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A PARAMETRIC STUDY ON TRANSPORTATION PROBLEM UNDER FUZZY ENVIRONMENT

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ABSTRACT

This paper presents a solution algorithm for solving the transportation problem having fuzzy parameters in the constraints. A parametric study is carried out for this problem. Some basic stability notions are defined and characterized for the problem of concern. These notions are the set of feasible parameters, the solvability set and the stability set of the first kind. Finally, an illustrative numerical example is given to clarify the theory and the solution algorithm.

***KEYWORDS*:** Fuzzy parameters, Transportation problem, α-level set, Stability.

1. INTRODUCTION

Transportation problems are generally concerned with the distribution of certain product from several sources to numerous locations at minimum cost. Suppose there are m warehouses where a commodity is stocked, and n markets where it is needed. Let the supply available in the warehouses be $a_1, a_1, \ldots, a_m$ and the demands at the markets be $b_1, b_1, \ldots, b_n$. The unit cost of shipping from warehouse i to market j is $\$c_{ij}$. (If a particular warehouse cannot supply a certain market, we set the appropriate c_{ij} at $+\infty$). The objective is to minimize the total cost of transportation from all the warehouses to all the markets.

The application of the general transportation problem (GTP) is not limited to transporting commodities between sources and destinations. The generalized transportation problem was first posed by Ferguson and Dantzig [7] in a paper discussing the allocation of aircraft to routes (see also [10,11,13]). Many papers are introduced for solving such problems. For example Charnes and Cooper [5], Garvin [8] and Hadley [9]. A dual method for solving GTP was presented by Balas [1] and an operator theory of parametric programming for GTP was presented by Balachandran and Thompson [2-4].

This paper is organized as follows: In section 2, some basic definitions on fuzzy set theory are introduced. In section 3, we formulate the GTP having the supply available in the warehouses a_i as fuzzy parameters (FGTP). Section 4 is devoted to a parametric study on the FGTP. In section 5, we propose a solution algorithm to solve the problem of concern. In section 6, an illustrative numerical example is given to clarify the theory and the solution algorithm. Finally, section 7 contains the conclusions.

2. FUZZY CONCEPTS

L. A. Zadeh advanced the fuzzy theory at the university of California in 1965. The theory proposes a mathematical technique for dealing with imprecise concepts and problems that have many possible solutions.

The concept of fuzzy mathematical programming on a general level was first proposed by Tanaka et al (1974) in the framework of the fuzzy decision of Bellman and Zadeh [14]. Now, we present some necessary definitions [6].

Definition 1

A real fuzzy number $\tilde{a}$ is a fuzzy subset of the real line R with membership function $\mu_{\tilde{a}}$ satisfies the following conditions:

(1) $\mu_{\tilde{a}}$ is a continuous mapping from R to the closed interval [0,1].

(2) $\mu_{\tilde{a}}(a) = 0 \ \forall \ a \in (-\infty, a_1]$.

(3) $\mu_{\tilde{a}}$ is strictly increasing and continuous on $[a_1, a_2]$.

(4) $\mu_{\tilde{a}}(a) = 1 \quad \forall \ a \in [a_2, a_3]$.

(5) $\mu_{\tilde{a}}(a)$ is strictly decreasing and continuous on $[a_3, a_4]$.

(6) $\mu_{\tilde{a}}(x) = 0 \quad \forall \ a \in [a_4, +\infty)$.

where a_1, a_2, a_3 and a_4 are real numbers, and the fuzzy number is denoted by

$\tilde{a} = [a_1, a_2, a_3, a_4]$.

Definition 2

The fuzzy number $\tilde{a} = [a_1, a_2, a_3, a_4]$ is a trapezoidal number, denoted by $[a_1, a_2, a_3, a_4]$, its membership function $\mu_{\tilde{a}}$ is given by (see Fig.1):

$$\mu_{\tilde{a}}(a) = \begin{cases} 0, & a \le a_1, \\ 1 - \left(\dfrac{a - a_2}{a_1 - a_2}\right)^2 & a_1 \le a \le a_2, \\ 1, & a_2 \le a \le a_3, \\ 1 - \left(\dfrac{a - a_3}{a_4 - a_3}\right)^2 & a_3 \le a \le a_4, \\ 0, & \text{otherwise} \end{cases}$$

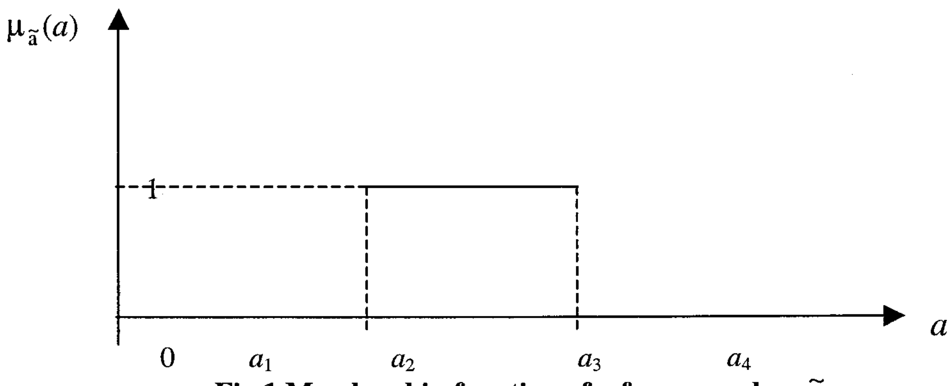

Fig.1 Membership function of a fuzzy number $\tilde{a}$.

Definition 3

The α-level set of the fuzzy number $\tilde{a}$ is defined as the ordinary set $L_\alpha(\tilde{a})$ for which the degree of their membership function exceeds the level $\alpha \in [0,1]$:

$$L_\alpha(\tilde{a}) = \{a \in \mathrm{R} \mid \mu_{\tilde{a}}(a) \ge \alpha\}.$$

3. PROBLEM FORMULATION

The problem of consideration is the following fuzzy general transportation problem (FGTP):

(FGTP): Min $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

Subject tos

$\sum_{j=1}^{n} x_{ij} \le \tilde{a}_i \qquad i = 1, 2, ..., m$ (supply restriction at warehouse i)

$\sum_{i=1}^{m} x_{ij} \ge b_j \qquad j = 1, 2, ..., n$ (demand requirement at market j)

$x_{ij} \ge 0 \qquad \forall$ pairs (i, j).

where $\tilde{a}_i, i = 1, 2, ..., m$ represent fuzzy parameters involved in the constraints with their membership functions are $\mu_{\tilde{a}_i}$. For a certain degree α together with the concept of α-level set of the fuzzy numbers $\tilde{a}_i$, therefore problem (FGTP) can be understood as the following nonfuzzy general transportation problem (P_1):

(P_1): Min $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

Subject to

$\sum_{j=1}^{n} x_{ij} \le a_i \qquad i = 1, 2, ..., m$

$\sum_{i=1}^{m} x_{ij} \ge b_j \qquad j = 1, 2, ..., n$

$a_i \in L_\alpha(\tilde{a}_i) \qquad i = 1, 2, ..., m,$

$x_{ij} \ge 0 \qquad \forall$ pairs (i, j)

where $L_\alpha(\tilde{a}_i)$ are the α-level set of the fuzzy numbers $\tilde{a}_i$. The above problem can be written in the following equivalent form:

(P_2) Min $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} \le a_i \qquad i=1,2,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j \qquad j=1,2,\ldots,n$$

$$h_i^0 \le a_i \le H_i^0 \qquad i=1,2,\ldots,m$$

$$x_{ij} \ge 0 \qquad \forall \text{ pairs } (i, j)$$

It should be noted that the constraint ($a_i \in L_\alpha(\tilde{a}_i)$) has been replaced by the constraint $h_i^0 \le a_i \le H_i^0$, $i=1,2,\ldots,m$ where h_i^0 and H_i^0 are lower and upper bounds on a_i and they are constants.

4. A PARAMETRIC STUDY ON PROBLEM (P_2)

The parametric study of the problem (P_2) where h_i^0 and H_i^0, $i=1, 2, \ldots, m$ are assumed to be parameters rather than constants can be understood as follows:
Let $X(h, H)$ denotes the decision space of problem (P_3), defined by:

$$X(h, H) = \Big\{(x_{ij}, a_i) \in R^{m(n+1)} \;\Big|\; a_i - \sum_{j=1}^{n} x_{ij} \ge 0,\ \sum_{i=1}^{m} x_{ij} - b_j \ge 0,\ H_i - a_i \ge 0,\ a_i - h_i \ge 0,$$
$$x_{ij} \ge 0, \qquad i=1, 2, \ldots, m;\ j = 1, 2, \ldots, n\Big\}$$

Some Basic Stability Botions for the Parametric Problem

In what follows we give the definitions of some basic notions for the parametric problem. These notions are the set of feasible parameters, the solvability set, and the stability set of the first kind (see [12]).

The Set of Feasible Parameters

The set of feasible parameters of the parametric problem, which is denoted by U, is defined by:

$$U = \{(h, H) \in R^{2m} \,|\, X(h, H) \ne \phi\}.$$

The Solvability Set

The solvability set of the parametric problem, which is denoted by V, is defined by:

$$V = \{(h, H) \in U \mid \text{problem } (P_3) \text{ has } \alpha\text{-optimal solutions}\}.$$

The Stability Set of the First Kind

Suppose that h^*, $H^* \in V$ with a corresponding α-optimal solution (x_{ij}^*, a_i^*) of parametric problem. The stability set of the first kind of parametric problem which is denoted by

$S\ (x_{ij}^*, a_i^*)$ is defined by:

$S\ (x_{ij}^*, a_i^*) = \{(h, H) \in V \mid (x_{ij}^*, a_i^*),\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n$ is α-optimal solution of parametric problem$\}$.

Utilization of the Kuhn-Tucker Conditions Corresponding To Parametric Problem

The Lagrange function of parametric problem can be written as follows:

$$L = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \lambda_k(a_i - \sum_{j=1}^{n} x_{ij}) + \eta_k(\sum_{j=1}^{n} x_{ij} - b_j) + \rho_k \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} + \sum_{i=1}^{m} \theta_i(a_i - h_i) + \sum_{i=1}^{m} \beta_i(H_i - a_i), \qquad k = 1, 2, \ldots, nm$$

then, the Kuhn-Tucker necessary optimality conditions corresponding to the parametric problem at the solution (x_{ij}^*, a_i^*), $j = 1, 2, \ldots, n;\ i = 1, 2, \ldots, m$ will take the form:

$$\frac{\partial L}{\partial x_{ij}} = \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{\partial c_{ij}x_{ij}}{\partial x_{ij}} + \lambda_k \frac{\partial\left(a_i - \sum_{j=1}^{n} x_{ij}\right)}{\partial x_{ij}} + \eta_k \frac{\partial\left(\sum_{i=1}^{m} x_{ij} - b_j\right)}{\partial x_{ij}} + \rho_k \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} = 0, \quad k = 1, 2, \ldots, mn$$

$$\frac{\partial L}{\partial a_i} = \lambda_i \frac{\partial\left(a_i - \sum_{j=1}^{n} x_{ij} \right)}{\partial a_i} + \theta_i - \beta_i = 0, \qquad i = 1, 2, ..., m$$

$$a_i - \sum_{j=1}^{n} x_{ij} \geq 0 \qquad i = 1, 2, ..., m$$

$$\sum_{i=1}^{m} x_{ij} - b_j \geq 0 \qquad j = 1, 2, ..., n$$

$$H_i - a_i \geq 0, \quad a_i - h_i \geq 0, \quad i = 1, 2, ..., m$$

$$x_{ij} \geq 0 \qquad \forall \text{ pairs } (i, j)$$

$$\lambda_i (a_i - \sum_{j=1}^{n} x_{ij}) = 0 \qquad i = 1, 2, ..., m$$

$$\eta_j (\sum_{i=1}^{m} x_{ij} - b_j) = 0 \qquad j = 1, 2, ..., n$$

$$\beta_i (H_i - a_i) = 0, \quad \theta_i (a_i - h_i) = 0, \quad i = 1, 2, ..., m$$

$$\rho_k x_{ij} = 0 \qquad \forall \text{ pairs } (i, j), \qquad k = 1, 2, ..., mn$$

$$\beta_i, \eta_k, \lambda_k, \theta_i, \rho_k \leq 0, \quad i = 1, 2, ..., m; \quad j = 1, 2, ..., n; \quad k = 1, 2, ..., mn$$

where all the relations of the above system are evaluated at (x_{ij}^*, a_i^*) and θ_i, β_i, ,η_k, ρ_k, λ_k are the Kuhn-Tucker multipliers. The set $T(x_{ij}^*, a_i^*)$ is the set of parameters h and H for which the Kuhn-Tucker necessary optimality conditions corresponding to parametric problem are utilized at (x_{ij}^*, a_i^*). Clearly, this set can be considered as a subset from the set $S(x_{ij}^*, a_i^*)$, i.e. $T\ (\mathrm{x}_{ij}^*, \mathrm{a}_i^*) \subseteq S(\mathrm{x}_{ij}^*, \mathrm{a}_i^*)$.

5. SOLUTION ALGORITHM

In this section, we describe a solution algorithm for solving the (FGTP). In this algorithm, we will use the Vogel's Approximation Method (VAM) define a penalty for each row and column. This penalty is the absolute difference between the smallest and the next smallest cost in a given row or column. Next, the row or column with the largest penalty is identified and the variable that has the smallest cost in that row or column is selected as the next basic variable.

Step 1 Determine the points (a_1, a_2, a_3, a_4) for the fuzzy number in the formulation problem (FGTP).

Step 2 Convert the problem (p_1) in the form of the problem (p_2).

Step 3 Formulate the problem (p_2) in the parametric form of parametric problem.

Step 4 Apply VAM to get the basic feasible solution.

6. AN ILLUSTRATIVE EXAMPLE

Consider a transportation problem with three warehouses and four markets. The market demands are $b_1 = 4$, $b_2 = 3$, $b_3 = 4$, $b_4 = 4$. The warehouse capacities are given as fuzzy numbers as follows: $\tilde{a}_1 = (1,2,5,7)$, $\tilde{a}_2 = (3,7,8,9)$, $\tilde{a}_3 = (1,3,5,6)$. The following table gives the unit cost of shipping:

	M_1	M_2	M_3	M_4
W_1	2	2	2	1
W_2	10	8	5	4
W_3	7	6	6	8

The transportation table is given by

x_{11} 2	x_{12} 2	x_{13} 2	x_{14} 1	W_1
x_{21} 10	x_{22} 8	x_{23} 5	x_{24} 4	W_2
x_{31} 7	x_{32} 6	x_{33} 6	x_{34} 8	W_3
Demands 4	3	4	4	

with $\alpha = 0.36$ and applying definition of membership function, we get
$1.2 \leq a_1 \leq 6.6$, $3.8 \leq a_2 \leq 8.8$, $1.4 \leq a_3 \leq 8.8$

The nonfuzzy problem can be written as follows:

$$\text{Min } Z = 2x_{11} + 2x_{12} + 2x_{13} + x_{14} + 10x_{21} + 8x_{22} + 5x_{23} + 4x_{24} + 7x_{31} + 6x_{32} + 6x_{33} + 8x_{34}$$

Subject to

$$a_1 - x_{11} - x_{12} - x_{13} - x_{14} \geq 0,$$
$$a_2 - x_{21} - x_{22} - x_{23} - x_{24} \geq 0,$$
$$a_3 - x_{31} - x_{32} - x_{33} - x_{34} \geq 0,$$

$$x_{11} + x_{21} + x_{31} \geq 4,$$
$$x_{12} + x_{22} + x_{32} \geq 3,$$
$$x_{13} + x_{23} + x_{33} \geq 4,$$
$$x_{14} + x_{24} + x_{34} \geq 4,$$
$$1.2 \leq a_1 \leq 6.6, \qquad 3.8 \leq a_2 \leq 8.8, \qquad 1.4 \leq a_3 \leq 8.8,$$
$$x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34} \geq 0.$$

So we can get the following results:

$x^*_{11} = 3$, $x^*_{12} = 0$, $x^*_{13} = 0$, $x^*_{14} = 0$, $x^*_{21} = 0$, $x^*_{22} = 0$, $x^*_{23} = 3$, $x^*_{24} = 4$, $x^*_{31} = 1$, $x^*_{32} = 3$, $x^*_{33} = 1$, $x^*_{34} = 0$ with the α-optimal parameters $a^*_1 = 3$, $a^*_2 = 7$, $a^*_3 = 5$.

The parametric form of the above problem is stated as follows:

$$\text{Min } Z = 2x_{11} + 2x_{12} + 2x_{13} + x_{14} + 10x_{21} + 8x_{22} + 5x_{23} + 4x_{24} + 7x_{31} + 6x_{32} + 6x_{33} + 8x_{34}$$

Subject to

$$a_1 - x_{11} - x_{12} - x_{13} - x_{14} \geq 0,$$
$$a_2 - x_{21} - x_{22} - x_{23} - x_{24} \geq 0,$$
$$a_3 - x_{31} - x_{32} - x_{33} - x_{34} \geq 0,$$
$$x_{11} + x_{21} + x_{31} \geq 4,$$
$$x_{12} + x_{22} + x_{32} \geq 3,$$
$$x_{13} + x_{23} + x_{33} \geq 4,$$
$$x_{14} + x_{24} + x_{34} \geq 4,$$
$$H_1 - a_1 \geq 0,\ H_2 - a_2 \geq 0,\ H_3 - a_3 \geq 0,\ a_1 - h_1 \geq 0, a_2 - h_2 \geq 0, a_3 - h_3 \geq 0,$$
$$x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34} \geq 0.$$

The Kuhn-Tucker necessary optimality conditions corresponding to the above parametric problem will have the form:

$\rho_1 + \eta_1 - \lambda_1 = -2$, $\rho_2 + \eta_2 - \lambda_2 = -2$, $\rho_3 + \eta_3 - \lambda_3 = -2$, $\rho_4 + \eta_4 - \lambda_4 = -1$,
$\rho_5 + \eta_5 - \lambda_5 = -10$, $\rho_6 + \eta_6 - \lambda_6 = -8$, $\rho_7 + \eta_7 - \lambda_7 = -5$, $\rho_8 + \eta_8 - \lambda_8 = -4$,
$\rho_9 + \eta_9 - \lambda_9 = -7$, $\rho_{10} + \eta_{10} - \lambda_{10} = -6$, $\rho_{11} + \eta_{11} - \lambda_{11} = -6$, $\rho_{12} + \eta_{12} - \lambda_{12} = -8$,
$\lambda_1 + \theta_1 - \beta_1 = 0$, $\lambda_2 + \theta_2 - \beta_2 = 0$, $\lambda_3 + \theta_3 - \beta_3 = 0$,
$\lambda_1 (a_1 - x_{11} - x_{12} - x_{13} - x_{14}) = 0$, $\lambda_2 (a_2 - x_{21} - x_{22} - x_{23} - x_{24}) = 0$,
$\lambda_3 (a_3 - x_{31} - x_{32} - x_{33} - x_{34}) = 0$, $\eta_1 (x_{11} + x_{21} + x_{31} - 4) = 0$, $\eta_2 (x_{12} + x_{22} + x_{32} - 3) = 0$,
$\eta_3 (x_{13} + x_{23} + x_{33} - 4) = 0$, $\eta_4 (x_{14} + x_{24} + x_{34} - 4) = 0$,
$\beta_1 (H_1 - a_1) = 0$, $\beta_2 (H_2 - a_2) = 0$, $\beta_3 (H_3 - a_3) = 0$,
$\theta_1 (a_1 - h_1) = 0$, $\theta_2 (a_2 - h_2) = 0$, $\theta_3 (a_3 - h_3) = 0$,

$\rho_1 x_{11} = \rho_2 x_{12} = \rho_3 x_{13} = \rho_4 x_{14} = \rho_5 x_{21} = \rho_6 x_{22} = \rho_7 x_{23} = \rho_8 x_{24} = \rho_9 x_{31} = \rho_{10} x_{32} = \rho_{11} x_{33}$
$= \rho_{12} x_{34} = 0.$
$a_1 - x_{11} - x_{12} - x_{13} - x_{14} \geq 0, \quad a_2 - x_{21} - x_{22} - x_{23} - x_{24} \geq 0, \quad a_3 - x_{31} - x_{32} - x_{33} - x_{34} \geq 0,$
$x_{11} + x_{21} + x_{31} \geq 4, \quad x_{12} + x_{22} + x_{32} \geq 3, \quad x_{13} + x_{23} + x_{33} \geq 4, \quad x_{14} + x_{24} + x_{34} \geq 4,$
$H_1 - a_1 \geq 0, \; H_2 - a_2 \geq 0, \; H_3 - a_3 \geq 0, \quad a_1 - h_1 \geq 0, \quad a_2 - h_2 \geq 0, \quad a_3 - h_3 \geq 0,$
$x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34} \geq 0.$
$\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8, \rho_9, \rho_{10}, \rho_{11}, \rho_{12}, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9, \lambda_{10}, \lambda_{11},$
$\lambda_{12}, \eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6, \eta_7, \eta_8, \eta_9, \eta_{10}, \eta_{11}, \eta_{12}, \theta_1, \theta_2, \theta_3, \beta_1, \beta_2, \beta_3 \leq 0.$

Where all relations of the above system are evaluated at the solution $(x_{ij}^*, a_i^*) =$ (3,0,0,0,0,0,3,4,1,3,1,0,3,7,5). From this system, it can be shown that:
$\rho_1 = \rho_7 = \rho_8 = \rho_9 = \rho_{10} = \rho_{11} = \rho_{12} = \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \lambda_8 = \lambda_9 = \lambda_{10}$ $= \lambda_{11}, = \lambda_{12} = \eta_2 = \eta_3 = \eta_4 = \eta_5 = \eta_6 = \eta_{12} = 0, \rho_2 = \rho_3 = -2, \rho_4 = -1, \rho_5 = -10, \rho_6 = -8, \eta_1 = -2, \eta_7 = -5, \eta_8 = -4, \eta_9 = -7, \eta_{10} = -6, \eta_{11} = -6, \theta_1 = \beta_1, \theta_2, = \beta_2, \theta_3 = \beta_3.$
The stability se of the first kind of problem $P3$ is given by
$S(x_{ij}^*, a_i^*) = \{h_1, h_2, h_3, H_1, H_2, H_3 \in \mathrm{R}^6 \mid h_1 = H_1 = 3, h_2 = H_2 = 7, h_3 = H_3 = 5\}.$

The set of feasible parameters is expressed as follows:

$U = \{h_1, h_2, h_3, H_1, H_2, H_3 \in \mathrm{R}^6 \mid H_1 \geq h_1 - 0.2, H_2 \geq h_2 - 6.6, H_3 \geq h_3 - 7.4\}.$

7. *CONCLUSIONS*

In this paper, we have proposed a solution algorithm for solving the generalized transportation problem with fuzzy parameters in the constraints. Some basic stability notions for the problem of concern have been defined. These notions are the set of feasible parameters, the solvability set and the stability set of the first kind. A parametric study has been carried out for this problem. An illustrative example has been given to clarify the theory and the solution algorithm.

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