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NEW ALGORITHMS FOR DIGITAL ANALYSIS OF POWER INTENSITY OF NON STATIONARY SIGNALS

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ABSTRACT

Discrete and continuos time-frequency algorithms of the power intensity of non-stationary signals are proposed. Results show that the power intensity waveform analysis performed well in both discrete and continuous time-frequency characteristics. Two methods for the digital implementation of the proposed algorithms are described and assessed.

INTRODUCTION

Power intensity estimation techniques have proven invaluable in the study of the data processing systems employed in communication, sonar, radar, biomedical and geophysical applications. Fast Fourier transform (FFT) offers high spectral resolution over a wide signal bandwidth but suffers from limited analysis flexibility due to stationary assumptions. The spectrogram is one useful method for the study of time-frequency characteristics of signals with marked changes in both time and frequency [1-2]. In this method the signal is subdivided into smaller records and each subrecord is assumed to be stationary. Each subrecord is multiplied by a data window and then FFT analysis is applied on each data record. This technique produces useful qualitative characteristics of the signal but may require careful interpretation of the quantitative feature of the plot. As such, average values of the fluctuation characteristics are obtained depending on the duration of the data window.

A new efficient technique for the study of the spectra of non-stationary signal is proposed. The technique is based on the FT of the intensity of the signal rather than the FT of the signal itself. A subdivision of the signal is not needed and hence the limitation due time-frequency uncertainty is avoided [3].

MATHEMATICAL ANALYSIS

In power spectrum estimation, the signal under consideration is processed in such away that the distribution of power among its frequency components is estimated. As such the phase relation between frequency components are suppressed. The information contained in the power spectrum is essentially that which is present in the autocerrelation sequence. This would suffice for complete statistical description of a Gaussian signal.

The Fourier transform of a weakly non-stationary signal as a function of time and frequency is given by [4]:

$$F\{s^2(t)\} = \int df\, S(f+\upsilon)S^*(f) \tag{1}$$

where $F[s^2(t)]$ is the Fourier transform of the signal.

If the signal is stationary, the statistical averaging of the intensity waveform $s^2(t)$ is constant. Consequently, the statistical averaging of $S(f+\upsilon)S^*(f)$ must vanish for all values $\upsilon \neq 0$, thus.

$$F_\upsilon\{s^2(t)\} = \int df\, S(f+\upsilon)S^*(f) \tag{2}$$

where E denotes the expected value or statistical averaging.

The Fourier transform is a function of both time and frequency indicating the nonstationary scharior of the signal, and the Fourier integral maps a signal in one domain to the other [5]. Equation (2) provides a continuos time-dependent output for a continuos frequency-dependent input. However, the implementation of the transform requires a mathematical representation of the function and the calculation of the integral expression. It is therefore desirable to perform a similar operation on discrete signals to facilitate implementation with digital hardware. Thus, the intensity waveform in discrete from may be written as:

$$S^2(m) = \sum_{n}\sum_{n^1} S_n S^*_{n^1} e^{2j\pi m(n-n^1)/N} \tag{3}$$

where $n = 0, \pm 1, \pm 2, \ldots$, is a complete orthogonal set of which Parseval's relation holds,

N = Finite duration length of sample signals

m = 1, 2,.....,(N-1)/2

The discrete Fourier transform of $S^2(m)$ is defined as:

$$\begin{aligned} DFT\{s^2(m)\} &= \frac{1}{N}\sum_m S^2(m)\, e^{-j\pi mn/N} \\ &= \sum_l \sum_l S_l S_{l^1}^{*} \frac{1}{N}\sum_m e^{2j\pi m(l-l^1-n)/N} \\ &= \sum_l S_{l+n} S_{l^1}^{*} \end{aligned} \tag{4}$$

This discrete-sum spectral representation for a finite-length sequence is referred to as the discrete Fourier transform (DFT). If the fluctuation is non-stationary, then

$$E[S_{l+n}\, S_{l^1}^{*}] = 0, \quad n \neq 0 \tag{5}$$

and hence we have:

$$DFT\ \{E[S^2(m)]\} = \sum_l E[S_{l+n} S_{l^1}^{*} \tag{6}$$

DIGITAL IMPLEMENTATION

An important Criterion for the digital implementation of the technique is the development of a statistical averaging method. Two algorithms are proposed for the computation of the DFT based on averaging over a time-duration and averaging over a set of decimated series.

A) Averaging over a time-duration $T_l = m\Delta t$: the time-duration procedure can be performed as follows:

1) Form L=N/M Section of data $S^{l(m)}$,
m=1, 2, 3,.........., M and l =1, 2, 3,........, L.

2) Compute the mean square value averaged over M points.

$$E[S^2(l)] = \frac{1}{M} \sum_{m=(l-1)M+1}^{m=LM} [S(m)]^2 \tag{7}$$

3) Compute

$$P_l = SFT\{E[S^2(l)]\} \quad l = 1,2,...,L/2 \tag{8}$$

4) The mean square value spectrum of the intensity waveform is

$$I_l = 2|P_l|^2, \quad l = 1,2,...,L/2 \tag{9}$$

where $l = 1,2,3,.....,lL/2$.

B) Averaging over a set of decimated series: The DFT sum represented by equation (6) is computed by carrying out the sum over the first and last half of the input signal separately. the sum is therefore produced by the following decimated procedure:

1) Form M series by decimating each M^{th} point ML=N

$$\begin{aligned} &S^{(1)}(l), l = 1, M+1, 2M+1, ..., (L-1)M+1 \\ &S^{(2)}(l), l = 2, M+2, ..., (L-1)M+2 \\ &S^{(M)}(l), l = M, 2M, ..., LM \end{aligned} \tag{10}$$

2) Compute the sample mean square value spectrum:

$$[S^{(m)}]^2, m = 1,...,M \text{ and } l = 1,...,L \tag{11}$$

3) Average over m points

$$DFT\{[S^{(m)}(l)]^2\}, \quad m = 1,2,...,M \qquad l = 1,2,...,L/2 \tag{12}$$

Both methods are based upon the assumption that the statistical characteristics of s(t) is stationary with an effective sampling interval $M\Delta t$ by averaging, i.e., both methods are identical with respect to the frequency resolution. The two methods are however different and should be applied simultaneously depending on the application and the form of the fluctuation signal under consideration.

DISCUSSION

Computer simulations were performed in order to evaluate the vialibility and effectiveness of the proposed optimum power intensity technique. Various plots of the power intensity are compared to conventional spectrograms. The surface plot of time dependent spectrogram of a crossed chirp signal shown in Fig.(1) may be compared to the power intensity of the same signal shown in Fig.(2). In addition, the contour plot of the cross chirp signal obtained using spectrogram analysis as shown in Fig.(3) is compared to the power intensity contour shown in Fig.(4). The time frequency resolution of the power intensity method produces more accurate results than that of the spectrogram. However, a large dc value is produced using the proposed method due to squaring. This dc value has no physical meaning and may be removed by simple filtering techniques.

Moreover, the power intensity method may be considered superior to spectrogram techniques in which subrecords are assumed stationary which makes certain signals hard to analyze with such spatial methods. The wavelet analyzer is the most obvious application of modern spectrum analysis of waveforms of this type of nonstationary signals. Hence, power intensity and wavelet spatial spectrum method is proposed as the most likely path for attainary increased analysis.

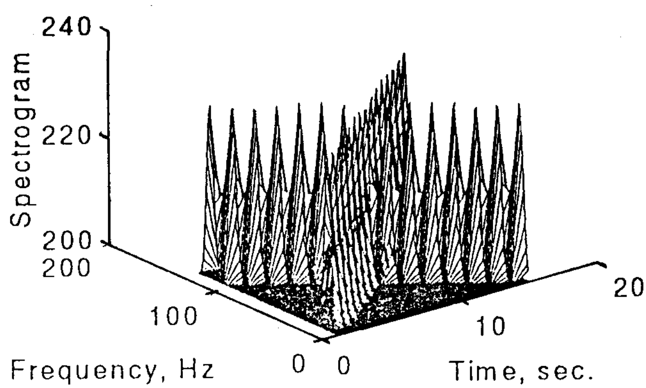

Fig. 1. Spectrogram surface plot of crossed-chirp signal

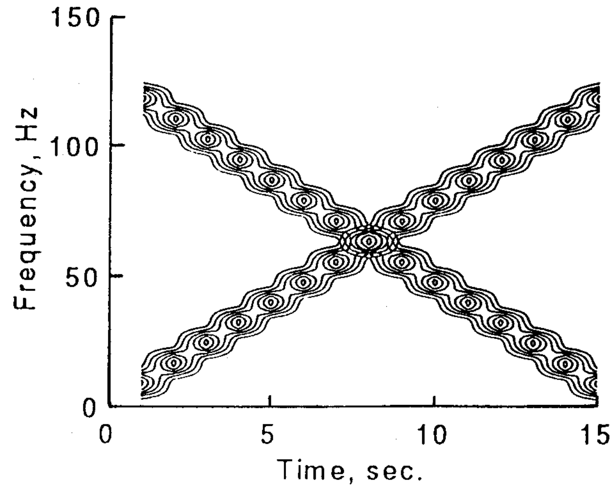

Fig. 2. Spectrogram contour plot of crossed-chirp signal

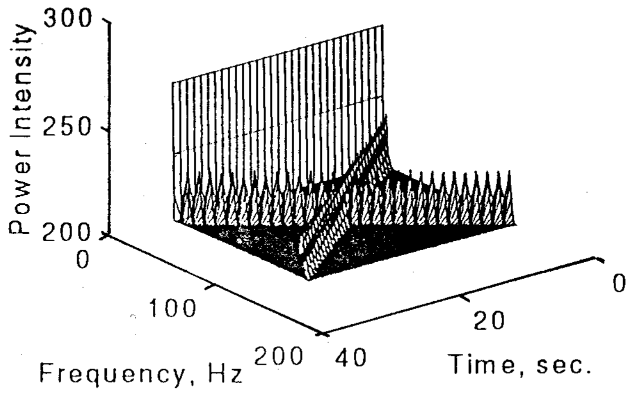

Fig. 3. Power intensity surface plot of crossed-chirp signal

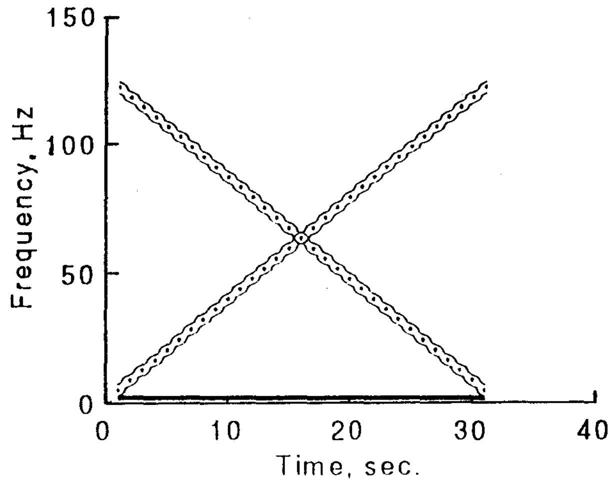

Fig. 4. Power intensity contour plot of crossed-chirp signal

CONCLUSIONS

A new method which describes the characteristics of nonstationary signals is proposed. The power intensity of the signal is considered rather than the signal itself. Two algorithms with the same effective frequency resolution are proposed for the digital implementation of the new method. The algorithms are however different in estimating the expected values and they should be used depending on the application and type of the signal under study.

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