

## COMPUTER AIDS TO ANALYSIS OF SEMI-RIGID STEEL SPACE FRAMES

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### ABSTRACT

This work describes the recent development of the semi-rigid connection technique and the application of this method to the analysis of space frame structure.

To carry out the above method of analysis accurately and efficiently, a computer system is presented. The basic concept of the computer program is defined as an assemblage of well-defined subroutines as modules into a package of dealing with such analysis.

The computer program takes into account the moment-rotation relationship of five of the most common types of connections. A moment-rotation relationship of a sixth connection type is created in this research work and inserted within the computer program.

It is concluded that in spite of the fact that the amount of different rotation between beam and column is relatively small in the loaded frame structures, this differential rotation has a significant effect on the geometrical behavior of frame structures and consequently on the design of their elements. It has been shown that the consideration of semi-rigid connections in the analysis of frame structures is of great importance.

### NOMENCLATURE

A	Cross-area of member.
$C_i$	Curve-fitting constants.
$D_k$	Curve-fitting parameters.
E	Modules of elasticity.
F	Force matrix.
$F_c$	Flexibility matrix of connection, end 1.
$F_d$	Flexibility matrix of connection, end 2.
$F_m$	Flexibility matrix of member.

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G	Modules of rigidity
H	Equilibrium matrix
H <sub>c</sub>	Equilibrium matrix of connection, end1
H <sub>d</sub>	Equilibrium matrix of connection, end2
H <sub>m</sub>	Equilibrium matrix of member.
I	Moment of inertia.
K	Stiffness matrix.
K'	The standardization factor, (Dimensions factor).
L	Member length.
M <sub>1</sub>	Moment carried by connection, end1
M <sub>2</sub>	Moment carried by connection, end2
M <sub>o</sub>	Initial connection moment.
M <sub>u</sub>	Ultimate moment capacity.
R <sub>k</sub>	The instantaneous connection stiffness at an arbitrary rotation O.
R <sub>ki</sub>	Initial connection stiffness.
T	Transformation matrix.
e	The deformation vector.
n	Shape parameter.
r	Stress- resultant vector.
a	Scaling factor.
θ	Differential rotation of connection.
θ <sub>1</sub>	Differential rotation of connection, end 1.
θ <sub>2</sub>	Differential rotation of connection, end 2.
θ <sub>k</sub>	Starting rotation of the linear component given from experimental M-O curve.
θ <sub>o</sub>	Plastic rotation.

## INTRODUCTION

The term "semi-rigid connection" is the nomenclature for a steel member connection that is neither rigid nor pinned. Researchers are interested though in standard types of connection, in common use at the ends of beams, which offer varying but potentially useful degrees of moment restraint to the beam.

The objective of the development of controlled moment connections is to produce better structures, either because they are more economic or because they offer improved performance. Economy does not mean simply lighter structures, economy means striking the optimum balance between steel weight and connection cost. Improved performance refers to some aspect of serviceability such as reduced deflection or reduced vibration of floor or roof beams [1].

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With the advent of computers, much effort has been devoted to the analysis of frames with semi-rigid connections. Numerical analysis of frames which implemented actual connection behavior in the solution procedures have made it possible for engineers and designers to get a better understanding of the response characteristics of real frames. Connection flexibility not only affects the behavior of beams and columns of the frame but it has definite influence on the overall strength and stability of the structure.

A large number of papers are written on semi-rigid framing techniques, the authors presented a few papers, which show the main principles of these techniques.

In 1975 [2], M.J. Frye and G. A. Morris, presented a procedure for analyzing steel frames with any combination of pinned connections, fixed connections, connections with any specified flexibility characteristics, or any of seven commonly used connection types. An iterative nonlinear analysis procedure was described in which repeated modifications are made to assumed flexibility characteristics for all connections in a structure. When a suitable set of connection flexibility characteristics has been arrived at, a single analysis is performed to determine the correct structural deflections and internal forces.

In 1984 [3], K. M. Ang and G. A. Morris presented a procedure for analyzing three - dimensional rectangular steel frames that incorporate any of five commonly used beam- column connection types. A method is described for expressing the moment- rotation behavior of all connections of given types (for example all end plate connections) in terms of a single standardized Ramberg - Osgood function. The method involves an examination of experimental information on the moment - rotation behavior of a given connection type to determine the influence of various size parameters, such as end plate thickness. It has been used to generate standardized moment - rotation functions for five common connection types and the functions were presented. An iterative successive approximation structural analysis procedure was described in which repeated approximations were made to assume stiffness characteristics of all connections in the structure. When the appropriate connection stiffness have been determine, a single linear analysis is performed to determine the correct structural displacement and internal forces. Thus, the nonlinear behavior of the connection was properly accounted for.

In 1985 [4], Nerhercot studied the joint action on the design of steel frame structures. He presented the effect of connection stiffness on the behavior of frames. He found that all the connections widely used in structural design are in fact semi-rigid according to experimental data of  $M-\theta$  tests of each connection. Then depending on simple design methods of non-sway frames presented, a new

design method of semi-rigid connections was suggested for use in sway and non-sway steel structural design. Before that, in 1980, Nethercot with the help of Jones, and Kirby [5] found that semi-rigidity of connections has a non-linear function of deformation.

In 1986, Al-Rifaie and Trikha [6] presented a step-wise linearized procedure based on the matrix displacement method for elastic-plastic analysis of steel framed structures in which the influence of the width of connections has been recognized by idealizing both the beam and column members by rigid ended elements. A member was allowed to develop a plastic hinge only at the extremities of the intermediate elastic part. Stiffness matrices for the resulting four possible element types have been given and the effect of shear deformation was ignored. The work was later extended by Al-Rifaie and Maseer [7] to include the effects of large deformations.

In 1987, Nethercot, Kirby and Davison [8], studied the moment rotation stiffness characteristics of steel beam-to-column connections. They made a series of tests of such connection suitable for rectangular frames using I-sections to distinguish the real benefit of each type of connections in terms of moment capacity and connection stiffness. In all tests, similar beam and column sections, test apparatus, instrumentation and test procedure were employed. They studied the following connection types

- 1- Web cleats.
- 2- Flange cleats.
- 3- Combined seat and web cleats.
- 4- Flush end plates.
- 5- Extended end plates.
- 6- Connection to the column flange and webs are also included.

In 1987, Lendsey [9], studied the factors that need to be considered in analyzing partially rigid connections frames and then suggested a design method for such types of frames.

In 1987, A.Azizinamini, J.H.Bradburn and J.B.Radziminski [10], studied the static moment-rotation characteristics of semi-rigid beam-to-column connections consisting of top and seat flange angles, and double web angles. Initial stiffness and complete moment-rotation curves were quantified for connections of varying geometry. From this study, it was found that, for the type of connection investigated, the geometric parameters that most significantly affected the moment-rotation behavior are: the depth of the beam section to which the connections

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elements are framed, the thickness of the flange angles, and the gage in the leg of the flange angle attached to the column.

In 1987, A.R.Kukreti, T.M.Murray and A.Abolmaali [11], presented a methodology to develop the moment-rotation relationship (analytically) for a steel bolted end-plate connection. The finite element method was used as a tool to analyze a typical connection and has been validated by comparing the analytical results with those obtained from physical tests of a few selected specimens. Although the methodology was developed for a flush end-plate connection, it can also be extended to study the behavior of other type of symmetrical steel bolted connections.

In 1987, E.M.Lui and W.F.Chen [12], presented numerical studies of frames using simplified and rigorous analysis methods. Observations regarding the effects of flexible connections on the strength, deflection and internal force distribution of steel framed structures are discussed.

In 1989, Chen and Kishi [13], gave a data base and modeling research of all attempts of investigation on semi-rigid connection according to LRFD (Load Resistance Factor Design) specifications and they tabulated their data base effort.

This work presents a simple method of semi-rigid analysis of plane and space frames taking into account the effect of shear deformation, then an efficient interactive computer program for this analysis is done.

The computer analysis program takes into account the moment-rotation relationships of six of the most common types of connections.

### MODELING OF CONNECTIONS

Several analytical models have been developed to represent connection flexibility. These models are generally either a sophisticated numerical simulation or an approximation based on test data. Early models used the initial stiffness of the connection as the key parameter in a linear  $M-\theta$  model. Although the linear model is very easy to use, it has a serious disadvantages. It is not suitable for a full range of rotation.

In the following, review of the literature is presented which deals with modeling of connection behavior.

### Polynomial Model

A popular model for the connection behavior is the polynomial function proposed by Frye and Morris . They used the method of least squares to determine the constants of the polynomial. The primary disadvantage of this model is that the first derivative of this function which indicates connection stiffness, may be discontinuous and/or possible negative, which is physically impossible. To overcome this problem, a modified procedure was developed by J.B.Radziminki and A.Azizina-mini. It has the form

$$\theta = C_1 (KM)^1 + C_2 (KM)^3 + C_3 (KM)^3 \quad (1)$$

Where  $C_i$  = curve-fitting constants; and  $K$  = the standardization factor (a dimensionless factor whose value depends on the size parameters for the particular connection considered);  $M$  = connection moment.

### Modified Exponential Model (12)

The Chen-Lui exponential model has been refined to accommodate linear components by Kishi and Chen and is referred to herein as the modified exponential model.

This model is represented by a function of the form

$$M = M_0 + \sum_{j=1}^m C_j \left[ 1 - \exp \left( \frac{|\theta|}{2j\alpha} \right) \right] + \sum_{k=1}^n D_k (\theta - |\theta_k|) H [|\theta| - |\theta_k|] \quad (2)$$

where  $M_0$  = initial connection moment;  $\alpha$  = scaling factor;  $C_j$ ;  $D_k$  = curve-fitting parameters;  $\theta_k$  = starting rotation of the linear component given from experimental  $M-\theta$ , curve and

$$H[\theta] = 1 \text{ for } \theta \geq 0 \quad (3)$$

$$H[\theta] = 0 \text{ for } \theta < 0 \quad (4)$$

Using the linear interpolation technique for original  $M-\theta$ , data, the weight function for each  $M-\theta$ , datum is nearly equal. The constant  $C_j$  and  $D_k$  for the exponential and linear terms of the function are determined by a matrix. individual terms of the matrix are obtained by the method of least squares for a given set of moment-rotation data similar to that of the Chen-Lui exponential model.

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The instantaneous connection stiffness  $R_k$  at an arbitrary rotation  $\theta$  can be evaluated by differentiating Eq.2 with respect to  $\theta$ .

When the connection is loaded, we have

$$R_k = R_{kt} = \left. \frac{dM}{d|\theta|} \right|_{|\theta|=|\theta_t|} = \sum_{j=1}^m \frac{C_j}{2j\alpha} \exp\left(\frac{|\theta|}{2j\alpha}\right) + \sum_{k=1}^n D_k H[|\theta| - |\theta_k|] \quad (5)$$

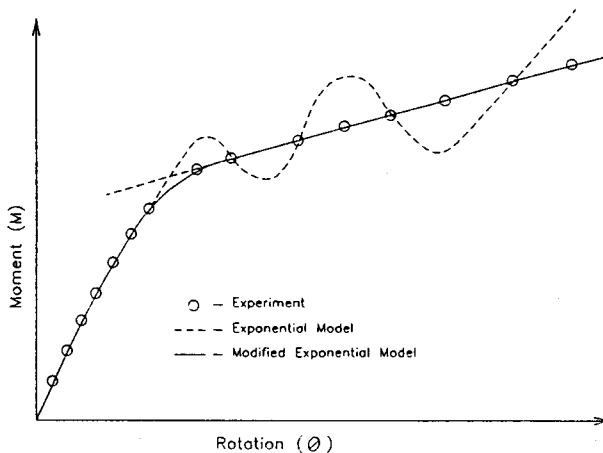
When the load is removed, we have

$$R_k = R_{kt} = \left. \frac{dM}{d|\theta|} \right|_{|\theta|=0} = \sum_{j=1}^m \frac{C_j}{2j\alpha} + D_k H[-|\theta_k|]_{k=1} \quad (6)$$

This model has the following merits:

1. The formulation is relatively simple and straightforward.
2. It can deal with connection loading and unloading for the full range of rotation in a second-order structural analysis.
3. The abrupt changing of the connection stiffness among the sampling data is only generated from inherent experimental characteristics.

The comparison between the Chen-Lui exponential model and the modified exponential model for numerical example test data including a linear component is shown in Fig. 1.



**Fig. 1. Comparison between results by exponential and modified exponential models for M-θ, data including a linear term.**

**Power Model with three Parameters (13)**

The modified exponential model mentioned earlier is curve-fitting equation obtained by using the least mean square technique for the test data. From a different viewpoint, Kishi and Chen (1989) developed another procedure to predict the moment-rotation characteristics of steel beam-to-column connections. In these procedure, the initial elastic stiffness and ultimate moment capacity of the connection are determined by a simple analytical model. Using those values so obtained, a three parameter power model was adopted to represent the moment-rotation relationship of the connection.

The generalized form of this model is

$$M = \frac{R_{ki} \theta}{\left\{1 + \left(\frac{\theta}{\theta_0}\right)\right\}^{1/n}} \tag{7}$$

Where  $\theta_0 = M_U / R_{KI}$ ;  $R_{KI}$  = initial connection stiffness;  $M_U$  = ultimate moment capacity;  $\theta_0$  = a reference plastic rotation; and  $n$  = shape parameter.

**Jones-Kirby-Nethercot B-Spline Model (1982) (5)**

In this model, the experimental (M- $\theta$ ) data are divided into a number of subsets, each spanning a small range of M. A cubic B-spline curve is then used to fit each and every subset of data with continuities of first and second derivatives enforced at their intersections. This model circumvents the problem of negative stiffness and represents the nonlinear (M- $\theta$ ) behavior extremely well. However, a large number of data are required in this curve-fitting process.

**Colson power model(14)**

This model uses a power function of the form

$$\theta = \frac{|M|}{R_n} \frac{1}{\left(1 - \left|\frac{M}{M_{\alpha}}\right|^n\right)} \tag{8}$$



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In (Fig.2)  $R_{ki}$  is the initial connection stiffness,  $M_{CU}$  is the ultimate moment capacity of the connection, and  $n$  is a parameter to account for the curvature of the  $(M-\theta)$  relationship. Since the model has only three parameters ( $R_{ki}$ ,  $M_{CU}$  and  $n$ ), it is not as accurate as the B-spline model. However, the number of data required for this model is drastically reduced.

### Ang-Morris power model (1984) (3)

The Ang-Morris power model has the form

$$\frac{\theta}{(\theta)_0} = \left| \frac{KM}{(KM)_0} \right| \left[ 1 + \left( \left| \frac{KM}{(KM)_0} \right| \right)^{n-1} \right] \quad (9)$$

where  $(\theta)_0$ ,  $(KM)_0$  and  $n$  are parameters defined in Fig.3.  $K$  is the standardization constant dependent upon the connection type and geometry. The Ang-Morris power model is a four-parameter model. It can represent the nonlinear  $(M-\theta)$  behavior of a variety of connections reasonably well.

In the present study the Ang-Morris power model was taken because it covers different types of connections, and has a good representation of the nonlinear behavior of moment-rotation relationship with a simple representation of connection parameters. A simple modification is made by adding a new common type of connection.

## BEHAVIOR OF CONNECTIONS

Before proceeding to the discussion of the analysis of semi-rigid connected frames, it is necessary to discuss how a typical loaded connection behaves. Also it will be shown how this behavior can be modeled mathematically.

A connection is a medium through which forces and moments are transmitted from one member to another. For a beam-to-column connection, a general set of forces that may be transmitted includes axial force, shearing force, bending moment and torsion. The effect of torsion can be neglected. Furthermore, for most connections, the axial and shearing deformations are usually small compared to the rotational deformation. Consequently, for practical purposes, only the connection's rotational deformation need to be considered.

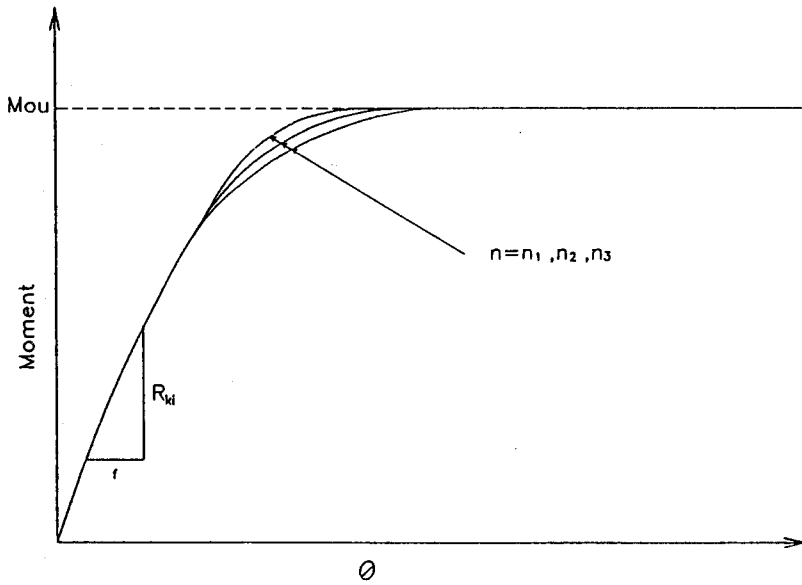


Fig. 2. Parameters of the colson's power model.

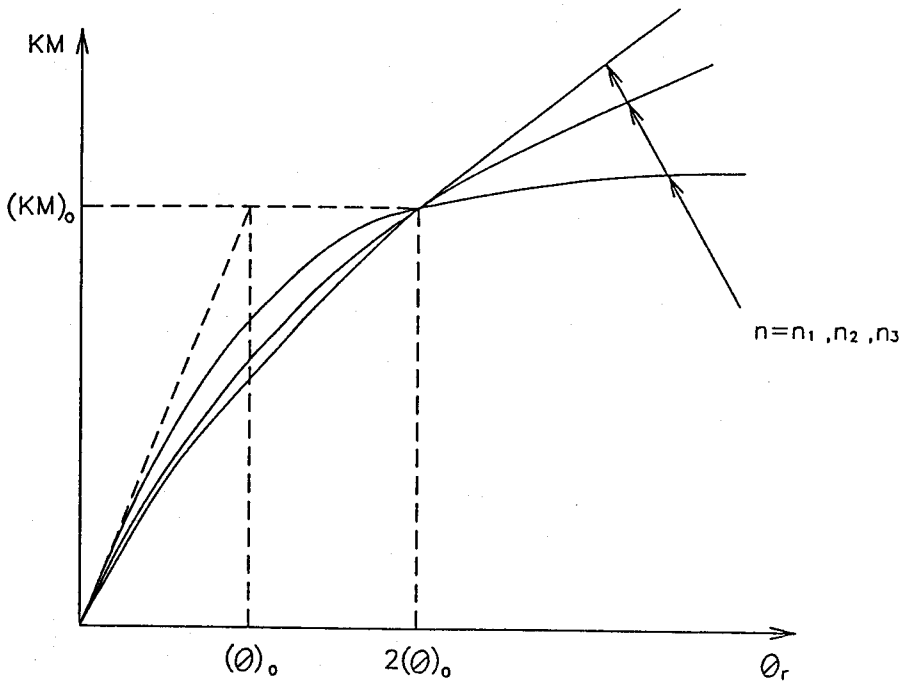


Fig. 3. Parameters of the Ang - Morris's power model.

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The rotational deformation is customarily expressed as a function of the moment in the connection. When a moment  $M$  is applied to a connection, it rotates by an amount  $\theta$ . As shown in Fig.4, the rotation represents the change in angle between the beam and the column from its original configuration. The angle  $\theta$  is a measure of the relative rotation ( $M-\theta$ ) behavior of a variety of commonly-used semi-rigid connections.

For example, the single web angle connection represents a very flexible connection and the strap angle connection represents a rather rigid connection. Several observations can be made from this Fig.5:

1. All types of connection exhibit a ( $M-\theta$ ) behavior that falls between the extreme cases of ideally pinned (the horizontal axis) and fully rigid (the vertical axis) conditions;
2. For the same moment, the more flexible the connection is, the larger is the value of  $\theta$ . conversely, for a specific value of  $\theta$ , a more flexible connection will transmit less moment between the adjoining members;
3. The maximum moment that a connection can transmit (herein referred to as the ultimate moment capacity) decreases with the more flexible connection end;
4. The ( $M-\theta$ ) relationship for the semi-rigid connections are typically nonlinear over virtually the entire range of loadings.

From Fig.5, it can be shown that a linear approximation of the initial portion of the curves is acceptable for the frame under its serviceability limit state, but it becomes unacceptable for the frame under its ultimate limit state.

As mentioned above when a connection is loaded, it rotates according to the curves shown in Fig.5. However, the connection will follow a different path which is almost linear with a slope equal to the initial slope of the ( $M-\theta$ ) curves when a connection is unloaded. This phenomenon is shown in Fig.6

The primary distortion of a steel beam-column connection is the rotational deformation  $\theta$  caused by the bending moment  $M$ . Because of the complexity of connection behavior, the moment-rotation function is virtually always determined experimentally. The five common beam-column connection types considered in this study are shown in Fig.7 and typical moment-rotation functions<sup>(3)</sup> are presented in Table 1. For each type of connection, the maximum deviation is calculated using the standardized function.

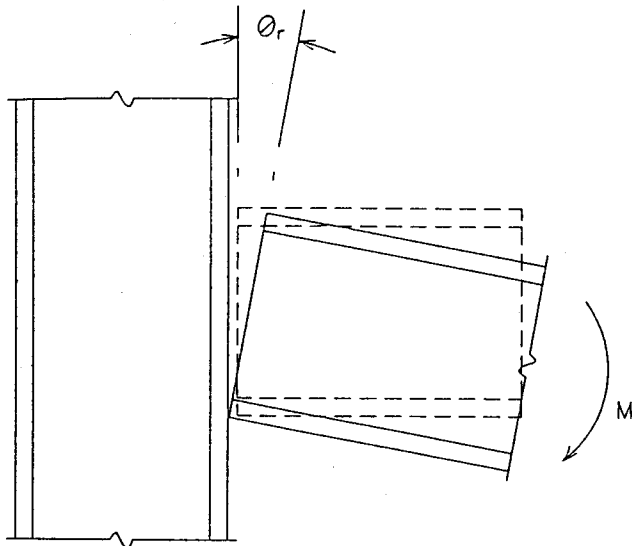


Fig. 4. Relative rotation of beam to column.

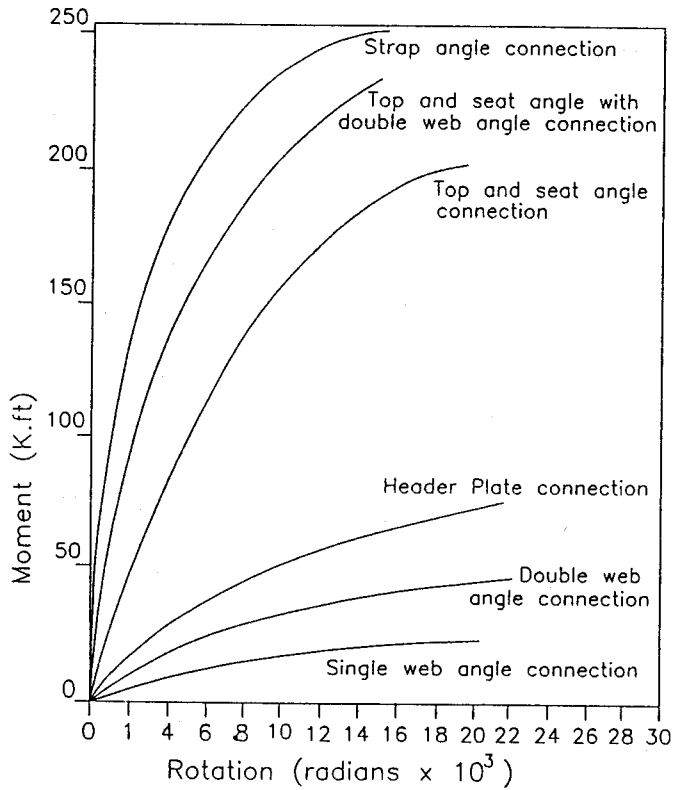


Fig. 5. Connection moment - rotation curves.

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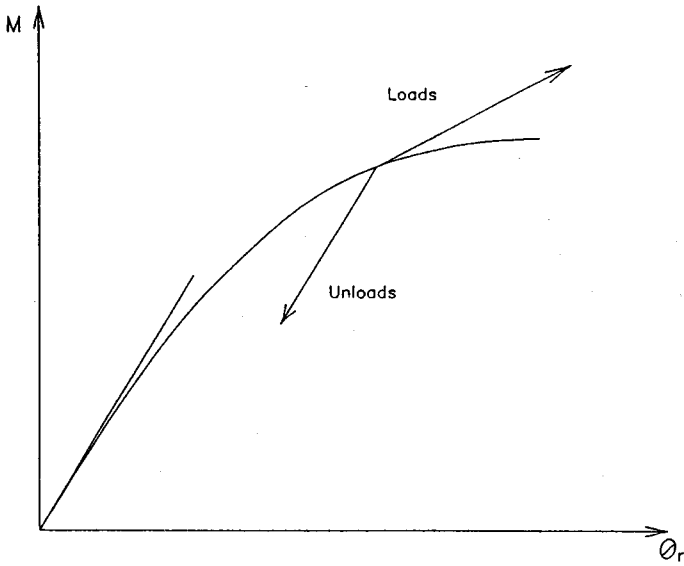


Fig. 6. Loading/unloading characteristic of a connection.

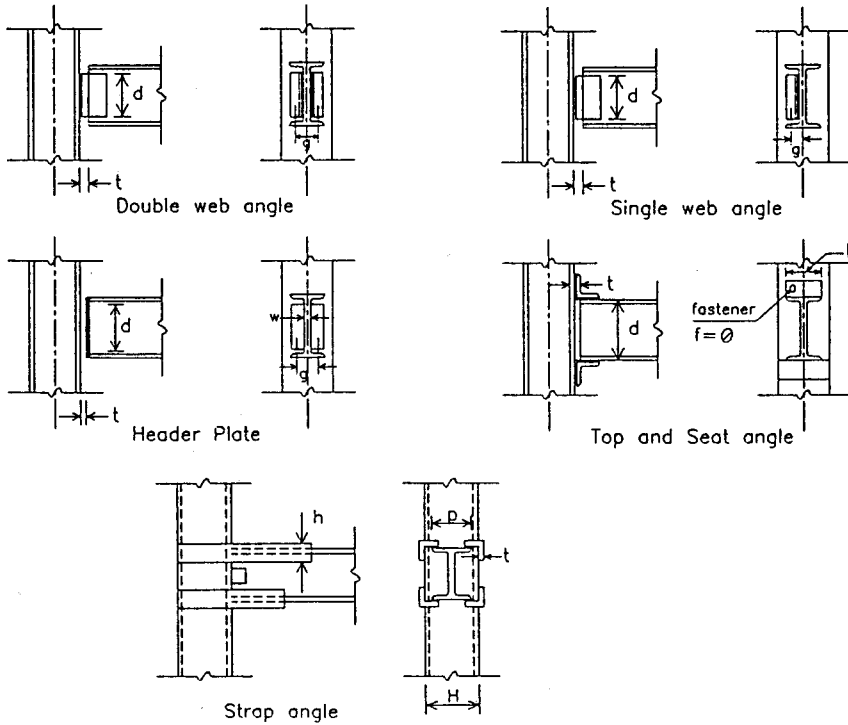


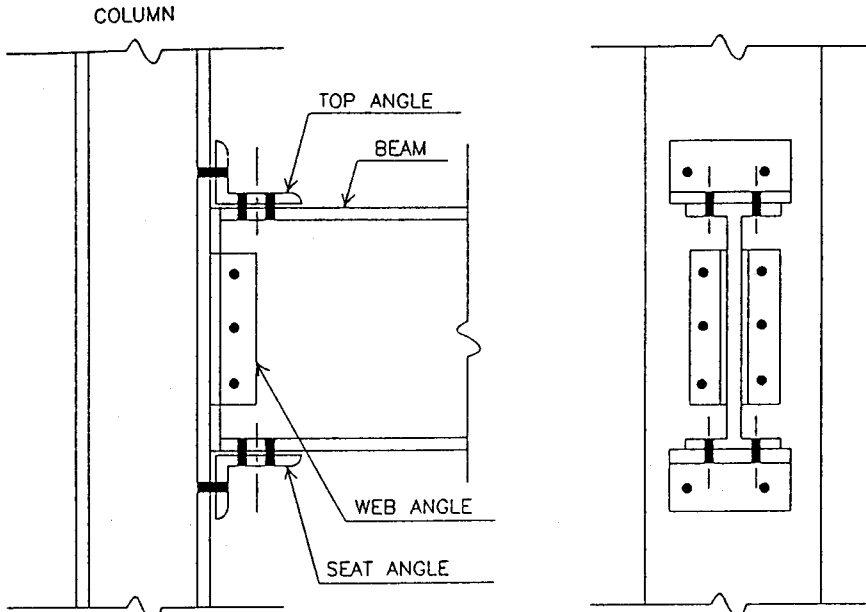
Fig. 7. Connection types and standarization parameters.

Table 1. Standardized connection moment – rotation functions

Connection type	Dimensions factor	Standardized moment-rotation equations	Maximum deviation of standardized curves form experimental curves(%)	Reference for experimental curves	Number of experimental curves produced
Single web angle connection	$K' = d^{-2.09} t^{-1.64} g^{2.06}$	$\frac{\theta}{1.03 \times 10^2} = \frac{K' \times M}{3275} \left[ 1 + \left( \frac{K' \times M}{3275} \right)^{29.3} \right]$	-11	Linspon (1968)	6
Double web angle connection	$K' = d^{-2.2} t^{0.08} g^{-0.28}$	$\frac{\theta}{3.98 \times 10^{-3}} = \frac{K' \times M}{0.63} \left[ 1 + \left( \frac{K' \times M}{0.63} \right)^{3.94} \right]$	-18	Batho & Rowan (1934) Lewitt et al. (1977) Sommer (1969)	7
Header plate connection	$K' = d^{-2.41} t^{-1.54} g^{2.12} w^{-0.4}$	$\frac{\theta}{7.04 \times 10^{-3}} = \frac{K' \times M}{186.77} \left[ 1 + \left( \frac{K' \times M}{186.77} \right)^{3.32} \right]$	-12	Sommer (1969)	7
Top and seat angle connection	$K' = d^{-1.6} t^{-0.54} p^{0.85} f^{-1.28}$	$\frac{\theta}{5.17 \times 10^{-3}} = \frac{K' \times M}{745.94} \left[ 1 + \left( \frac{K' \times M}{745.94} \right)^{4.61} \right]$	-4	Hechtman and Jonhston (1947)	7
Strap angle connection	$K' = h^{0.059} t^{-0.85} \left( \frac{H}{p} \right)^{-1.06}$	$\frac{\theta}{4.58 \times 10^{-5}} = \frac{K' \times M}{753.26} \left[ 1 + \left( \frac{K' \times M}{753.26} \right)^{4.98} \right]$	-5	Burn and Picard (1976) Beaulieu and Giroux (1974)	5

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In the present study, an additional moment-rotation equation is developed for one of the most common type of connections which is called (top and seat angle with double web angle) by combined the equation of the two separate type of connections (top and seat angle connection and double web angle connection) , as shown in Fig.8.



**Fig. 8. Top and seat angle with double web angle connection.**

This combination can be used with any two types of connections by using the following procedure.

Let the moment-rotation equation of first type of connection is

$$\theta\theta_1 = f(M_1)$$

And for the second type of connection is

$$\theta\theta_2 = g(M_2)$$

Then there are two boundary conditions which are

$$\theta\theta_1 = \theta\theta_2 = \theta\theta$$

$$M_1 + M_2 = M$$

Where

$\theta\theta_1$  = rotation of first type of connection;

$\theta\theta_2$  = rotation of second type of connection;

- $\theta$  = connection rotation ;
- $M_1$  = moment carried by first type of connection;
- $M_2$  = moment carried by second type of connection;
- $M$  = applied moment on connection(known).

Then by solving the above equations we can get the connection rotation  $\theta$  due to applied moment  $M$ .

## SETTING UP THE STIFFNESS MATRIX FOR SEMI-RIGID CONNECTION MEMBERS

### Semi -Rigid Joint Connections (Plane Element)

The case considered is a member attached to the joints at its ends by flexible connections, each of which transmits a moment proportional to the difference between the rotation of the end of the member and the rotation of the joint to which it is attached (differential rotation). Such joints occur in bolted frames, and in welded frames after the onset of plasticity. In both these cases, the moment-rotation relationship is unlikely to be linear, and the aim is to find the stiffness matrix of the complete system of the member plus the two connections.

Fig.9 shows a straight uniform member of length  $L$  and flexural rigidity  $EI$ , whose ends 1 and 2 are attached to the joints of a plane frame connections which exert moments  $EI \cdot K_1/L$  and  $EI \cdot K_2/L$ , respectively, (where  $k_1$  and  $k_2$  are defined in equation (10), per unit difference in rotation. The length of each connection is ignored therefore, the translations of each end of the member are the same as those of the corresponding joint although the rotation may differ.

Using the following expressions

$$r = K.e, \quad e = F.r$$

where  $r$ , the stress resultant vector, defines the state of stress in the element, and  $e$  the deformation vector, defines the corresponding deformations.

If  $c$  and  $d$  indicate the connections and  $m$  indicates the member, to find the flexibility of the complete member.



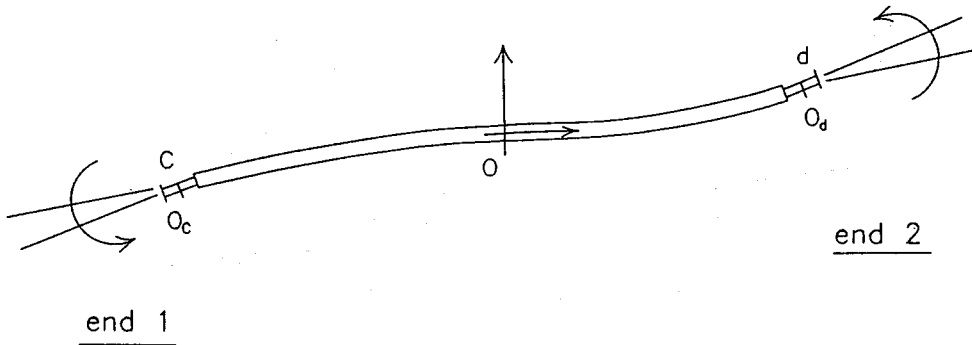


Fig. 9. A member with semi-rigid end-connection

$$F_c = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & L/EIk_1 \end{bmatrix}$$

$$F_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & L/EIk_2 \end{bmatrix}$$

$$F_m = \begin{bmatrix} L/EA & 0 & 0 \\ 0 & L^2/EI & 0 \\ 0 & 0 & L/EI \end{bmatrix}$$

$$H_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & L/2 & 1 \end{bmatrix}$$

$$H_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -L/2 & 1 \end{bmatrix}$$

$$F_m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And

$$k_1 = \frac{M_1 L}{EI \theta_1} \tag{10a}$$

$$k_2 = \frac{M_2 L}{EI \theta_2} \tag{10b}$$

Where

- $F_c$  = Flexibility matrix of connection, end 1.
- $H_c$  = Equilibrium matrix of connection, end 1.
- $F_d$  = Flexibility matrix of connection, end 2.
- $H_d$  = Equilibrium matrix of connection, end 2.
- $F_m$  = Flexibility matrix of member.

$H_m$  = Equilibrium matrix of member.

$M_1$  = Moment carried by connection, end 1.

$M_2$  = Moment carried by connection, end 2.

$\theta_1$  = Differential rotation of connection, end 1.

$\theta_2$  = Differential rotation of connection, end 2.

No transformation matrices are required, since the coordinate axes for the segments coincide with the coordinate axes for the member. Thus we have

$$F = H_C^t F_C H_C + H_m^t F_m H_m + H_d^t F_d H_d \quad (11)$$

Evaluating this expression, we obtain

$$F = \begin{bmatrix} L/EA & 0 & 0 \\ 0 & (L^3/12EI)(1+3/K_1+3/k_2) & (L^2/2EI)(1/K_1-1/k_2) \\ 0 & (L^2/2EI)(1/K_1+1/k_2) & (L/EI)(1+1/K_1+1/k_2) \end{bmatrix}$$

which on inversion gives

$$K = F^{-1}$$

$$K = \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3k} (k_1k_2 + k_1 + k_2) & \frac{6EI}{L^2k} (k_1 - k_2) \\ 0 & \frac{6EI}{L^2k} (k_1 - k_2) & \frac{EI}{Lk} (k_1k_2 + 3k_1 + 3k_2) \end{bmatrix}$$

where  $k = k_1k_2 + 4(k_1+k_2) + 12$ . The  $K_{ij}$  matrices for the complete member now follow in the usual way.

$$K_{ij} = H_i \cdot K \cdot H_j^t$$

$$[K] = \begin{bmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{bmatrix}$$

Then the stiffness matrix of plane member according to sign convention shown in Fig. 10 is.

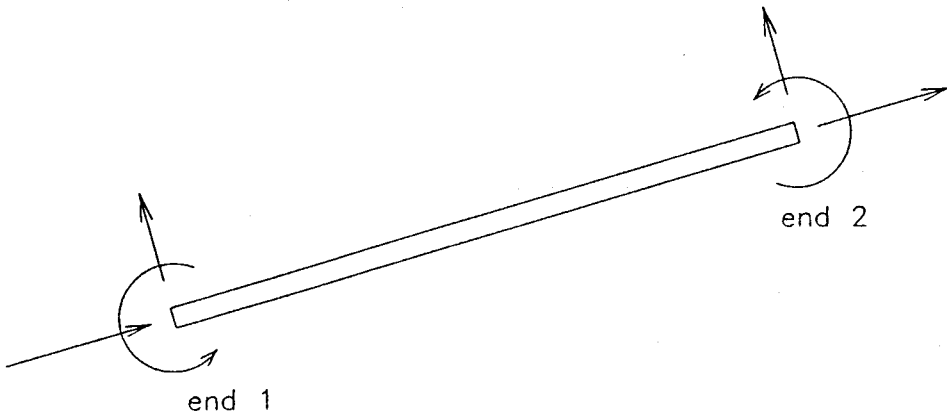


Fig. 10. Sign convention of plane element.

$$[K] = \begin{bmatrix} \frac{4EI}{L} \beta_1 & \frac{2EI}{L} \beta_3 & \frac{6EI}{L^2} \beta_4 & \frac{6EI}{L^2} \beta_4 & 0 & 0 \\ \frac{2EI}{L} \beta_3 & \frac{4EI}{L} \beta_2 & \frac{6EI}{L^2} \beta_5 & \frac{6EI}{L^2} \beta_5 & 0 & 0 \\ \frac{6EI}{L^2} \beta_4 & \frac{6EI}{L^2} \beta_5 & \frac{12EI}{L^3} \beta_6 & \frac{12EI}{L^3} \beta_6 & 0 & 0 \\ \frac{6EI}{L^2} \beta_4 & \frac{6EI}{L^2} \beta_5 & \frac{12EI}{L^3} \beta_6 & \frac{12EI}{L^3} \beta_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{EA}{L} & \frac{EA}{L} \\ 0 & 0 & 0 & 0 & \frac{EA}{L} & \frac{EA}{L} \end{bmatrix} \quad (12)$$

Where

$$\begin{aligned} \beta_1 &= (k_1 k_2 + 3k_1) / k \\ \beta_2 &= (k_1 k_2 + 3k_2) / k \\ \beta_3 &= (k_1 k_2) / k \\ \beta_4 &= (k_1 k_2 + 2k_1) / k \\ \beta_5 &= (k_1 k_2 + 2k_2) / k \\ \beta_6 &= (k_1 k_2 + k_1 + 2k_2) / k \end{aligned}$$

A usual special case of this result occurs when one  $k$  value is equal to zero and the other is equal to infinity. This corresponds to a member with a pin joint at one end and a rigid joint at the other.

(a)  $k_1=0, k_2 = \infty$ . (pin joint at end 1. rigid joint at end 2.)

Substituting these value we obtain

$$[K] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{3EI}{L} & \frac{3EI}{L^2} & \frac{3EI}{L^2} & 0 & 0 \\ 0 & \frac{3EI}{L^2} & \frac{3EI}{L^3} & \frac{3EI}{L^3} & 0 & 0 \\ 0 & \frac{3EI}{L^2} & \frac{3EI}{L^3} & \frac{3EI}{L^3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{EA}{L} & \frac{EA}{L} \\ 0 & 0 & 0 & 0 & \frac{EA}{L} & \frac{EA}{L} \end{bmatrix}$$

(b)  $K_1 = ?$ ,  $k_2 = 0$ . (pin joint at end 2. Rigid joint at end 1.)

$$[K] = \begin{bmatrix} \frac{3EI}{L} & 0 & \frac{3EI}{L^2} & \frac{3EI}{L^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3EI}{L^2} & 0 & \frac{3EI}{L^3} & \frac{3EI}{L^3} & 0 & 0 \\ \frac{3EI}{L^2} & 0 & \frac{3EI}{L^3} & \frac{3EI}{L^3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{EA}{L} & \frac{EA}{L} \\ 0 & 0 & 0 & 0 & \frac{EA}{L} & \frac{EA}{L} \end{bmatrix}$$

The reader may notice that the last term of the matrix  $K_{22}$  in case (a) (and  $K_{11}$  in case (b)) has been reduced from  $4EI/L$  to  $3EI/L$  by the presence of the pin joint. This corresponds to the modification of rotational stiffness normally made in the moment distribution process when a member has a pin joint at one end. Finally it may be noted that if  $k_1$  and  $k_2$  are both zero then all the flexural terms in the stiffness matrices become zero, which corresponds to the fact that a member which is pinned at both ends may be treated as a member of zero flexural rigidity.

Only bending deformations were taken into account for the flexural terms on the derivation of the stiffness matrix. Small additional deformations also arise due to shear. These are usually negligible, but for completeness, the shear deformations have been taken into account [15], the overall stiffness matrix is shown in *Appendix A*.

## **Computer Aids to Analysis of Semi –Rigid Steel .....**

### **Semi-Rigid Joint Connections (Space Element)**

So many papers were written on semi-rigid framing techniques. In spite of the importance of space frame and its application, there are a few papers dealing with space frame analysis using semi-rigid techniques <sup>(3,14,16)</sup>. The above mentioned papers don't present a complete solution of the case, therefore the authors present a simple and detailed procedure for space frame analysis with semi-rigid connections.

Now the effect of semi-rigid on the space element stiffness matrix is considered with the following assumption and limitations.

#### **ASSUMPTION AND LIMITATIONS**

The imposed assumption and limitation are the following.

- a) Axial and direct shearing deformations in connections are assumed to have infinite stiffness. Thus, only the bending moment-rotation behavior of the connection in the plane of major axis (xy local plane) is modeled as a nonlinear function.
- b) All members are prismatic, straight and linearly elastic.
- c) Possible buckling is ignored.
- d) Small deflection theory is employed.
- e) Connections are of negligible size. Hence, connection deformations are concentrated at the ends of the beams.

Then, the stiffness matrix of semi-rigid space element according to sign convention shown in Fig.11, with taken into account the above assumption and limitation will be.

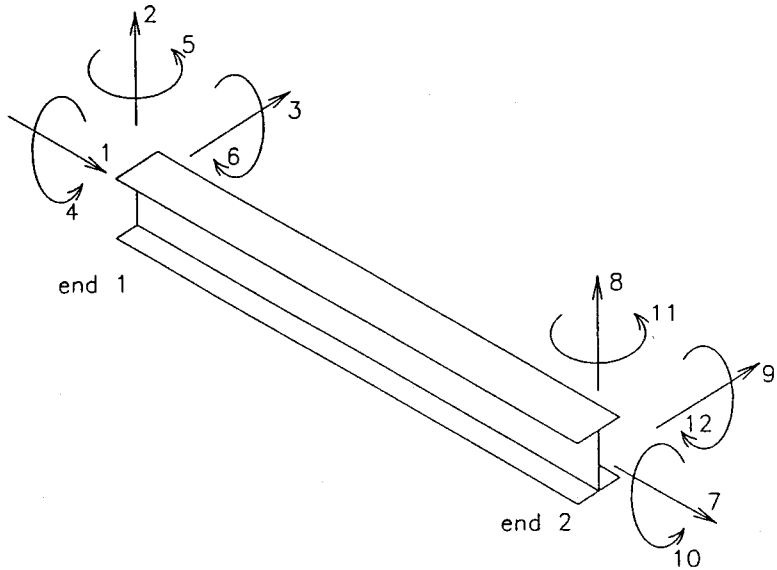


Fig. 11. Sign convention of space element.

$$K_{11} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12 EI_z \beta_6}{L^3} & 0 & 0 & 0 & \frac{6 EI_z \beta_4}{L^2} \\ 0 & 0 & \frac{12 EI_y}{L^3} & 0 & \frac{6 EI_y}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & 0 & \frac{6 EI_y}{L^2} & 0 & \frac{2 EI_y}{L} & 0 \\ 0 & \frac{6 EI_z \beta_4}{L^2} & 0 & 0 & 0 & \frac{2 EI_z \beta_1}{L} \end{bmatrix}$$

$$K_{22} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} \beta_6 & 0 & 0 & 0 & \frac{6EI_z}{L^2} \beta_5 \\ 0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 \\ 0 & \frac{6EI_z}{L^2} \beta_5 & 0 & 0 & 0 & \frac{4EI_z}{L} \beta_2 \end{bmatrix}$$

$$K_{12} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} \beta_6 & 0 & 0 & 0 & \frac{6EI_z}{L^2} \beta_4 \\ 0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 \\ 0 & \frac{6EI_z}{L^2} \beta_4 & 0 & 0 & 0 & \frac{2EI_z}{L} \beta_3 \end{bmatrix}$$

$$K_{21} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} \beta_6 & 0 & 0 & 0 & \frac{6EI_z}{L^2} \beta_5 \\ 0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 \\ 0 & \frac{6EI_z}{L^2} \beta_5 & 0 & 0 & 0 & \frac{2EI_z}{L} \beta_3 \end{bmatrix}$$

and

$$[k] = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

Then the overall stiffness matrix of semi-rigid space element with including shear deformation effect is listed in *Appendix B*.

## APPLICATIONS

In the following examples 1 and 2, six types of connection are used with the following properties:

1. For single and double Web angle connection:
2. Web angle size = L4x 4x 3/8"
3. Web angle length = 9"
4. For Header plate connection, using header plate dimension
5. Header plate length = 6"
6. Header plate width = 4"
7. Header plate thick = 0.5"
8. For Top and Seat angle connection:
9. Top and seat angle size L4x4x3/8"



## Computer Aids to Analysis of Semi-Rigid Steel .....

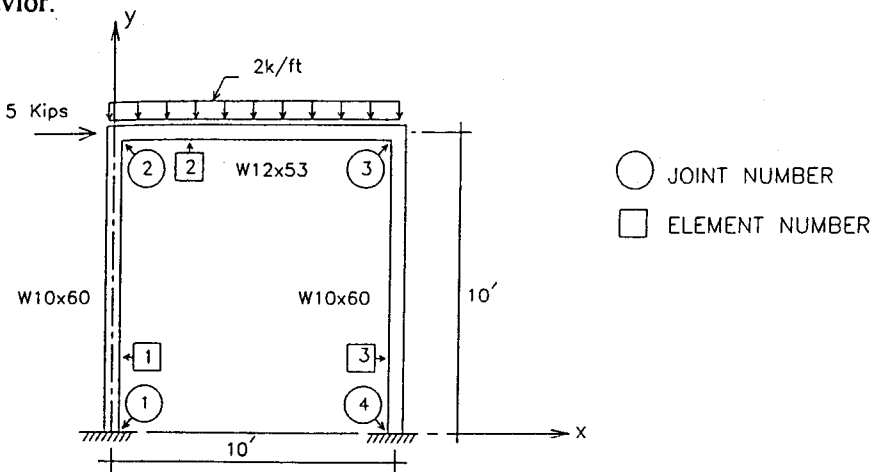
10. Top and seat angle length = 7"
11. Bolts diameter  $\phi = 0.875"$
12. For Top and Scat angle with double web angle connection, using the same properties of the two individual types.
13. For Strap connection: Strap angle size  $L2x2x1/4"$
14. The moment -rotation relationship of the above types of connection is shown in Fig.5.
15. The maximum differential rotation of beam-to-column connection in steel frames is dependent on the type of loading. It varies within a narrow range of 0.016 radians for a point load at mid-span to 0.021 radians for uniformly distributed loads. The algorithm is not very sensitive to this value, so a value of 0.02 radians was found to work well for commonly encountered loading. (17)

### Example 1

In this example, the steel plane frame shown in Fig. 12 is analyzed firstly as rigid and then as semi-rigidly connected, with different types of connections.

The frame is analyzed five times, firstly assuming all beam-column connections to be rigid, then in the second solution they are assumed to have header plate type of connection. In the third solution, the connections are assumed to be top and seat angle, in the fourth solution all connections are assumed to be top and seat angle with double web angle, and finally in the fifth solution they are assumed as strap angle.

Full comparison of the maximum bending moment of frame's members for various type of connection between the rigid analysis and semi-rigid analysis for all cycles is shown in Table 2. In this table, an absolute values of maximum bending moment are listed in order to promote the understanding of the frame's behavior.



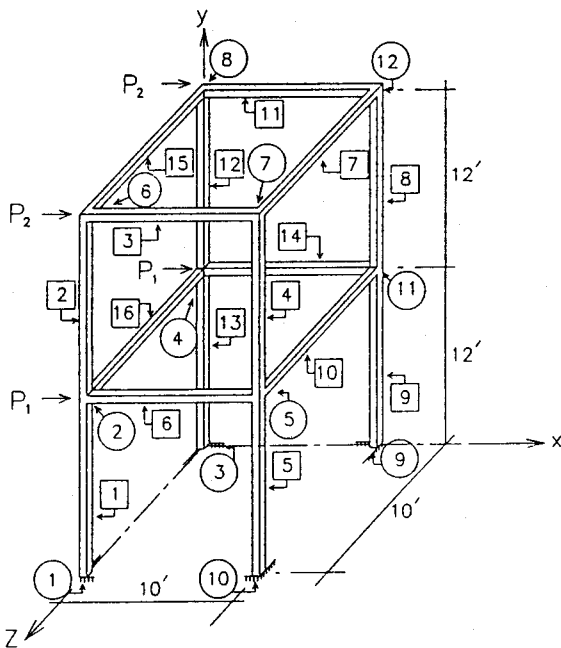
**Fig. 12. Plane frame of example 6.1.**

Table 2. Comparison of rigid and semi-rigid maximum bending moment of example 1, k.ft

Member No.	Solution 1	Solution 2		Solution 3		Solution 4		Solution 6	
	Max. Moment	Max. Moment	$\frac{M_{semi-rigid}}{M_{rigid}}$	Max. moment	$\frac{M_{semi-rigid}}{M_{rigid}}$	Max. Moment	$\frac{M_{semi-rigid}}{M_{rigid}}$	Max. moment	$\frac{M_{semi-rigid}}{M_{rigid}}$
1	22.053	26.025	1.180	23.6361	1.071	23.407	1.061	21.763	0.986
2	22.060	18.572	0.841	20.227	0.916	20.379	0.923	21.744	0.985
3	8.127	11.976	1.473	9.502	1.169	9.389	1.155	8.317	1.023

**Example 2**

The two story steel space frame with external loading is shown in Fig.13. Normally, the beam-column connections would be assumed to be act as rigid and semi-rigid.



ALL COLUMNS W10x60  
 ALL BEAMS W12x53  
 $P_1 = 5$  Kips  
 $P_2 = 7$  Kips  
 LOAD ON BEAMS 6-10-14-16  
 $W_y = 2$  K/ft  
 LOAD ON BEAMS 3-7-11-15  
 $W_y = 1$  K/ft  
 COLUMNS MAJOR AXIS ON Z AXIS

Fig. 13. Space frame of example 6.2.

## Computer Aids to Analysis of Semi-Rigid Steel .....

As in example 1, the frame is analyzed six times with various types of connection: five times as in example 1, and in the sixth solution the program will choose the suitable connection types according to the connection moment capacity or according to maximum permissible differential rotation<sup>(17)</sup>.

As in the previous example, a full comparison of the maximum bending moment of frame's members for various type of connection between the rigid analysis and semi-rigid analysis for all cycles is shown in Table 3.

**Table 3. Comparison of rigid and semi-rigid maximum bending moment of example 2, k.ft.**

Member No.	Solution 1		Solution 2		Solution 3		Solution 4		Solution 5		Solution 6	
	Max. Moment	Max. Moment	M <sub>semi-rigid</sub> /M <sub>rigid</sub>	Max. Moment	M <sub>semi-rigid</sub> /M <sub>rigid</sub>	Max. Moment	M <sub>semi-rigid</sub> /M <sub>rigid</sub>	Max. Moment	M <sub>semi-rigid</sub> /M <sub>rigid</sub>	Max. Moment	M <sub>semi-rigid</sub> /M <sub>rigid</sub>	Max. Moment
1	47.423	63.689	1.342	55.630	1.173	54.942	1.158	48.992	1.033	69.051	1.456	
2	29.272	21.029	0.718	27.111	0.926	27.460	0.938	26.913	0.919	24.435	0.834	
3	29.272	21.013	0.717	27.110	0.926	27.459	0.938	26.600	0.908	22.794	0.778	
4	15.045	5.833	0.387	12.214	0.811	12.656	0.841	15.779	1.048	5.117	0.340	
5	40.621	56.900	1.390	48.789	1.192	48.142	1.176	42.730	1.044	60.496	1.478	
6	53.989	28.757	0.532	40.585	0.751	41.544	0.769	50.125	0.928	29.525	0.546	
7	6.978	7.225	1.035	6.389	0.915	6.312	0.904	6.0911	0.990	7.823	1.122	
8	15.405	5.834	0.378	12.213	0.792	12.656	0.821	15.779	1.024	5.117	0.332	
9	40.921	56.909	1.390	48.787	1.192	48.142	1.176	42.730	1.044	60.497	1.478	
10	13.214	14.912	1.128	13.417	1.015	13.276	1.004	13.079	0.989	14.953	1.131	
11	29.272	21.013	0.717	27.109	0.926	27.459	0.938	26.600	0.908	22.794	0.778	
12	29.272	21.030	0.718	27.110	0.926	27.460	0.938	26.914	0.919	24.436	0.834	
13	47.424	63.690	1.342	55.628	1.173	54.942	1.158	48.992	1.033	69.052	1.456	
14	53.989	28.757	0.532	40.584	0.751	41.455	0.769	50.126	0.928	29.525	0.546	
15	6.978	7.225	1.035	6.389	0.915	6.312	0.904	5.554	0.990	7.832	1.122	
16	13.214	14.912	1.128	13.147	1.015	13.276	1.004	13.079	0.989	14.953	1.131	

Columns
Beams

## CONCLUSIONS

In this research work, a semi-rigid analysis of plane and space frames was carried out, and an efficient interactive computer program for this analysis is presented.

A complete computer package for the analysis of frames with semi-rigid connections is available.

The most important conclusions are:

The analysis of space frame was based on two ideas, firstly the idea is presented by Livesley<sup>(18)</sup>, a simple modeling of the member with semi-rigid connections by dividing the member to three parts, the main member and the two connections, and a combination technique of the three parts was discussed. Secondly, the three-dimensional members were analyzed as semi-rigid in the plane of major axis<sup>(3)</sup>, the other components was treated as rigid, this was based on the fact that all members designed to be carried the principals forces in the plane of major axes of the members. Research work is needed to study the behavior of the connections in the plane of minor axis.<sup>(3,14,16)</sup>

On these two ideas the analysis computer program of space frame was built and the program output comes logical and compatible with the real geometrical behavior.<sup>(2,3)</sup>

As long as, the results of the frame analysis are concerned, the program requires one cycle to analysis the plane and space frames as rigidly connected, and the other cycles treated the frame connections as semi-rigid, the accuracy of results after the second cycle is around 65%, the third cycle results accuracy 96%, and the fourth cycle results are comes with an accuracy 99%.

In spite of the amount of differential rotation between beam and column is relatively small (not more than 0.002 rad.), this differential rotation have a significant effect in the geometrical behavior of frame structures.

When semi-rigid connections are considered in the analysis, a significant reduction in the beam's maximum bending moment is obtained. Although, this reduction in the maximum beam's bending moment will cause an increase in column's bending moments, this increment will cause little effect in design process, the axial stresses will control the selection process of columns section.

## Computer Aids to Analysis of Semi -Rigid Steel .....

As presented above, including the effect of semi-rigid in the analysis of frame structures will results an economical design solution. This economical solution can be obtained through the selection of an economical suitable connection's type, and through the reduction in beam's maximum moment which leads to the selection of an economical beam.

### REFERENCES

1. **Chen W.F., 1987** "Joint Flexibility In Steel Frames," Elsevier Applied Science Publishers LTD..
2. **Frye,M.J, and Morris, G.A, 1976** "Analysis Of Flexibly Connected Steel Frames," Canadian Journal Of Civil Engineers, Vol.2, No.3, pp.280-291, September
3. **Ang, K.M., And Morris, G.A., 1984**"Analysis Of Three-Dimensional Frames With Flexible Beam-Column Connections, "Canadian Journal Of Civil Engineers, Vol.11, pp.245-254.
4. **Nethercot D.A. , 985** "Joint Action And The Design Of Steel Frames," The Structural Engineer, Vol. 63A,No.12, pp.371-379.
5. **Jones S.W., P.A.kirby and d.a.nethercot, 1980** "Effect of Semi-rigid Connection on Steel Column Strength," Journal of Constructional Steel Research, Vol.1, No.1, pp.38-40.
6. **Al-Rifaie, W.N., and Trikha, D.N. ,1986** "Elastic-Plastic Analysis of Frames with Wide Flange Members," Int. Conf. On Steel Structures, Yugoslavia, Sep. 28-Oct.1.
7. **Maseer, A.H.1989** "Elastic-Plastic Analysis of Steel Frames with Large Deformations," M.Sc Thesis, Military Engineering College, Baghdad, Oct.
8. **Davison J.B., P.A.Kirby, and D.A.Nerthcot, 1987** "Rotational Stiffness Characteristics of Steel Beam-to-Column Connections," J.Construct .Steel Research, Vol.8,pp.17-54.
9. **Lindsey, S.D., 1987** "Design of Frames with PR Connections, "J.Construct. Steel Research, Vol.8, pp.251-260.

10. Azizinamini A., Bradburn J.H. and Rasaziminiski J.B., 1987 "Initial Stiffness of Semi-rigid Steel Beam-to-Column Connections," J. Construct. Steel Research, Vol.8, pp.71-90.
11. Kukreti A.R., Murray, T.M. and Abolmaali A. , 1987 "End-Plate Connection Moment-Rotation Relationship," J.Construct. Steel Research, Vol.8, pp.137-157.
12. Lui E.M., And Chen W.F., 1987" Steel Frame Analysis With Flexible Joints," J. Construct. Steel Research, Vol.8, pp.161-202.
13. Chen W.F., and Kishi N.,1989 "Semi-rigid Steel Beam-to-Column Connections: Data Base and Modeling," Journal of Structural Eng. Vol.115,N.1, pp.105-117, January.
14. Colson A., 1987 "Three - dimensional Physical and Mathematical of Connections," Laboratoire de Mecanique et Technologie Ecole Normale Supérieure,Cachan,France.
15. Hall A., and Kabaila A.P., 1982 " Basic Concepts of Structural Analysis ," Pitman Publishing , 1<sup>st</sup> Edition.
16. Janes J., Jaspert J.P., and Maquoi R., 1987 "Strength and Behaviour of In-plane Weak axis Joints of 3-D Joints," Laboratoire de Mecanique et Technologie Ecole Normale Supérieure, Cachan, France.
- 17.Ioannides S.A., 1987 "Frame Analysis Including Semi-rigid Connections and P-Delta Effect," Laboratoire de Mecanique et Technologie Ecole Normale Supérieure, Cachan, France.
18. Livesley, R.K., 1975 "Matrix Methods of Structural Analysis," Pergmon SInternational Library, 2<sup>nd</sup> Edition.

APPENDIX A

Stiffness Matrix of Semi-Rigid Plane Elements

The stiffness matrix of semi-rigid plane element K with taken into account effect of shear deformation was listed bellow according to the sign convention of Fig.10

$$[K] = \begin{bmatrix} \frac{4EI}{L} \beta_1 \Psi_1 & \frac{2EI}{L} \beta_3 \Psi_1 & -\frac{6EI}{L^2} \beta_4 \Phi_1 & \frac{6EI}{L^2} \beta_4 \Phi_1 & 0 & 0 \\ \frac{2EI}{L} \beta_3 \Psi_1 & \frac{4EI}{L} \beta_2 \Psi_1 & -\frac{6EI}{L^2} \beta_5 \Phi_1 & \frac{6EI}{L^2} \beta_5 \Phi_1 & 0 & 0 \\ -\frac{6EI}{L^2} \beta_4 \Phi_1 & \frac{6EI}{L^2} \beta_5 \Phi_1 & \frac{12EI}{L^3} \beta_6 \Phi_1 & -\frac{12EI}{L^3} \beta_6 \Phi_1 & 0 & 0 \\ \frac{6EI}{L^2} \beta_4 \Phi_1 & \frac{6EI}{L^2} \beta_5 \Phi_1 & -\frac{12EI}{L^3} \beta_6 \Phi_1 & \frac{12EI}{L^3} \beta_6 \Phi_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{EA}{L} & \frac{EA}{L} \\ 0 & 0 & 0 & 0 & \frac{EA}{L} & \frac{EA}{L} \end{bmatrix}$$

Where

$$\alpha_1 = \frac{6EI}{L^2 GA_Y} \dots\dots\dots(A-1)$$

$$\phi_1 = \frac{1}{2\alpha_1 + 1} \dots\dots\dots(A-2)$$

$$\Psi_1 = \frac{\alpha_1 / 2 + 1}{2\alpha + 1} \dots\dots\dots(A-3)$$

In the above expressions,  $A_Y$  is the shear area corresponding to shear force in the y direction, y being the principal axes of the section.

APPENDIX B

Stiffness Matrix of Semi-Rigid Space Elements

The stiffness matrix of semi-rigid plane element [K] with taken into account effect of shear deformation is listed bellow according to the sign convention of Fig.11

$$K_{11} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} \beta_6 \phi 1 & 0 & 0 & 0 & \frac{6EI_z}{L^2} \beta_4 \phi 1 \\ 0 & 0 & \frac{12EI_y}{L^3} \phi 2 & 0 & -\frac{6EI_y}{L^2} \phi 2 & 0 \\ 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & 0 & -\frac{6EI_y}{L^2} \phi 2 & 0 & \frac{4EI_y}{L} \psi 2 & 0 \\ 0 & \frac{6EI_z}{L^2} \beta_4 \phi 1 & 0 & 0 & 0 & \frac{4EI_z}{L} \beta_1 \psi 1 \end{bmatrix}$$

$$K_{22} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} \beta_6 \phi 1 & 0 & 0 & 0 & -\frac{6EI_z}{L^2} \beta_5 \phi 1 \\ 0 & 0 & \frac{12EI_y}{L^3} \phi 2 & 0 & \frac{6EI_y}{L^2} \phi 2 & 0 \\ 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & 0 & \frac{6EI_y}{L^2} \phi 2 & 0 & \frac{4EI_y}{L} \psi 2 & 0 \\ 0 & -\frac{6EI_z}{L^2} \beta_5 \phi 1 & 0 & 0 & 0 & \frac{4EI_z}{L} \beta_2 \psi 1 \end{bmatrix}$$



Computer Aids to Analysis of Semi-Rigid Steel .....

$$K_{12} = \begin{bmatrix} -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_z}{L^3}\beta_6\phi_1 & 0 & 0 & 0 & \frac{6EI_z}{L^2}\beta_4\phi_1 \\ 0 & 0 & -\frac{12EI_y}{L^3}\phi_2 & 0 & -\frac{6EI_y}{L^2}\phi_2 & 0 \\ 0 & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & 0 & \frac{6EI_y}{L^2}\phi_2 & 0 & \frac{2EI_y}{L}\psi_2 & 0 \\ 0 & -\frac{6EI_z}{L^2}\beta_4\phi_1 & 0 & 0 & 0 & \frac{2EI_z}{L}\beta_3\psi_1 \end{bmatrix}$$

$$K_{21} = \begin{bmatrix} -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_z}{L^3}\beta_6\phi_1 & 0 & 0 & 0 & -\frac{6EI_z}{L^2}\beta_5\phi_1 \\ 0 & 0 & -\frac{12EI_y}{L^3}\phi_2 & 0 & \frac{6EI_y}{L^2}\phi_2 & 0 \\ 0 & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & 0 & -\frac{6EI_y}{L^2}\phi_2 & 0 & \frac{2EI_y}{L}\psi_2 & 0 \\ 0 & -\frac{6EI_z}{L^2}\beta_3\phi_1 & 0 & 0 & 0 & \frac{2EI_z}{L}\beta_3\psi_1 \end{bmatrix}$$

and

$$[k] = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

where

$$\alpha_2 = 6EI_Y / (L^2GA_Z)$$

$$\phi_2 = 1 / (2\alpha_2 + 1)$$

$$\Psi_2 = (\alpha_2 / 2 + 1) / (2\alpha_2 + 1)$$

## Abbas and AL-Waily

In the above expressions,  $A_y$  and  $A_z$  are the shear areas corresponding to shear force in the  $y$  and  $z$  directions respectively,  $y$  and  $z$  being the principal axes of the section.