{F_j - F_{j+1}}{n} \right| \quad (12)$$

(C) The *Average Absolute Deviation of n cash flows*:

$$\frac{\sum_{j=1}^{n-1} |F_j - \bar{F}|}{n} \quad (13)$$

(D) The *Standard Deviation of n cash flows*:

$$\sqrt{\frac{\sum_{j=1}^n (F_j - \bar{F})^2}{n}} \quad (14)$$

A group of one hundred sets of problems (cases) were generated. Each set is composed of four investment opportunities. For each alternative, cash flows for eight years were randomly generated. The four measures were computed. The total number of times where each measure selects the same alternative as the one selected by the variance is registered. The results are summarized in Table (2) below.

	Measure of Risk	Number of times measure selects the same proposal as the one selected by the standard deviation
A	Consecutive Absolute Difference	35
B	Relative Absolute Difference	22
C	Average Absolute Difference	86

Table (2)

The number of times each measure selects the same alternative as the one selected by the standard deviation

It is clear from Table (2) that the average absolute deviation measure was much closer than the absolute and relative differences in the selection of alternatives. The absolute difference missed 65% of the cases and was only correct 35% of the times. The Relative difference missed 78% of the cases and was correct 22% of the times. Although such results may not be generalized, they are good enough to determine that the average absolute deviation measure is superior to the absolute and relative differences in measuring risk. This triggers the next test.

In order to get a better feel of how close the average absolute deviation is in selecting investment proposals as compared to the standard deviation, a program RISKAUTO [29] was used. This program uses data generated by BUDGEN [17], tests them using RISK [16] and risk measure. A success is registered when a risk measure selects the same alternative as the one selected by the standard deviation. RISKAUTO provides three values: MINDIFF (the minimum difference), MAXDIFF (the maximum difference), and AVGDIFF (the average difference).

To illustrate, consider this example. The results of one of the test sets with 6 investment alternatives. An alternative may consist of more than one investment opportunity. A group of investment opportunities that do not violate any of the problem constraints may hence be called an investment alternative³ are shown in Table (3). The Standard and average absolute deviation of the annual cash flows were calculated for each alternative.

Investment Alternatives						
Year	1	2	3	4	5	6
1	5,000	1,000	6,000	1,500	500	5,000
2	5,600	1,500	4,500	1,000	1,400	3,000
3	4,500	100	1,500	1,400	900	5,000
4	1,800	1,500	2,800	1,800	1,500	2,000
5	500	1,500	1,200	900	800	1,500
6	1,800	2,000	1,500	1,200	1,200	2,600
7	2,000	180	6,000	1,000	700	8,000
8	1,500	200	5,000	2,000	500	7,000
9	2,000	2,000	1,000	1,200	400	5,000
10	1,500	1,000	200	1,000	600	2,000
$\bar{\sigma}$	1,738	728	2,208	371	373	2,238
A	1,448	602	1,924	300	290	1,890

Table (3)
Cash flows, Standard Deviation and Average Absolute
Deviation of 6 investment alternatives

According to Table (3), the best alternative according to the standard deviation is the fourth ($\bar{\sigma}=371$), and the best alternative according to the average absolute deviation is the fifth ($A=290$).

⁽³⁾ An alternative may consist of more than one investment opportunity. A group on investment opportunities that do not violate any of the problem constraints may hence be called an investment alternative.

Such a case will be registered as a failure for the average absolute deviation for selecting an alternative other than that selected by the standard deviation. In this case, we calculate the difference between the standard deviation of the fourth and fifth alternatives and find it to be 2 (0.54%). This difference is then saved and compared to other differences for other test sets. From these differences the three values MINDIFF, MAXDIFF and AVGDIFF are registered.

Three groups of problems were tested based on the above test procedure. The first consisted of investment opportunities (alternatives) with lives equal to 10 years, the second 15 years and the third 20 years. In each group, different problem sizes were tested, ranging from sets with 3 alternatives each, through sets with 20 alternatives each. On each problem size, 100 sets of randomly generated problems were tested from which the three values, MINDIFF, MAXDIFF and AVGDIFF were calculated. In addition, the total number of successes were reported as the number of matches. A total of 5,400 test cases were generated, tested and reported. Table (4) below summarizes the results of these tests.

No. of inv. alt.	Number of Annual Cash flows											
	Cash flows = 10				Cash flows = 15				Cash flows = 20			
	No. of matches	Difference			No. of matches	Difference			No. of matches	Difference		
min		max	avg	min		max	avg	min		max	avg	
3	90	1.29	13.38	5.64	93	1.97	10.45	6.54	93	0.25	16.56	6.11
4	91	1.71	13.11	3.94	86	0.43	10.99	3.18	94	0.11	3.72	1.65
5	91	1.49	9.90	3.79	92	0.04	9.64	4.30	89	0.25	9.68	2.81
6	85	0.05	14.78	6.51	84	0.32	13.34	4.51	91	0.67	7.92	3.24
7	90	0.32	15.45	4.17	88	0.38	9.37	5.58	94	0.24	13.74	5.17
8	85	1.00	27.54	7.82	84	0.14	15.11	3.82	90	0.33	7.21	3.44
9	84	0.29	10.71	5.16	90	0.16	8.14	3.41	85	0.21	12.28	2.74
10	87	0.37	20.60	5.34	85	0.37	14.41	6.05	90	0.24	10.22	5.70
11	84	0.43	26.08	9.83	87	0.01	10.94	4.55	94	0.72	9.48	4.77
12	80	0.10	17.28	6.28	89	0.01	10.72	3.59	84	0.16	9.82	3.64
13	78	1.28	22.02	8.25	75	0.11	14.63	4.28	89	1.67	13.41	5.60
14	77	0.25	91.51	6.81	82	0.59	13.04	6.21	84	1.40	20.74	6.72
15	83	0.67	17.07	9.39	86	0.00*	12.99	6.32	84	0.05	13.69	5.15
16	81	0.03	17.53	5.87	84	0.40	12.35	4.63	85	0.27	7.31	2.91
17	82	0.10	20.62	8.20	80	0.02	9.89	4.32	79	0.13	15.18	5.30
18	74	0.06	15.40	6.35	82	0.23	10.28	4.53	78	0.11	10.46	4.29
19	79	0.06	24.89	7.63	84	0.09	14.30	4.91	83	0.70	10.80	3.58
20	77	0.47	26.06	7.88	78	0.36	28.18	5.06	82	0.36	18.15	5.26
min	74	0.03	9.90	3.79	75	0.01	8.14	3.18	78	0.05	3.72	1.65
max	91	1.71	27.54	9.83	93	1.97	28.18	6.54	94	1.67	20.74	6.72
avg	83	0.55	18.44	6.60	85	0.33	12.71	4.77	87	0.44	11.69	4.34

* Zeros in the table indicate very small positive numbers.

Table (4)

Test cases showing the relation between the average absolute deviation and the standard deviation as measures for risk

According to Table (4), the average absolute deviation successfully selected the same alternative as the standard deviation in 83% of the cases (on the average) in the 10 year category. The range of success was from a low 74% (18 alternatives) to a high 91% (4 and 5 alternative). The average success was 85% for the 15 year category, with a low of 75% and a high of 93%. The average success was 87% for the 20 year category, with a low

of 78% and a high of 94%. The MINDIFF, MAXDIFF, and AVGDIFF values are shown in the second, third and fourth columns respectively of each category. The MINDIFF value for the 10 year category scored an average of 0.55% and the MAXDIFF for the same group scored an average of 18.44% with an overall average of 6.6%. For the second group, the MINDIFF was of 0.33% on the average, and the MAXDIFF was 12.71%, with an overall average of 4.77%. Finally the third category values were 0.44%, 11.69% and an average 4.34%. The difference between the standard deviation of the alternative selected by the *average absolute deviation* measure ranges between a low 0.01% and a high of 28.18% (both from the second category). The overall average of differences between the two standard deviations is 5.24% (computed by taking the average of the averages).

According to these results, it is safe to conclude that the *average absolute deviation* is a reasonable approximation of the standard deviation as a risk measurement as tested on actual problems. And since the *average absolute deviation* measure could be transformed to linear relations it will be adapted to measure the cash fluctuations. The standard deviation although a more accurate measure of dispersion will not be used for it is based on nonlinear relations.

4.2.2 Simplifying the Objective Function of the

Minimization problem

Two objectives in the *multi-objective* model are to be satisfied simultaneously, the maximization of the total net present values of selected group of investments, and the minimization of the overall risk measured in terms of the fluctuations in annual cash flow.

Hence,

$$\begin{aligned}
 A &= \sum_{j=1}^n \left| \sum_{i \in I} f_{ij} x_i - \sum_{i \in I} \bar{F}_i x_i \right| \\
 &= \sum_{j=1}^n \left| f_{0j} x_0 + f_{1j} x_1 + \dots + f_{mj} x_m \right. \\
 &\quad \left. - \bar{F}_0 x_0 - \bar{F}_1 x_1 - \dots - \bar{F}_m x_m \right| \\
 &= \sum_{j=1}^n \left| x_0 (f_{0j} - \bar{F}_0) + x_1 (f_{1j} - \bar{F}_1) + \dots + x_m (f_{mj} - \bar{F}_m) \right|
 \end{aligned}$$

To ease the notation, let $S_{ij} = (f_{ij} - \bar{F}_i)$, and since $x_0=1$, then A becomes:

$$A = \sum_{j=1}^n \left| \left(\sum_{i=1}^m S_{ij} x_i \right) + S_{0j} \right| \tag{17}$$

hence, the objective function becomes:

$$\text{Minimize } A = \sum_{j=1}^n \left| \left(\sum_{i=1}^m S_{ij} x_i + S_{0j} \right) \right| \tag{18}$$

Let Y_j , be defined as follows:

$$Y_j = \left(\sum_{i=1}^m S_{ij} x_i \right) + S_{0j}$$

Because Y_j may take positive and negative values, it is redefined in terms of two non-negative variables Y_j^+ and Y_j^- as follows:

$$Y_j = Y_j^+ - Y_j^- \quad ; \text{ where } Y_j^+, Y_j^- \geq 0 \tag{19}$$

The mathematical model of the capital rationing minimization problem (CRMIN) may now be presented.

$$\text{(CRMIN)} \quad \text{Minimize } A = \sum_{j=1}^n (Y_j^+ + Y_j^-) \tag{20}$$

subject to:

$$Y_j^+ - Y_j^- - \sum_{i=1}^m S_{ij} x_i = S_{0j} \quad ; 1 \leq j \leq n \quad (21)$$

$$\sum_{i \in I} c_i x_i \leq C$$

$$x_0 = 1$$

$$x_i = 0, 1 \quad ; \forall i \in I, i \neq 0$$

$$Y_j^+ \geq 0, Y_j^- \geq 0 \quad ; 1 \leq j \leq n \quad (22)$$

It is possible not to consider the *multi-objective* model which is presented in the next section.

4.3 The Capital Rationing Goal Programming Model (CRGP)

The goal programming model *CRGP* may be written as follows:

$$(CRGP) \quad \text{Maximize } W_1 z_1 - W_2 z_2$$

subject to:

$$\sum_{i=1}^m P_i x_i - z_1 \geq 0 \quad (23)$$

$$\sum_{j=1}^n (Y_j^+ + Y_j^-) - z_2 \leq 0 \quad (24)$$

$$Y_j^+ - Y_j^- - \sum_{i=1}^m S_{ij} x_i = S_{0j} \quad ; 1 \leq j \leq n$$

$$\sum_{i \in I} c_i x_i \leq C$$

$$x_0 = 1$$

$$x_i = 0, 1 \quad ; 1 \leq i \leq m$$

$$Y_j^+, Y_j^-, z_1, z_2 \geq 0 \quad ; 0 \leq j \leq n$$

Where W_1, W_2 are weights that may be used to emphasize the importance of one objective over the other⁽⁴⁾. Constraint (23) results

⁽⁴⁾ A value of 1 may be used for both weights if no special preference is desired.

in the maximization of the total net present value of the current and selected investment opportunities, resulted from the positive coefficient of z_1 in the objective function Constraint (24) results in the minimization of the cash fluctuations, due to the negative coefficient of z_2 in the objective function.

The following two simple examples illustrate how the model performs in selecting the group of investment opportunities such that the total NPV is maximized and yearly cash fluctuations are minimized.

Example 2

Consider the following problem with two investment opportunities each with a life of two years. The problem data is summarized in Table (5) below. Total capital available is 800. A discount rate of 12% was used in the calculation of the net present values.

		Investment Opportunities			$S_{ij} = f_{ij} - \bar{F}_j$		
		Current	First	Second	S_{0j}	S_{1j}	S_{2j}
Cost		(500)	(632)	(708)			
Year 1		1,000	2,700	1,000	-1,000	850	-1,000
Year 2		3,000	1,000	3,000	1,000	-850	1,000
Mean		2,000	1,850	2,000			
NPV		2,486	2,300	2,300			

Table (5)

Example 2 data

The mathematical model of the above problem may then take the form:

Maximize $z_1 - z_2$

$$\begin{aligned}
 \text{subject to:} \quad & 23,00 x_1 + 2,300 x_2 - z_1 \geq 0 \\
 & Y_1^+ + Y_2^- + Y_2^+ + Y_2^- - z_2 \leq 0 \\
 & 630 x_1 + 708 x_2 \leq 800 \\
 & Y_1^+ - Y_1^- - 850 x_1 + 1,000 x_2 = -1,000 \\
 & Y_2^+ - Y_2^- + 850 x_1 - 1,000 x_2 = 1,000 \\
 & x_1, x_2 = 0,1 \\
 & Y_1^+, Y_1^-, Y_2^+, Y_2^- \geq 0 \\
 & z_1, z_2 \geq 0
 \end{aligned}$$

This problem was solved using LINDO [30] as the mixed integer problem optimization program. Data was fed to the program via two matrices: G , H , and the vector S_0 . See Appendix B for a definition of each matrix and the constraint matrix of the problem. The solution is given in Table (6).

Variable	Value
x_1	1
x_2	0
z_1	2300
z_2	300
Y_1^+	0
Y_1^-	150
Y_2^+	150
Y_2^-	0

Table (6)
Variable values of Example 2

According to Table (6), the first alternative that yields the minimum risk desired, is selected.

Example 3

In example 2 both investment opportunities had the same NPV. But the selection of the first investment resulted in a lower risk caused by lower fluctuation of the annual cash flows of current and selected investments. In this example, the NPV of both investment opportunities are also equal but the risk is higher in the first alternative. Therefore, the model should select the second investment. Total capital available is 800. A discount rate of 12% was used in the calculation of the net present values. The problem data is shown in Table (7).

		Investment Opportunities			$S_{ij} = f_{ij} - F_j$		
		Current	First	Second	S_{0j}	S_{1j}	S_{2j}
Cost		(500)	(638)	(709)			
Year 1		1,000	800	3,000	-1,000	-1,200	-1,088
Year 2		3,000	3,200	825	1,000	1,200	-1,088
Mean		2,000	2,000	1,913			
NPV		2,486	2,346	2,346			

Table (7)

Example 3 Data

The mathematical model of the above problem is given by:

$$\text{Maximize } z_1 - z_2$$

subject to:

$$2,346 x_1 + 2,346 x_2 - z_1 \geq 0$$

$$Y_1^+ + Y_2^- + Y_2^+ + Y_1^- - z_2 \leq 0$$

$$638 x_1 + 709 x_2 \leq 800$$

$$Y_1^+ - Y_1^- + 1,200 x_1 - 1,088 x_2 = -1,000$$

$$Y_2^+ - Y_2^- - 1,200 x_1 + 1,088 x_2 = 1,000$$

$$x_1, x_2 = 0,1$$

$$Y_1^+, Y_1^-, Y_2^+, Y_2^- \geq 0$$

$$z_1, z_2 \geq 0$$

The solution is given in Table (8)

Variable	Value
x_1	0
x_2	1
z_1	2,346
z_2	176
Y_1^+	88
Y_1^-	0
Y_2^+	0
Y_2^-	88

Table (8)

Variable values of Example 3

5- Computational Results

The *CRGP* was tested on data from the Egyptian Government industrial sector [31]. Results were compared by the actual decisions concerning capital investments and found to be similar. Detailed discussion on the use of goal programming and *CRGP* may be found in [31].

The size of *CRGB*, is given by the number of variables and the number of constraints for any given n and m .

There are three types of variables: x , Y and z . The x variables (m , binary) represent the investment opportunities, the Y variables (2 for each year, continuous) are used for the transformation of the absolute value relation in the constraint set, and the z variables (two, continuous) are used to attain the two defined goals of the model. Thus, the total number of variables in the problem is equal to $2n+m+2$. There is a total of $n+3$ constraints.

APPENDIX A

A.1 Capital Requirements

It is possible in many cases that an investment opportunity requires funds both in local and foreign currencies. Thus, a constraint for each type of capital is required as follows:

$$\sum_{i \in I} cl_i x_i \leq CL \quad \text{A-1}$$

$$\sum_{i \in I} cf_i x_i \leq CF \quad \text{A-2}$$

A.2 Minimum Number of Selected Investments

This constraint states that a minimum number of investments are to be undertaken. It is assumed however, that this predetermined minimum value does not violate the resource availability constraints, otherwise, the problem would have no feasible solutions.

$$\sum_{i \in I'} x_i \geq \Omega \quad ; I' \subseteq I \quad \text{A-3}$$

A.3 Maximum Number of Selected Investments

If "o" is the maximum number of investments to be selected in any given year, then:

$$\sum_{i \in I''} x_i \leq \delta \quad ; I'' \subseteq I \quad \text{A-4}$$

A.4 Mutually Exclusive Investment Opportunities

Two investment opportunities "s" and "t" are said to be mutually exclusive if the selection of one prevents the selection of the other. The associating constraint will then take the form:

$$x_s + x_t \leq 1 \quad \text{A-5}$$

This constraint is to be repeated for each pair (group) of mutually exclusive proposals.

A.5 Contingent Investment Opportunities

Investment opportunity "v" is said to be contingent upon the selection of investment opportunity "w" if it can not be undertaken unless investment opportunity "w" is selected. This constraint will then take the form:

$$x_v - x_w \leq 0 \quad \text{A-6}$$

A.6 Limited Resources

Limited resources does not come only in the form of capital. Several authors [1-3] had indicated the nature and form of such constraints. If B is the limited resource (labor, materials, etc..), and b_i is the amount of that resource required when investment opportunity i is selected then:

$$\sum_{i \in I} b_i x_i \leq B \quad \text{A-7}$$

A.7 Added Value

Two or more investment opportunities may yield an extra added value when selected together. This can be easily incorporated in the objective function via a nonlinear relation (no additional constraints are required). The following may be added to the objective function of the maximization problem:

$$u \cdot \prod_{k \in I''} x_k \quad \text{A-8}$$

When $I''' \subseteq I$ is the set consisting of investment opportunities that would generate the added value u .

There is an alternative linear formulation to this case. If x_k and x_l generate an added value u , if selected together, then a new variable x' may be added to the set of investment opportunities I . The objective function coefficient of x' would equal to $P_k + P_l + u$

Two constraints need be added to the constraints set to prevent x_k and x_l from being selected with x' .

$$x' + x_k < 1$$

$$x' + x_l < 1$$

Note that since $P_k + P_l < P_k + P_l + u$, x_k and x_l will not be selected together, because the maximization problem would force the selection of x' instead or simply only one of the two variables is selected, in which case the $x' = 0$.

APPENDIX B

The constraint matrix of CRGP takes the following form:

x_1	x_2	...	x_m	Y_1^+	Y_1^-	Y_2^+	Y_2^-	Y_n^+	Y_n^-	Z_1	Z_2	RHS
P_1	P_2	...	P_m	0	0	0	0	0	0	-1	0	0
0	0	...	0	1	1	1	1	1	1	0	-1	0
c_1	c_2	...	c_m	0	0	0	0	0	0	0	0	C
$-S_{11}$	$-S_{21}$...	$-S_{m1}$	1	-1	0	0	0	0	0	0	S_{01}
$-S_{12}$	$-S_{22}$...	$-S_{m2}$	1	-1	0	0	0	0	0	0	S_{02}
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
$-S_{1n}$	$-S_{2n}$...	S_{mn}	1	-1	0	0	0	0	0	0	S_{0n}

Data is supplied to the program in two matrices, G , H , and the vector S_o . The matrix G is 2 by m and the matrix H is $m+1$ by n . To define G , let g_{ki} be the value of the entry in row k and column i of G , then:

$$g_{1i} = P_i \quad ; \quad 0 \leq i \leq m$$

and

$$g_{2i} = c_i \quad ; \quad 0 \leq i \leq m$$

Let h_{ij} be the value of the entry in row i and column j of H , then:

$$h_{ij} = S_{ij} \quad ; \quad 0 \leq i \leq m$$

$$h_{in} = S_{in} \quad ; \quad 0 \leq j \leq n$$

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