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"ON LOSS PROBABILITIES FOR DELAYED - ACCESS CIRCUIT - SWITCHED MULTIPLEXERS"

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ABSTRACT

Multiplexing schemes for packetized circuit-switched sources are characterized by an avoidable accessing delay. In the present work it is shown that, contrary to conventional immediate-access multiplexing schemes, one can introduce two different definitions of loss probability in the delayed-access case. Explicit analytical expressions are given for each definition, and a numerical example is presented which highlights the difference between them.

I. INTRODUCTION

In recent years efforts have intensified in order to implement the long advocated concept of Integrated Services Digital Network (ISDN), [1,2]. With such a network, services like voice, data, video, and text would be accommodated at each user-network interface point, and moreover, these services would all share the same transmission medium. Meanwhile, the idea of sharing transmission resources can be appealing to many Gulf States, since they are linked to almost the same destinations, and they invariably use their telecommunication facilities to provide similar services. To achieve high network throughput within this framework, hybrid multiplexing schemes that integrate circuit-switched sources (such as voice), and packet-switched sources (such as data), have been proposed. These multiplexing schemes generally require the packetization of circuit-switched sources, as for example in the SENET scheme [3] and the NEC 1.2 Gb/s loop [4]. Though packetization makes it possible to apply adaptive policies for sharing the channel bandwidth, it also results in delaying the access of the circuit-switched sources to the network. The name "delayed-access" multiplexers is therefore given to such hybrid schemes.

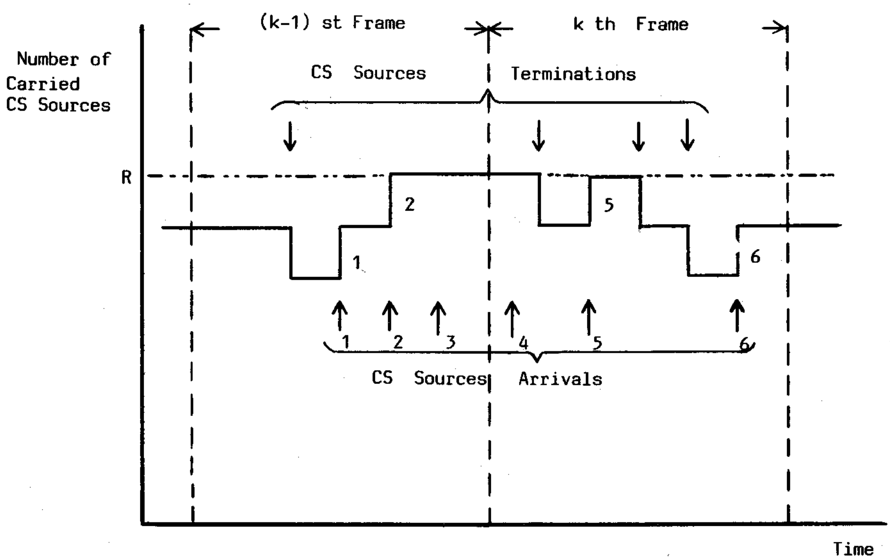
Analysis of delayed-access multiplexers has been considered by a number of authors [5,6]. Nonetheless, little attention has been given to the significance of the access delay on the loss probability of circuit-switched (CS) sources. In the present work this issue is addressed in detail, and it is shown that two different definitions of loss probability can be made. In addition, a number of analytical results are given, and a numerical example is presented.

II. MODEL FOR CIRCUIT-SWITCHED MULTIPLEXERS WITH DELAYED- ACCESS

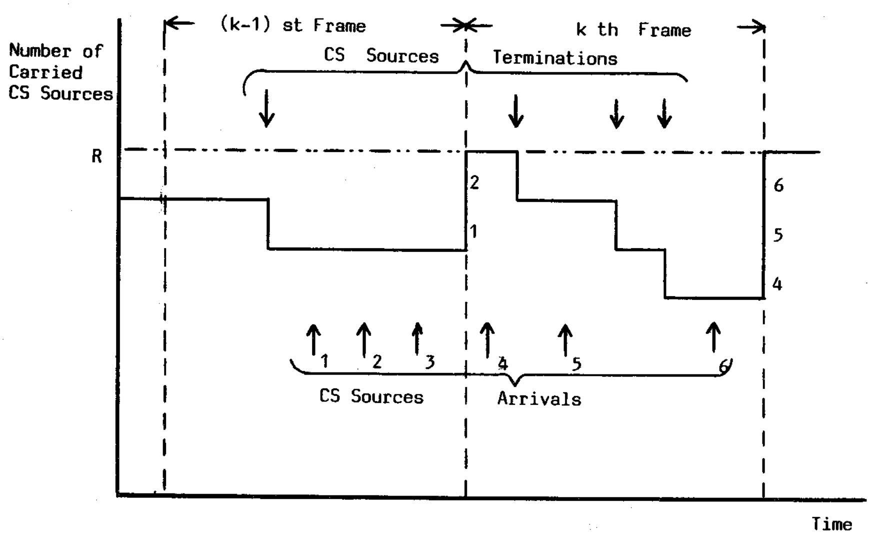
The performance of circuit-switched (CS) multiplexers with accessing delay can differ considerably from that of multiplexers with immediate access. This fact is illustrated in Figs. 1−a and 1−b, which compare the total number of carried CS sources in an immediate access and a delayed-access multiplexing schemes. Here, it is assumed that the total capacity of the transmission channel is R, where

$$R = \frac{\text{Channel Transmission Rate (bits/s)}}{\text{Transmission Rate Per CS Source (bits/s)}} \tag{1}$$

Moreover, it is assumed that the time horizon is subdivided into frames of fixed duration, such that decisions for accessing an incoming source are made either immediately (in the immediate-access case), or at the end of the frame (in the delayed-access case). It is evident from Fig. 1 that the sample functions for the two systems behave differently despite the fact that the arrival and termination statistics are the same. In particular, whereas the immediate-access system accommodates only two sources (5 and 6) during the k th frame period, the delayed-access system is capable of accommodating three sources (4,5,6) at the end of the k th frame period. With this simple illustration it becomes obvious that previous results derived for the immediate-access system do not directly extend to its delayed-access counterpart. In the following, we present a mathematical model that is appropriate for analyzing the latter system.



(a)



(b)

Fig. (1) – Sample functions for the number of carried circuit-switched sources over the transmission channel: (a) case of immediate-access multiplexer, (b) case of delayed-access multiplexer.

Consider a typical sample function for the number of carried CS sources in a channel of maximum capacity R sources. (See Fig. 2 and the accompanying notation). Further, assume that the arrival process $\{A(k)\}$ and termination process $\{T(k)\}$ are mutually independent, and that each constitutes an independent sequence. It then follows that the number of CS sources continuing into the k th frame, C(k), is statistically dependent only on C(k−1), and that C(k) acts as the system state with $\{C(k)\}$ constituting a one-dimensional Markov chain. Under statistical equilibrium conditions, one can therefore express the steady-state probabilities for $C(k) \triangleq C$, as:

$$P_C(j) = \sum_{r=0}^{R} \sum_{i=0}^{r} P_{T|N}(r-j|r) \cdot P_A(r-i) \; P_C(i) \tag{2}$$

$$+ \sum_{r=R+1}^{\infty} \sum_{i=0}^{R} P_{T|N}(R-j|R) \cdot P_A(r-i) \; P_C(i)$$

$$j = 0, 1, \ldots, R$$

where

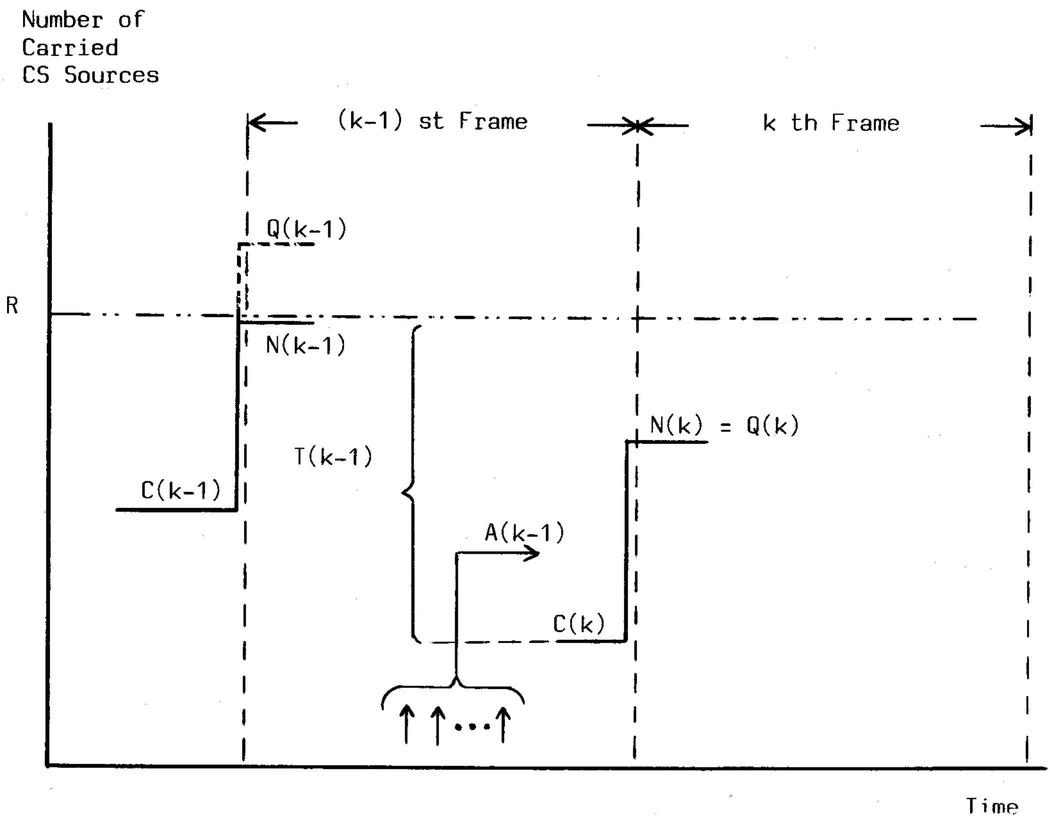
$$P_C(x) = \text{Prob}\{C = x\}$$

$$P_{T|N}(x|y) = \text{Prob}\{T = x \mid N = y\}$$

$$P_A(x) = \text{Prob}\{A = x\}$$

and C, T, N, and A are, respectively, the steady-state variables denoting the number of CS sources continuing into the present frame, the number of CS sources terminating within the present frame, the number of CS sources carried at the beginning of the present frame, and the number of CS sources arriving within the present frame. Using matrix notation and the constraint that $\sum_{j=0}^{R} P_C(j) = 1$, the (R+1) equations in (2) can be expressed as:

$$P_C^{\,T} = P_{T|N}^{\,T}\big|_{N=R} \cdot U \cdot (I - F)^{-1} \tag{3}$$



T (k-1) = Number of CS sources terminating in (k−1) st frame.
C (k) = Number of CS sources continuing into the k th frame.
= N (k−1) − T (k−1).
A (k−1) = Number of CS sources arriving in (k−1) st frame.
Q (k) = Number of CS sources requesting service at start of the k th frame.
N (k) = Number of CS sources carried at the start of the k th frame.

$$= \begin{cases} Q(k) & \text{if} \quad Q(k) \leqslant R \\ R & \text{if} \quad Q(k) > R \end{cases}$$

Fig. (2) – Basic terminology relevent to the analysis of a delayed-access multiplexer.

where

$$P_C^T = [\ P_C(\emptyset)\quad P_C(1)\quad \ldots\quad P_C(R)\ ]$$

$$P_{T|N}^T\ |_{N=R} = [\ P_{T|N}(\emptyset|R)\quad P_{T|N}(1|R)\quad \ldots\quad P_{T|N}(R|R)\ ]$$

$$U(i,j) = \begin{cases} 1 & i + j = R \\ \emptyset & \text{otherwise} \end{cases} \qquad i,j = \emptyset, 1, \ldots, R$$

$$F(i,j) = \sum_{r=\emptyset}^{R} [P_{T|N}(r-j|r) - P_{T|N}(R-j|R)].P_A(r-i)$$

From $[P_C(i)]$ one an deduce $[P_Q(j)]$ and $[P_N(j)]$ using the following relations:

$$P_Q(j) = \sum_{i=\emptyset}^{j} P_C(i)\ P_A(j-i) \qquad \emptyset \leqslant j < \infty \tag{4}$$

$$\begin{aligned} P_N(j) &= P_Q(j) & \emptyset \leqslant j \leqslant R-1 \\ &= 1 - \sum_{i=\emptyset}^{R-1} P_C(i) \sum_{r=\emptyset}^{R-1-i} P_A(r) & j = R \end{aligned} \tag{5}$$

In the above, Q denotes the steady-state number of CS sources requesting access at the start of a frame, whereas N is the steady state number of CS sources actually accessed.

III. EXPRESSIONS FOR LOSS PROBABILITY

To express the loss probability of a delayed-access multiplexer, two definitions may be adopted:

D1 : P_{L1} = Probability that a source which arrives during a frame is blocked, at steady-state (6)

$$D2 : P_{L2} = \frac{\text{Average number of blocked sources during one frame (at s.s)}}{\text{Average number of arriving sources during one frame (at s.s)}} \tag{7}$$

Definition D1 is the direct interpretation of the term "blocking probability" and is useful in applications where one is concerned with the blocking of individual calls, as for example in routing problems. Definition D2, on the other hand, gives the aggregate effect of the system on the total incoming traffic. It is useful in the determination of the average number of carried sources per frame when the average number of arriving sources is known.

Making use of the previously defined notation, one can express (6) and (7), explicitly, as:

$$P_{L1} = \sum_{i=\emptyset}^{R} P_C(i) \sum_{a=R-i+1}^{\infty} P_A(a) . \frac{a-(R-i)}{a} \Big/ (1 - P_A(\emptyset)) \tag{8}$$

$$P_{L2} = \sum_{i=\emptyset}^{R} P_C(i) \sum_{a=R-i+1}^{\infty} P_A(a) . [a - (R-i)] \Big/ \sum_{j=1}^{\infty} j\, P_A(j)$$

$$= 1 - \sum_{j=1}^{R} j\, [P_N(j) - P_C(j)] \Big/ \sum_{j=1}^{\infty} j\, P_A(j) \tag{9}$$

It is to be noticed that the term ($1-P_A(0)$) appears in the denominator of (8) since we condition our probability on the event that at least one source has arrived during the frame period. Meanwhile, in obtaining equation (9), one exploits the fact that the average number of lost sources per frame equals the average number of arrivals per frame less the average number of newly carried sources per frame.

It is interesting to investigate expressions (8) and (9) in the limiting case when the frame duration tends to zero. Under this condition, the multiplexer reduces to one of the immediate-access type, which implies that Eqs. (8) and (9) should reduce to the well known Erlang B formula [7]. Indeed, assuming the arrival process to be Poisson with rate λ arrivals/s, that the source service time is exponentially distributed with inverse mean holding time μ s^{-1}, and the frame duration to be dt, we have:

$$P_A(i) = \begin{cases} 1 - \lambda \cdot dt & i = 0 \\ \lambda \cdot dt & i = 1 \\ 0 & i > 1 \end{cases} \tag{10}$$

$$P_N(j) = \begin{cases} P_C(0)\ (1 - \lambda \cdot dt) & j = 0 \\ P_C(j) \cdot (1 - \lambda \cdot dt) + P_C(j-1) \cdot \lambda \cdot dt & 1 \leq j \leq R-1 \\ P_C(R) + P_C(R-1) \cdot \lambda \cdot dt & j = R \end{cases} \tag{11}$$

Substituting from (10) and (11) into (8), one obtains:

$$\begin{aligned} P_{L1} &= P_C(R) \cdot P_A(1) \,/\, (1 - P_A(0)) \\ &= P_N(R) \qquad\qquad \text{as } dt \longrightarrow 0 \\ &= E(\lambda/\mu,\ R) \end{aligned}$$

where E(a,r) is the Erlang B formula for a channel with capacity "r" subject to traffic of intensity "a". Likewise, one obtains from (9):

$$P_{L2} = 1 - \frac{\sum_{j=1}^{R-1} j\left[-P_C(j)+P_C(j-1)\right].\lambda.dt + R.P_C(R-1).\lambda.dt}{\lambda . dt}$$

$$= P_N(R) \qquad \text{as} \quad dt \longrightarrow \emptyset$$
$$= E(\lambda/\mu, R)$$

The important conclusion is that for an immediate-access multiplexer, $P_{L1} = P_{L2} = E(\lambda / \mu, R)$ under the Poisson assumption, whereas for the delayed-access multiplexer the two loss probabilities are generally different. This last point is further investigated in the following section through the use of a numerical example.

IV. A NUMERICAL EXAMPLE

We consider a T1−carrier system having a transmission rate of 1.544 Mb/s and assume that all CS sources using this channel operate at rate 64 kb/s. Consequently, the channel capacity = 1544/64 = 24. Furthermore, suppose that each source provides a traffic of 0.2 Erlang, which is generated by a Poisson arrival process of rate 0.001 s^{-1} and an exponential service time of mean value 200 seconds. Then, assuming the total number of sources connected to the system to be M, the total arrival rate becomes $\lambda = M \times .001\ s^{-1}$ and the inverse of mean service time is $\mu = .005\ s^{-1}$. Moreover, if we suppose that each CS source is packetized into packets of length K bits, the frame duration for this system would be $T_F = K/64000$ s. It is now possible to express the statistics of the arrival and departure processes for the above system in terms of λ, μ, and T_F, as follows:

$$P_A(n) = e^{-\lambda T_F} \frac{(\lambda T_F)^n}{n!}, \quad n = \emptyset, 1, 2, \ldots \tag{12}$$

$$P_{T|N}(x|y) = \frac{y\,!}{x\,!\,.\,(y-x)\,!}\,(1 - e^{-\mu T_F})^{x}\,.$$

$$(e^{-\mu T_F})^{y-x}, \quad x=0,\ldots,R \quad x \leq y \leq R$$

$$= 0 \qquad \text{otherwise,} \qquad (13)$$

By choosing suitable values for M and K, one can readily calculate the vectors P_C and P_N by substituting from (12) and (13) into (3) and (5), respectively. Subsequently, the two loss probabilities P_{L1} and P_{L2} can be determined from (8) and (9). The obtained results are summarized in Table 1 below.

The table shows that both P_{L1} and P_{L2} coincide with the Erlang B formula when the frame duration T_F is very small. As T_F increases, P_{L1} initially decreases until it reaches a minimum after which it increases again. Moreover, the value of T_F at which the minimum is attained is large for small λ and is small for large λ. On the other hand, P_{L2} shows a continuous increase in its value as T_F increases from zero, irrespective of the value of λ. Such behavior can be explained as follows.

Recalling the sample functions of Fig. 1, we see that under some conditions the delayed-access multiplexer can accommodate more calls (and hence has smaller loss probability) than the immediate-access system. This is possible because as the incoming sources are delayed, there is a higher chance for the sources already occupying the channel to be terminated, thereby making it possible to accommodate more sources in the next frame. Such fact is illustrated in Fig. 3—a, which shows that as T_F increases, the average number of sources continuing into the next frame decreases, and consequently the proportion of available channel bandwidth increases. However, as T_F becomes relatively large, the number of sources arriving within T_F becomes so large that only a small fraction of them are serviced within the next rfame. This leads to an increase in the loss probability, especially when λ is large.

TABLE (1)

Loss probabilities for the T1 – carrier system when using a delayed-access multiplexing scheme

Frame Duration, T_F (s)	Total Arrival Rate, λ (s^{-1})													
	.050		.075		.100		.125		.200		.500		.1.00	
	P_{L1} x10^4	P_{L2} x10^4	P_{L1} x10^2	P_{L2} x10^2	P_{L1} x10	P_{L2} x10	P_{L1}	P_{L2}	P_{L1}	P_{L2}	P_{L1}	P_{L2}	P_{L1}	P_{L2}
.001	0.7317	0.7317	0.8393	0.8393	0.6609	0.6609	0.1679	0.1679	0.4299	0.4299	0.7630	0.7630	0.8806	0.8806
.1	0.7295	0.7317	0.8368	0.8393	0.6589	0.6609	0.1674	0.1679	0.2486	0.4299	0.7607	0.7630	0.8780	0.8806
1.0	0.7104	0.7321	0.8148	0.8396	0.6416	0.6611	0.1630	0.1680	0.4174	0.4299	0.7418	0.7631	0.8584	0.8807
10.0	0.5747	0.7688	0.6616	0.8708	0.5232	0.6774	0.1341	0.1708	0.3571	0.4337	0.7133	0.7663	0.8681	0.8829
20.0	0.5243	0.8728	0.6139	0.9531	0.4932	0.7177	0.1286	0.1775	0.3625	0.4419	0.7440	0.7716	0.8794	0.8858
Immediate -Access Loss Prob	0.7317x10^{-4}		0.8393x10^{-2}		0.6609x10^{-1}		0.1679		0.4299		0.7630		0.8806	

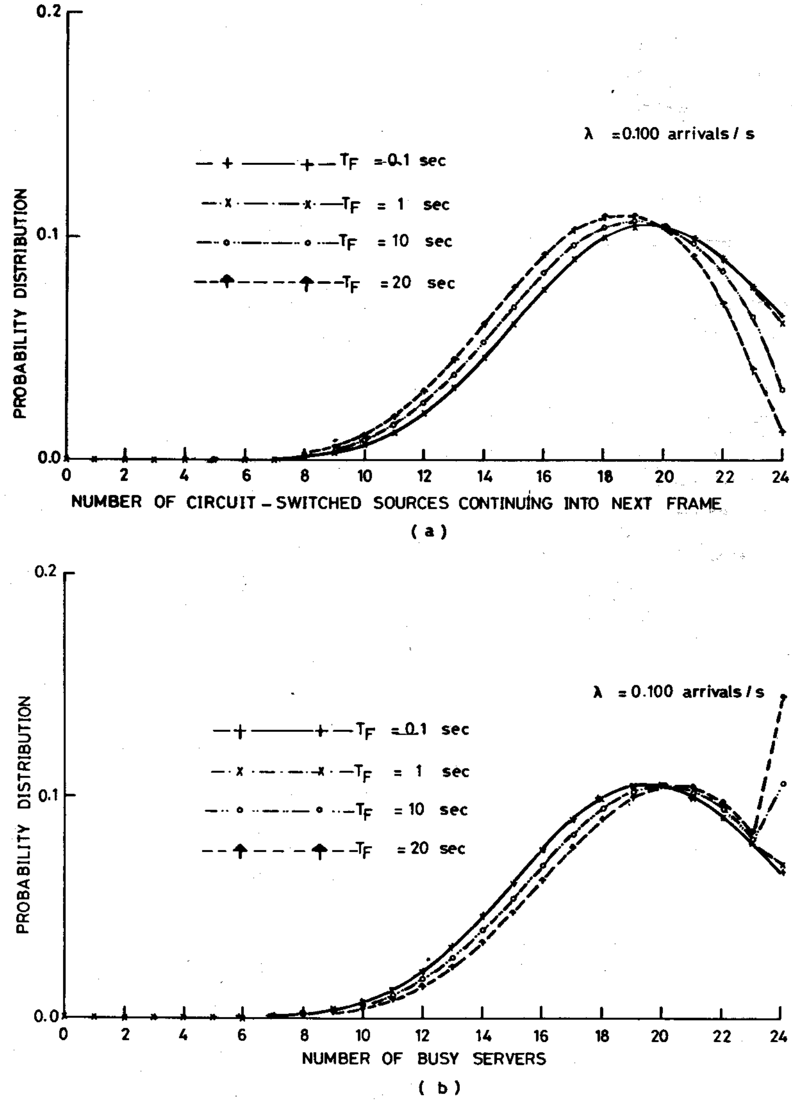


Fig. (3) – Effect of frame duration on the distribution curves for a delayed-access circuit-switched multiplexer: (a) probability distribution for the number of sources continuing into the next frame, (b) probability distribution for the number of occupied channels.

Meanwhile, a comparison of the R.H.S. in the first line of Eq. (9) with that of Eq. (8) shows that, while we divide by

$\sum_{j=1}^{\infty}$ j P_A (j) in the case of P_{L2}, the corresponding term is

$a \sum_{j=1}^{\infty}$ P_A (j) in the case of P_{L1}. In other words, P_{L2} uses a constant weighting

factor for all values of "a" in the summation, whereas P_{L1} uses a weight that decreases with the value of "a". Consequently, one would expect P_{L2} to be larger, in general, than P_{L1}. On the other hand, the second line of Eq. (9) shows that P_{L2} is inversely proportional to the difference between the average of N and that of C, but is dirctly proportional to T_F. Figures 3−a and 3−b show the dependence of the probability distributions for C and N on T_F, and the numerical computation of C and N can then confirm that as T_F increases from zero, $(N-C)/T_F$ decreases and hence P_{L2} increases.

V. CONCLUSIONS

In the present work it has been shown that two different definitions of loss probabilities are possible for a delayed-access multiplexer: a "true" loss probability and an "aggregate" loss probability. Our investigation has revealed that the true loss probability initially decreases with the frame duration of the packetized source, and then increases with further increase in the duration. The aggregate loss probability, on the other hand, increases monotonically with the frame duration.

It is to be remarked that for conventional multiplexing schemes, in which incoming sources are accessed immediately, the two definitions of loss probabilities yield the same value. Therefore, it is immaterial which definition we use for system analysis purposes. For delayed-access multiplexers, however, it is important that we use the "true" loss probability definition since it provides more accurate information regarding the blocking of individual sources. Moreover, it is clear from our investigation that one can select an optimum value for the frame duration such that it yields the smallest loss probability for a given traffic volume.

Meanwhile, considering the special case of packetized voice and assuming typical permissible system delays, one may find the difference between loss probabilities for the immediate-access and delayed-access schemes to be small. This fact, however, should not overshadow the importance of using the appropriate expression for the loss probability of delayed-access schemes; whenever nonvoice applications are considered (as for example in image transmission), and also whenever the blocking probabilities of individual routes are to be determined.

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